Two-Dimensional Electron-Hole Systems in a Strong Magnetic Field

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Two-dimensional systems containing N_e electrons and N_h holes ($N_e > N_h$) strongly correlated through Coulomb interactions in the presence of a large magnetic field are studied by exact numerical diagonalization. Low lying states are found to contain neutral (X^0) and negatively charged (X^-) excitons and higher charged exciton complexes (X_k^-, a bound state of k neutral excitons and an electron). Representing these states in terms of angular momenta and binding energies of the different exciton complexes, and the pseudopotentials describing their interactions with electrons and with one another, permits numerical studies of systems that are too large to investigate in terms of individual electrons and holes. Laughlin incompressible ground states of such a multi-component plasma are found. A generalized composite Fermion picture based on Laughlin type correlations is proposed. It is shown to correctly predict the lowest band of angular momentum multiplets for different charge configurations of the system for any value of the magnetic field.

I. INTRODUCTION

In two-dimensional electron-hole systems in the presence of a strong magnetic field, neutral excitons X^0 and spin-polarized charged excitonic ions X_k^- (X_k^- consists of k neutral excitons bound to an electron) can occur [1–4]. The complexes X_k^- should be distinguished from spin-unpolarized ones (e.g. spin-singlet biexciton or charged exciton) that are found at lower magnetic fields [5] but unbind at very high fields as predicted by hidden symmetry arguments [6]. The excitonic ions X_k^- are long lived Fermions whose energy spectra display Landau level structure [2–4]. In this work we investigate, by exact numerical diagonalization within the lowest Landau level, small systems containing N_e electrons and N_h holes ($N_e > N_h$) confined to the surface of a Haldane sphere [7,8]. For $N_h = 1$ these systems serve as simple guides to understanding photoluminescence [1–3,5,9–12]. For larger values of N_h it is possible to form a multi-component plasma containing electrons and X_k^- complexes [4]. We propose a model [13] for determining the incompressible quantum fluid states [14] of such plasmas, and confirm the validity of the model by numerical calculations. In addition, we introduce a new generalized composite Fermion (CF) picture [15] for the multi-component plasma and use it to predict the low lying bands of angular momentum multiplets for any value of the magnetic field.

The single particle states of an electron confined to a spherical surface of radius R containing at its center a magnetic monopole of strength $2S\phi_0$, where $\phi_0 = hc/e$ is the flux quantum and $2S$ is an integer, are denoted by $|S, l, m\rangle$ and are called monopole harmonics [7,8]. They are eigenstates of $\hat{l}^2$, the square of the angular momentum operator, with an eigenvalue $\hbar^2 l(l + 1)$, and of $\hat{l}_z$, the z component of the angular momentum, with an eigenvalue $\hbar m$. The energy eigenvalue is given by $(\hbar\omega_c/2S)[l(l+1) - S^2]$, where $\hbar\omega_c$ is the cyclotron energy. The $(2l + 1)$-fold
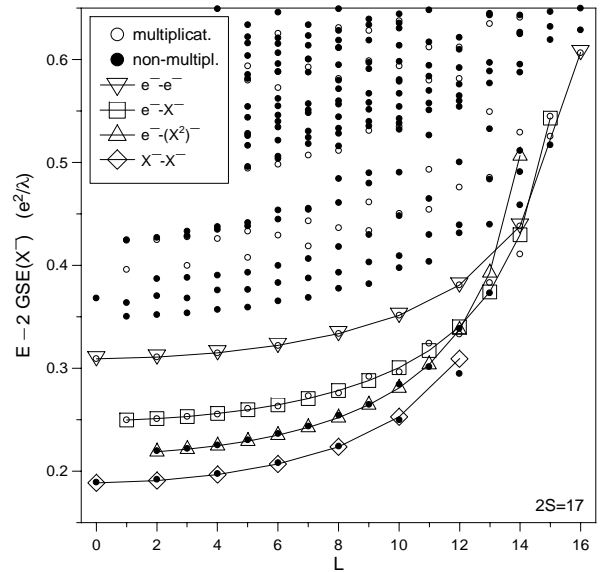

FIG. 1. Energy spectrum of four electrons and two holes at $2S = 17$. Open circles – multiplicative states; solid circles – non-multiplicative states; triangles, squares, and diamonds – approximate pseudopotentials.

degenerate Landau levels (or angular momentum shells) are labelled by $n = l - S = 0, 1, 2, \ldots$

