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Ground States of Quantum Hall Three-Body "Short-Range" Repulsion and Mean Field Approximation: Correlation Functions and Overlaps

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We analyze properties of trial wave functions in fractional quantum Hall effect, generated by the "short-range", three-body repulsion and its mean field approximation. Ground states of electron repulsion Hamiltonians at filing factor $\nu = 1/2$ are evaluated as a description of physically observed state $\nu = 5/2$. We analyze overlaps between the Moore–Read state and its mean field approximation for different number of particles and compare both states with ground state of the Coulomb interaction in the first excited Landau level. Our study also includes examination of electron–electron correlation functions and electron densities of excited states for both interactions.

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1. Introduction

Ongoing study on a topic of particle–hole symmetry [1–3] in fractional quantum Hall effect (FQHE) resulted in a recent paper [4] on the mean field approximation (MFA) of the three-body interaction Hamiltonians. This discussion is especially relevant in the context of debate regarding competition between the Moore-Read Pfaffian (Pf) [5] and the anti-Pfaffian (APf) [6, 7] states as the description of experimentally observed $\nu = 5/2$ FQHE state [8–13]. Although both Pf and APf states exhibit the same energy they are not topologically equivalent (edge states).

The Moore–Read Pfaffian state is the famous ground state of three-body "short range" repulsion, defined by three-body Haldane pseudopotentials where only the first one is nonzero ($V_3^{(3)} > 0, V_{m>3}^{(3)} = 0$):

$$H = \sum_{l=3} V_l^{(3)} P_{3Q-l}^{(3)},$$

where the 3-body Haldane pseudopotential $V_l^{(3)}$ is the energy of three electrons in the state with total angular momentum $3Q - l$ and $P_{3Q-l}^{(3)}$ are projection operators. Surprisingly, even though Pf and APf are described entirely in terms of three-body interaction, they seem to capture many features of ground states of two-body Coulomb interaction Hamiltonians in half-filled first excited Landau level (LL1). Remarkably, the Moore–Read state can also be characterized as a Jack polynomial which makes it fall into the category of the Jack states and allows for application of tools known from the symmetric functions theory [14–22]. This is especially useful when one generates coefficients of Pf wave function for large systems, as the Jack states can be computed with relatively fast recursion formula [17, 18, 23, 24].

In this paper we analyze properties of the Moore–Read state and ground state of "short-range" MFA Hamiltonian i.e. we apply MFA to the three-body Hamiltonian and generate simpler, two-body interaction. This new interaction hopefully gives ground state with similar properties as ground state of initial Hamiltonian (see Sect. 2). We base our work on the recent paper [4], where authors apply MFA to three-body Hamiltonians. As a result, the authors obtain corresponding values of two-body pseudopotentials. Then they tested usefulness of MFA with comparison of energy spectra. Original work of this paper is the further test of MFA introduced in [4]. We examine overlaps of Pf wave function and its MFA for different number of particles, also we compare both states with ground states of the Coulomb repulsion in half-filled LL1 (as both are considered as trial functions describing this state). Then we present electron densities and correlation functions. Even though paper [4] provided values of two-body MFA for different sphere sizes, we use the thermodynamical limit case (which coincides with values for the disc geometry) — two-body pseudopotentials with only two nonzero values of pseudopotential in a ratio 3:1 ($V_1 = 3, V_3 = 1, V_{m>3} = 0$).

2. Mean field approximation

A formalism proposed in [4] allows for reduction of three-body interaction into a two-body interaction, which contains certain physical characteristics of higher order interaction. In general, for a fully spin polarized state of electrons in spherical geometry, with $2Q$ magnetic flux quanta (flux quantum is $\phi_0 = hc/e$), three-body interaction can be written as

$$\mathcal{V}^{(3)} = \frac{1}{3!3!} \sum_{\{q_i;k_i\}} V^{(3)}_{q_1,q_2,q_3;k_1,k_2,k_3} c^\dagger_{q_3} c^\dagger_{q_2} c^\dagger_{q_1} c_{k_1} c_{k_2} c_{k_3}, \quad (1)$$

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where q_i, k_i are quantum numbers of particles and $V^{(3)}_{q_1,q_2,q_3;k_1,k_2,k_3}$ is the energy of triplet of particles at orbitals $q_1, q_2, q_3; k_1, k_2, k_3$. MFA method involves replacing one factor $c^\dagger_{q_1} c_{k_1}$ with $\langle c^\dagger_{q_1} c_{k_1} \rangle$ (ground state expectation value). For uniform, rotationally symmetric ground states, expectation value takes the form $\langle c^\dagger_{q_1} c_{k_1} \rangle = \nu\, \delta_{q_1,k_1}$. Then MF Hamiltonian equals (up to a constant)

$$\mathcal{V}^{(2)} = \sum_{q_1,q_2;k_1,k_2} V^{(2)}_{q_1,q_2;k_1;k_2} c^\dagger_{q_1} c^\dagger_{q_2} c_{k_1} c_{k_2}, \tag{2}$$

where matrix elements are given by the partial trace of the three-body interaction Hamiltonian

$$V^{(2)}_{q_1,q_2;k_1;k_2} = \nu \sum_{l=-Q}^{Q} \langle q_1, q_2, l | \, \mathcal{V}^{(3)} \, | k_1, k_2, l \rangle \, . \tag{3}$$

It had been checked numerically that this interaction give a rotationally symmetric Hamiltonian [4]. For rotationally symmetric Hamiltonians two-body pseudopotentials can be obtained via diagonalization of $\mathcal{V}^{(2)}$ for two particle system. Obviously, pseudopotentials vary for different forms of three-body interactions ($V^{(3)}_{q_1,q_2,q_3;k_1,k_2,k_3}$) and there is dependence on number of flux quanta as well. However "short-range" repulsion in the thermodynamical limit ($2Q \to \infty$) produces simple values of $V_1 = 3$ and $V_3 = 1$.

3. Results

We denote the ground states of MFA Hamiltonian of "short-range" three-body repulsion Hamiltonian by Ψ_{MF}. Pf wave function is denoted by Ψ_{MR} and the ground state of Coulomb interaction in half-filed LL1 by Ψ_{LL1}. Table I contains values of overlaps of mentioned wave functions for different values of system sizes.

TABLE I

Overlaps between ground states of examined interactions for different numbers of particles. Consecutive columns are: number of particles N, flux quanta $2Q$, dimensionality of Hilbert space, overlaps between Moore–Read state and Ψ_{MF}, overlaps between ground state of Coulomb interaction in LL1 and Ψ_{MF} and Moore–Read state, respectively. Overlaps calculated in spherical geometry.

N	$2Q = 2N-3$	dim	$\langle \Psi_{MR} \| \Psi_{MF} \rangle$	$\langle \Psi_{LL1} \| \Psi_{MF} \rangle$	$\langle \Psi_{LL1} \| \Psi_{MR} \rangle$
6	9	18	0.97655	0.45909	0.61182
8	13	151	0.98432	0.66963	0.77736
10	17	1514	0.96093	0.54359	0.71708
12	21	16660	0.89334	0.42309	0.67146
14	25	194668	0.90290	0.31985	0.48139

Overlaps of Ψ_{MF} and Ψ_{MR} are rather high for all of the examined system sizes, for the largest system this value is slightly above 0.9. This suggests that MFA gives indeed good approximation of Ψ_{MR}. Overlaps with Ψ_{LL1} are lower, for all of the system sizes MF state gives worst overlaps than the Pf state, however values do not fall below 0.3. We stress that values of overlaps decrease with the number of particles, however dimensionality of Hilbert spaces rapidly increases with number of particles.