II. FOUR ELECTRON–TWO HOLE SYSTEM

In Fig. 1 we display the energy spectrum obtained by numerical diagonalization of the Coulomb interaction of a system of four electrons and two holes at $2S = 17$. The states marked by open and solid circles are multiplicative [6] (containing one or more decoupled X^0's) and non-multiplicative states, respectively. For $L < 12$ there are four rather well defined low lying bands. Two of them begin at $L = 0$. The lower of these consists of two X^- ions interacting through a pseudopotential $V_{X^- - X^-}(L)$. The

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upper band consists of states containing two decoupled X^0's plus two electrons interacting through $V_{e^- e^-}(L)$. The band that begins at $L = 1$ consists of one X^0 plus an X^- and an electron interacting through $V_{e^- X^-}(L)$, while the band which starts at $L = 2$ consists of an X_2^- interacting with a free electron.

Knowing that the angular momentum of an electron is $l_{e^-} = S$, we can see that $l_{X_k^-} = S - k$, and that decoupled excitons do not carry angular momentum ($l_{X^0} = 0$). For a pair of identical Fermions of angular momentum l the allowed values of the pair angular momentum are $L = 2l - j$, where j is an odd integer. For a pair of distinguishable particles with angular momenta l_A and l_B, the total angular momentum satisfies $|l_A - l_B| \leq L \leq l_A + l_B$. The states containing two free electrons and two decoupled neutral excitons fit exactly the pseudopotential for a pair of electrons at $2S = 17$; the maximum pair angular momentum is $L^{\mathrm{MAX}} = 16$ as expected. The states containing two X^-'s terminate at $L = 12$. Since the X^-'s are Fermions, one would have expected a state at $L^{\mathrm{MAX}} = 2l_{X^-} - 1 = 14$. This state is missing in Fig. 1. By studying two X^- states for low values of S, we surmise that the state with $L = L^{\mathrm{MAX}}$ does not occur because of the finite size of the X^-. Large pair angular momentum corresponds to the small average separation, and two X^-'s in the state with L^{MAX} would be too close to one another for the bound X^-'s to remain stable. We can think of this as a "hard core" repulsion for $L = L^{\mathrm{MAX}}$. Effectively, the corresponding pseudopotential parameter, $V_{X^- X^-}(L^{\mathrm{MAX}})$ is infinite. In a similar way, $V_{e^- X^-}(L)$ is effectively infinite for $L = L^{\mathrm{MAX}} = 16$, and $V_{e^- X_2^-}(L)$ is infinite for $L = L^{\mathrm{MAX}} = 15$.

Once the maximum allowed angular momenta for all four pairings AB are established, all four bands in Fig. 1 can be roughly approximated by the pseudopotentials of a pair of electrons (point charges) with angular momenta l_A and l_B, shifted by the binding energies of appropriate composite particles. For example, the $X^- - X^-$ band is approximated by the $e^- - e^-$ pseudopotential for $l = l_{X^-} = S - 1$ plus twice the X^- energy. The agreement is demonstrated in Fig. 1, where the squares, diamonds, and two kinds of triangles approximate the four bands in the four-electron–two-hole spectrum. The fit of the diamonds to the actual $X^- - X^-$ spectrum is quite good for $L < 12$. The fit of the $e^- - X^-$ squares to the open circle multiplicative states is reasonably good for $L < 14$, and the $e^- - X_2^-$ triangles fit their solid circle non-multiplicative states rather well for $L < 13$. At sufficiently large separation (low L), the repulsion between ions is weaker than their binding and the bands for distinct charge configurations do not overlap.

There are two important differences between the pseudopotentials $V_{AB}(L)$ involving composite particles and those involving point particles. The main difference is the hard core discussed above. If we define the relative

TABLE I. Binding energies ε_0, ε_1, ε_2, and ε_3 of X^0, X^-, X_2^-, and X_3^-, respectively, in the units of e^2/λ.