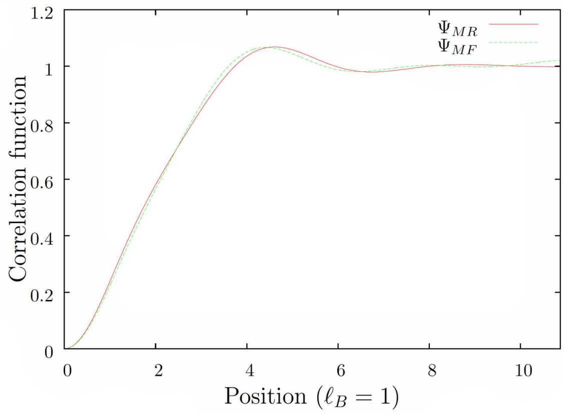

Fig. 1. Electron–electron correlation functions for the ground states of Pf state and its mean field approximation in units of magnetic length ℓ_B. System size $N = 14$, $2Q = 25$.

Correlation functions for ground states of "short range" three-body repulsion Hamiltonian and its MFA are presented in Fig. 1 (system size $N = 14$, $2Q = 25$). Presented data are measured in the units of magnetic length $\ell = \sqrt{\frac{\hbar e}{B}}$. One notices both correlation functions are very similar and vary only slightly. Moreover, local extrema and inflection points share similar positions. This, together with already discussed very high overlaps (see Table I) suggest that the Moore–Read state and its MFA exhibit very similar physical features.

Two consecutive figures (Figs. 2 and 3) represent electron density for ground states of three-body "short-range" repulsion and MFA for states of quasiholes and quasiparticles (system size $2Q = 24, N = 15$ and $2Q = 24$, N=14, respectively). Both pairs of wave functions are in $L^2 = 12$

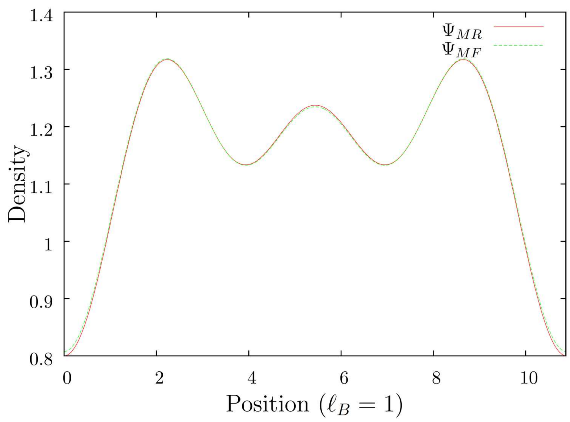

Fig. 2. Electron density of Pf and MFA on a sphere $2Q = 24$ for $N = 15$ particles. Units of magnetic length ℓ_B. Both ground states are eigenvectors of L^2 operators with $L^2 = 12$. Overlaps of states give very high value of 0.9907.

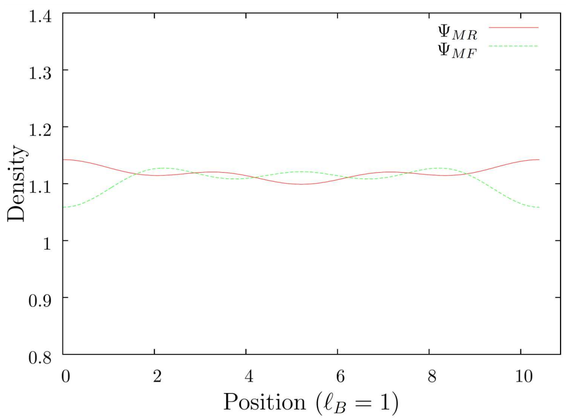

Fig. 3. Electron density of Pf and MFA on a sphere $2Q = 24$ for $N = 14$ particles (2 quasi-particles). Units of magnetic length ℓ_B. Both ground states are eigenvectors of L^2 operators with $L^2 = 12$. Overlaps of states give value of 0.9088.

angular momentum channel. Densities of the states at $2Q = 24, N = 15$ are very similar, moreover overlap is very high: 0.9907. In the case of ground states at $2Q = 24$ for $N = 14$ densities do not undergo the same shapes in Fig. 3. However, differences are not significant in absolute terms. Deviations from the average value of density are insignificant and never reach more than 10%. Overlaps between those states are lower than in previous case and reach 0.9088.