$2S$	ε_0	ε_1	ε_2	ε_3
10	1.3295043	0.0728357	0.0411069	0.0252268
15	1.3045679	0.0677108	0.0395282	0.0262927
20	1.2919313	0.0647886	0.0381324	0.0260328

angular momentum $\mathcal{R} = l_A + l_B - L$ for a pair of particles with angular momentum l_A and l_B, then the minimum allowed relative angular momentum (which avoids the hard core) is found to be given by

$$\mathcal{R}_{AB}^{\mathrm{min}} = 2\min(k_A, k_B) + 1, \qquad (1)$$

where $A = X_{k_A}^-$ and $B = X_{k_B}^-$. The other difference involves polarization of the composite particle. A dipole moment is induced on the composite particle by the electric field of the charged particles with which it is interacting. By associating an "ionic polarizability" with the excitonic ion X_k^-, the polarization contribution to the pseudopotential can easily be estimated. When a number of charges interact with a given composite particle, the polarization effect is reduced from that caused by a single charge, because the total electric field at the position of the excitonic ion is the vector sum of contributions from all the other charges, and there is usually some cancellation. We will ignore this effect in the present work and simply use the pseudopotentials $V_{AB}(L)$ obtained from Fig. 1 to describe the effective interaction.

III. EIGHT ELECTRON–TWO HOLE SYSTEM

As an illustration, we first present the results of exact numerical diagonalization performed on the ten particle system ($8e^-$ and $2h^+$). We expect low lying bands of states containing the following combinations of complexes: (i) $4e^- + 2X^-$, (ii) $5e^- + X_2^-$, (iii) $5e^- + X^- + X^0$, and (iv) $6e^- + 2X^0$. The total binding energies of these configurations are: $\varepsilon_{\mathrm{i}} = 2\varepsilon_0 + 2\varepsilon_1$, $\varepsilon_{\mathrm{ii}} = 2\varepsilon_0 + \varepsilon_1 + \varepsilon_2$, $\varepsilon_{\mathrm{iii}} = 2\varepsilon_0 + \varepsilon_1$, and $\varepsilon_{\mathrm{iv}} = 2\varepsilon_0$. Here ε_0 is the binding energy of an X^0, ε_1 is the binding energy of an X^0 to an electron to form an X^-, and ε_k is the binding energy of an X^0 to an X_{k-1}^- to form an X_k^-. Some estimates of these binding energies (in magnetic units e^2/λ where λ is the magnetic length) as a function of $2S$ are given in Tab. I. Clearly, $\varepsilon_0 > \varepsilon_1 > \varepsilon_2 > \varepsilon_3$. The total energy depends upon not only the total binding energy, but the interactions between all the charged complexes in the system as well. All groupings (i)–(iv) contain an equal number of $N = N_e - N_h$ singly charged complexes. However, both angular momenta of involved complexes and the relevant hard cores are different. Which of the groupings has a state with the lowest total repulsion and
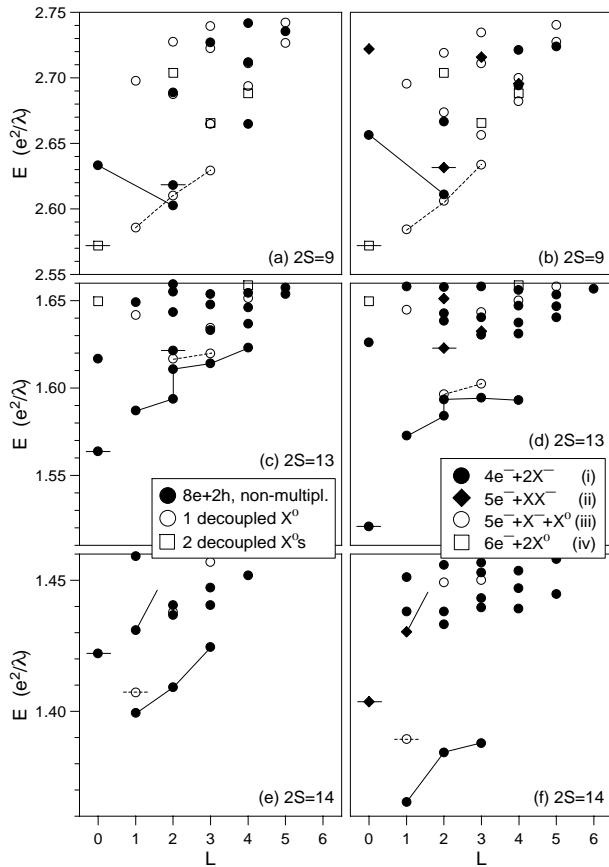
FIG. 2. Left: low energy spectra of the $8e + 2h$ system on a Haldane sphere at $2S = 9$ (a), $2S = 13$ (c), and $2S = 14$ (e). Right: approximate spectra calculated for all possible groupings containing excitons (charged composite particles interacting through pseudopotentials as in Fig. 1). Lines connect corresponding states in left and right frames.
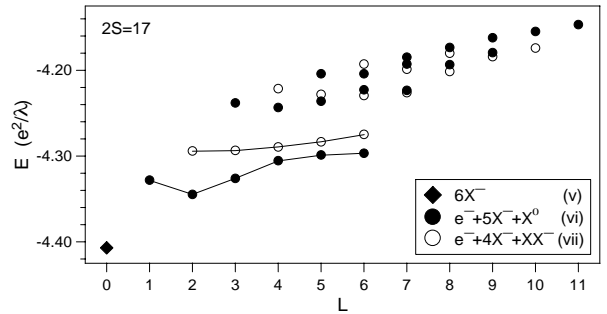

FIG. 3. Low energy spectra of different charge configurations of the $12e + 6h$ system on a Haldane sphere at $2S = 17$: $6X^-$ (diamonds), $e^- + 5X^- + X^0$ (filled circles), and $e^- + 4X^- + X_2^-$ (open circles).

binding energy, i.e. the absolute (possibly incompressible) ground state of the electron-hole system, depends on $2S$. It follows from the mapping between electron-hole and spin-unpolarized electron systems [6] that the multiplicative state $Ne^- + N_h X^0$ in which all holes are bound into decoupled X^0's is the absolute ground state (only) at the values of $2S$ corresponding to the filling factor $\nu = 1 - 1/m = 2/3, 4/5, \ldots$ of N (excess) electrons. At other values of $2S$, a non-multiplicative state containing an X^- is likely to have lower energy.

In Fig. 2, we show the low energy spectra of the $8e + 2h$ system at $2S = 9$ (a), $2S = 13$ (c), and $2S = 14$ (e). Filled circles mark the non-multiplicative states, and the open circles and squares mark the multiplicative states with one and two decoupled excitons, respectively. In frames (b), (d) and (f) we plot the low energy spectra of different charge complexes interacting through appropriate pseudopotentials (see Fig. 1), corresponding to four possible groupings (i)–(iv). As marked with lines, by comparing left and right frames, we can identify low lying states of type (i)–(iv) in the electron-hole spectra.

The fitting of energies in left and right frames at $2S = 13$ and 14 is noticeably worse than at $2S = 9$. It is also much worse than almost a perfect fit obtained for the three charge system ($6e + 3h$ vs. $3X^-$, $e^- + X^- + X_2^-$, etc.) [4]. This is almost certainly due to treating the polarization effect of the six charged particle system improperly by using the pseudopotential obtained from the two charged particle system (Fig. 1). A better fit is obtained by ignoring the polarization effect, and only including the hard core effect on the pseudopotentials of a pair of point charges with angular momentum l_A and l_B.

IV. LARGER SYSTEMS

It is unlikely that a system containing a large number of different species (e.g. e^-, X^-, X_2^-, etc.) will form the absolute ground state of the electron-hole system. However, different charge configurations can form low lying excited bands. An interesting example is the $12e + 6h$ system at $2S = 17$. The $6X^-$ grouping (v) has the maximum total binding energy $\varepsilon_{\mathrm{v}} = 6\varepsilon_0 + 6\varepsilon_1$. Other expected low lying bands correspond to the following groupings: (vi) $e^- + 5X^- + X^0$ with $\varepsilon_{\mathrm{vi}} = 6\varepsilon_0 + 5\varepsilon_1$ and (vii) $e^- + 4X^- + X_2^-$ with $\varepsilon_{\mathrm{vii}} = 6\varepsilon_0 + 5\varepsilon_1 + \varepsilon_2$.

Although we are unable to perform an exact diagonalization for the $12e + 6h$ system in terms individual electrons and holes, we can use appropriate pseudopotentials and binding energies of groupings (v)–(vii) to obtain the low lying states in the spectrum. The results are presented in Fig. 3. There is only one $6X^-$ state (the $L = 0$ Laughlin $\nu_{X^-} = 1/3$ state [4]) and two bands of states in each of groupings (vi) and (vii). A gap of $0.0626\,e^2/\lambda$ separates the $L = 0$ ground state from the lowest excited state.