4. Conclusion

Our study of three-body 'short-range" repulsion and its MFA confirms similarities between ground states of those interactions at half-filled Landau levels. We find that overlaps are quite high (see Table I) and ground states correlation functions are similar. When one changed number of flux quanta and particles (by adding/subtracting particle/flux quanta), ground states of examined interactions revealed similar electron densities and high overlap for system size $N = 15$ and $2Q = 24$. For system system of $N = 14$ and $2Q = 24$ overlaps were lower and densities revealed different shapes, however differences were not big in absolute terms. We also compared both Ψ_{MR} and Ψ_{MF} with ground state of the Coulomb repulsion in half-filled LL1. Examined overlaps decreased with system size, but for all of the examined cases stayed above 0.3. From both functions Pf give better overlaps and is more likely to give proper description of this state.

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References

[1] A. Wójs, C. Töke, J.K. Jain, *Phys. Rev. Lett.* **105**, 196801 (2010).

[2] J.K. Jain, *Phys. Rev. Lett.* **63**, 199 (1989).

[3] J.K. Jain, *Composite Fermions*, Cambridge University Press, New York 2007.

[4] G.J. Sreejith, Yuhe Zhang, J.K. Jain, *Phys. Rev. B* **96**, 125149 (2017).

[5] G. Moore, N. Read, *Nucl. Phys. B* **360**, 362 (1991).

[6] M. Levin, B. Halperin, B. Rosenow, *Phys. Rev. Lett.* **99**, 236806 (2007).

[7] S. Lee, S. Ryu, C. Nayak, M. Fisher, *Phys. Rev. Lett.* **99**, 236807 (2007).

[8] W. Bishara, C. Nayak, *Phys. Rev. B* **80**, 121302 (2009).

[9] A. Wójs, C. Töke, J.K. Jain, *Phys. Rev. Lett.* **105**, 096802 (2010).

[10] E.H. Rezayi, S.H. Simon, *Phys. Rev. Lett.* **106**, 116801 (2011).

[11] M.R. Peterson, C. Nayak, *Phys. Rev. B* **87**, 245129 (2013).

[12] K. Pakrouski, M.R. Peterson, T. Jolicoeur, V.W. Scarola, C. Nayak, M. Troyer, *Phys. Rev. X* **5**, 021004 (2015).

[13] M.P. Zaletel, R.S.K. Mong, F. Pollmann, E.H. Rezayi, *Phys. Rev. B* **91**, 045115 (2015).

[14] B.A. Bernevig, F.D.M. Haldane, *Phys. Rev. B* **77**, 184502 (2008).

[15] B.A. Bernevig, F.D.M. Haldane, *Phys. Rev. Lett.* **100**, 246802 (2008).

[16] B.A. Bernevig, F.D.M. Haldane, *Phys. Rev. Lett.* **102**, 066802 (2009).

[17] A. Bernevig, N. Regnault, *Phys. Rev. Lett.* **103**, 206801 (2009).

[18] R. Thomale, B. Estienne, N. Regnault, A. Bernevig, *Phys. Rev. B* **84**, 045127 (2011).

[19] B. Kuśmierz, Y.-H. Wu, A. Wójs, *Acta Phys. Pol. A* **126**, 1134 (2014).

[20] B. Kuśmierz, Y.-H. Wu, A. Wójs, *Acta Phys. Pol. A* **129**, A73 (2016).

[21] B. Kuśmierz, A. Wójs, *Acta Phys. Pol. A* **130**, 1183 (2016).

[22] B. Kuśmierz, A. Wójs, *Acta Phys. Pol. A* **130**, 607 (2016).

[23] L. Lapointe, A. Lascoux, J. Morse, *Electr. J. Combin.* **7**, N1 (2000).

[24] F.K. Sogo, *J. Math. Phys.* **35**, 2282 (1994).