In Fig. 4 we present the spectra of the $6X^-$ charge configurations for $2S = 21, 23, 25,$ and 27. The dashed lines are obtained by adding to the ground state energy the binding energy difference appropriate for the next low-
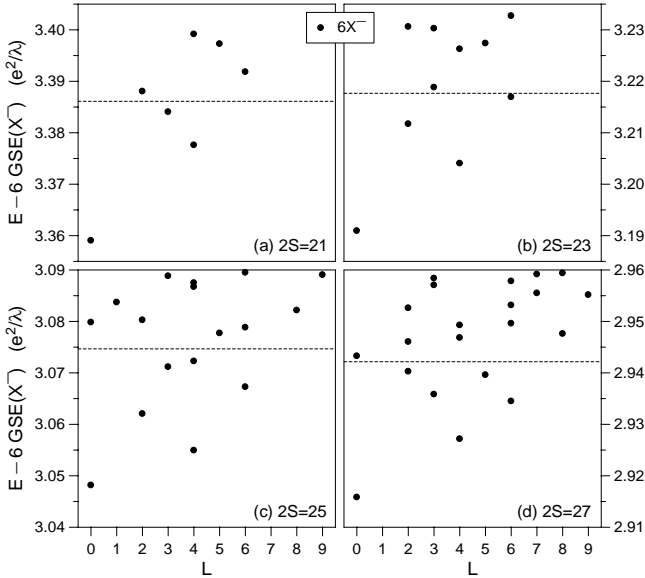
FIG. 4. Low energy spectra of the $6X^-$ charge configuration of the $12e + 6h$ system on a Haldane sphere at $2S = 21$, 23, 25, and 27. Dashed lines – estimated lower bounds of higher bands.

est charge configuration; no states other than the plotted six X^- states are expected below these lines. The $L = 0$ ground states observed at different $2S$ correspond to $\nu_{X^-} = 2/7$, $2/9$, $6/29$, and $1/5$. The $\nu_{X^-} = 1/5$ state is a Laughlin state and $\nu_{X^-} = 2/7$ and $2/9$ are states in Jain sequence. The $\nu_{X^-} = 6/29$ state is a CF hierarchy state [7] corresponding to two quasiparticles (QP's) of the $\nu_{X^-} = 1/5$ state forming a $\nu_{QP} = 1/5$ state at the next level of the CF hierarchy. Without knowing the nature of the QP-QP interaction vs. pair angular momentum L, there is no guarantee that the CF hierarchy picture (which assumes the validity of the mean field approximation) is valid. Fig. 4c seems to indicate that it is, since the $L = 0$ state has a lower energy than the other two states at $L = 0$ and 4, predicted for two QP's each with $l_{QP} = 5/2$. Our study of the pseudopotential of QP's in the Laughlin $\nu = 1/5$ state at $\nu_{QP} = 1/5$ very strongly suggests that it behaves like the Coulomb pseudopotential, so that the MFCF picture should work.

V. GENERALIZED LAUGHLIN WAVEFUNCTION

It is known that if the pseudopotential $V(\mathcal{R})$ decreases quickly with increasing $\mathcal{R}$, the low lying multiplets avoid (strongly repulsive) pair states with one or more of the smallest values of $\mathcal{R}$ [17]. For the (one-component) electron gas on a plane, avoiding pair states with $\mathcal{R} < m$ is achieved with the factor $\prod_{i<j}(x_i - x_j)^m$ in the Laughlin $\nu = 1/m$ wavefunction. For a system containing a number of distinguishable types of Fermions interacting

through Coulomb-like pseudopotentials, the appropriate generalization of the Laughlin wavefunction will contain a factor $\prod(x_i^{(a)} - x_j^{(b)})^{m_{ab}}$, where $x_i^{(a)}$ is the complex coordinate for the position of ith particle of type a, and the product is taken over all pairs. For each type of particle one power of $(x_i^{(a)} - x_j^{(a)})$ results from the anti-symmetrization required for indistinguishable Fermions and the other factors describe Jastrow type correlations between the interacting particles. Such a wavefunction guarantees that $\mathcal{R}_{ab} \geq m_{ab}$, for all pairings of various types of particles, thereby avoiding large pair repulsion [13,16]. Fermi statistics of particles of each type requires that all m_{aa} are odd, and the hard cores defined by Eq. (1) require that $m_{ab} \geq \mathcal{R}_{ab}^{\min}$ for all pairs.

VI. GENERALIZED COMPOSITE FERMION PICTURE

In order to understand the numerical results obtained in the spherical geometry (Figs. 2 and 3), it is useful to introduce a generalized CF picture by attaching to each particle fictitious flux tubes carrying an integral number of flux quanta ϕ_0. In the multi-component system, each a-particle carries flux $(m_{aa} - 1)\phi_0$ that couples only to charges on all other a-particles and fluxes $m_{ab}\phi_0$ that couple only to charges on all b-particles, where a and b are any of the types of Fermions. The effective monopole strength [10,15,17,18] seen by a CF of type a (CF-a) is

$$2S_a^* = 2S - \sum_b (m_{ab} - \delta_{ab})(N_b - \delta_{ab}) \qquad (2)$$

For different multi-component systems we expect generalized Laughlin incompressible states (for two components denoted as $[m_{aa}, m_{bb}, m_{ab}]$) when all the hard core pseudopotentials are avoided and CF's of each kind fill completely an integral number of their CF shells (e.g. $N_a = 2l_a^* + 1$ for the lowest shell). In other cases, the low lying multiplets are expected to contain different kinds of quasiparticles (QP-a, QP-b, . . .) or quasiholes (QH-a, QH-b, . . .) in neighboring filled shells.

Our multi-component CF picture can be applied to the system of excitonic ions, where the CF angular momenta are given by $l_{X_k^-}^* = |S_{X_k^-}^*| - k$. As an example, let us first analyze the low lying $8e + 2h$ states in Fig. 2. At $2S = 9$, for $m_{e^-e^-} = m_{X^-X^-} = 3$ and $m_{e^-X^-} = 1$ we predict the following low lying multiplets in each grouping: (i) $2S_{e^-}^* = 1$ and $2S_{X^-}^* = 3$ gives $l_{e^-}^* = l_{X^-}^* = 1/2$. Two CF-$X^-$'s fill their lowest shell ($L_{X^-} = 0$) and we have two QP-e^-'s in their first excited shell, each with angular momentum $l_{e^-}^* + 1 = 3/2$ ($L_{e^-} = 0$ and 2). Addition of L_{e^-} and L_{X^-} gives total angular momenta $L = 0$ and 2. We interpret these states as those of two QP-e's in the incompressible [331] state. Similarly, for other groupings

we obtain: (ii) $L = 2$; (iii) $L = 1$, 2, and 3; and (iv) $L = 0$ ($\nu = 2/3$ state of six electrons).

At $2S = 13$ and 14 we set $m_{e^- e^-} = m_{X^- X^-} = 3$ and $m_{e^- X^-} = 2$ and obtain the following predictions. First, at $2S = 13$: (i) The ground state is the incompressible [332] state at $L = 0$; the first excited band should therefore contain states with one QP-QH pair of either kind. For the e^- excitations, the QP-e^- and QH-e^- angular momenta are $l^*_{e^-} = 3/2$ and $l^*_{e^-} + 1 = 5/2$, respectively, and the allowed pair states have $L_{e^-} = 1$, 2, 3, and 4. However, the $L = 1$ state has to be discarded, as it is known to have high energy in the one-component (four electron) spectrum [18]. For the X^- excitations, we have $l^*_{X^-} = 1/2$ and pair states can have $L_{X^-} = 1$ or 2. The first excited band is therefore expected to contain multiplets at $L = 1$, 2^2, 3, and 4. The low lying multiplets for other groupings are expected at: (ii) $L = 2$ and 3; (iii) $2S^*_{X_2^-} = 3$ gives no bound X_2^- state; setting $m_{e^- X^-} = 1$ we obtain $L = 2$; and (iv) $L = 0$, 2, and 4. Finally, at $2S = 14$ we obtain: (i) $L = 1$, 2, and 3; (ii) incompressible [3*2] state at $L = 0$ ($m_{X^- X^-}$ is irrelevant for one X^-) and the first excited band at $L = 1$, 2, 3, 4, and 5; (iii) $L = 1$; and (iv) $L = 3$.

For the $12e + 6h$ spectrum in Fig. 3 the following CF predictions are obtained: (v) For $m_{X^- X^-} = 3$ we obtain the Laughlin $\nu = 1/3$ state with $L = 0$. Because of the hard core of $V_{X^- X^-}$, this is the only state of this grouping. (vi) We set $m_{X^- X^-} = 3$ and $m_{e^- X^-} = 1$, 2, and 3. For $m_{e^- X^-} = 1$ we obtain $L = 1$, 2, 3^2, 4^2, 5^3, 6^3, 7^3, 8^2, 9^2, 10, and 11. For $m_{e^- X^-} = 2$ we obtain $L = 1$, 2, 3, 4, 5, and 6. For $m_{e^- X^-} = 3$ we obtain $L = 1$. (vii) We set $m_{X^- X^-} = 3$, $m_{e^- X_2^-} = 1$, $m_{X^- X_2^-} = 3$, and $m_{e^- X^-} = 1$, 2, or 3. For $m_{e^- X^-} = 1$ we obtain $L = 2$, 3, 4^2, 5^2, 6^3, 7^2, 8^2, 9, and 10. For $m_{e^- X^-} = 2$ we obtain $L = 2$, 3, 4, 5, and 6. For $m_{e^- X^-} = 3$ we obtain $L = 2$. In groupings (vi) and (vii), the sets of multiplets obtained for higher values of $m_{e^- X^-}$ are subsets of the sets obtained for lower values, and we would expect them to form lower energy bands since they avoid additional small values of $\mathcal{R}_{e^- X^-}$. However, note that the (vi) and (vii) states predicted for $m_{e^- X^-} = 3$ (at $L = 1$ and 2, respectively) do not form separate bands in Fig. 3. This is because the $V_{e^- X^-}$ pseudopotential increases more slowly than linearly as a function of $L(L+1)$ in the vicinity of $\mathcal{R}_{e^- X^-} = 3$. In such case the CF picture fails [17].

The agreement of our CF predictions with the data in Figs. 2 and 3 (marked with lines) is really quite remarkable and strongly indicates that our multi-component CF picture is correct. We were indeed able to confirm predicted Jastrow type correlations in the low lying states by calculating their coefficients of fractional parentage [17,19]. We have also verified the CF predictions for other systems that we were able to treat numerically. If exponents m_{ab} are chosen correctly, the CF picture works well in all cases.

VII. SPECIAL CASE: MANY ELECTRON–ONE HOLE SYSTEMS

In an investigation of photoluminescence, the eigenstates of a system containing up to $N_e = 7$ electrons and a single hole have been studied as a function of d, the separation between the surfaces on which electrons and the hole are confined [10–12]. For d larger than a few magnetic lengths λ, the low energy spectra can be understood quite simply [10] in terms of the lowest band of multiplets of N_e electrons weakly coupled to the hole. There is clear evidence for bound states of the hole to one or more Laughlin [14] quasielectrons. For $d < \lambda$ there has been no convincing interpretation of the low lying states, although Apalkov et al. [12] suggested an explanation in terms of "dressed" X^0 excitons.

At $d = 0$ there are two types of states which contain excitons, viz. multiplicative states containing $N_e - 1$ electrons and one X^0, and non-multiplicative states containing $N_e - 2$ electrons and one X^-. The multiplicative states are particularly simple; their energies are simply the energies of $N_e - 1$ interacting electrons less the binding energy ε_0 of an X^0. The non-multiplicative states are an example of a two-component plasma and can be understood in our generalized CF picture.

For $N_e = 7$, the $6e^- + X^0$ and $5e^- + X^-$ states can be found in the $8e + 2h$ spectra shown in Fig. 2, where they correspond to the $6e^- + 2X^0$ and $5e^- + X^- + X^0$ multiplicative states marked with open symbols. We have shown that the predictions of our model work very well for this system. In particular, it is clear from Fig.2ab that while the $7e + 1h$ ground state at $2S = 9$ is the (multiplicative) incompressible $\nu = 2/3$ state of six electrons, the low lying states at $L = 1$, 2, and 3 all contain an X^- and thus their nature is very different.

Similarly, at $2S = 15$, the pseudopotential calculation for the $5e^- + X^-$ grouping (as in Fig.2bdf) as well as the CF prediction for $m_{e^- e^-} = 3$ and $m_{e^- X^-} = 2$ undoubtly preclude the interpretation of the low energy band at $L = 1$, 2, 3, and 4 (see figures in Refs. [10,12]) in terms of a "dressed" exciton in favor of the $5e^- + 1X^-$ configuration. In the CF picture of those states, one electron binds to the X^0 forming an X^- and leaving behind a quasihole (QH-e^-) in the Laughlin $\nu = 1/3$ state. The X^- (with $l^*_{X^-} = 3/2$) and the QH-e^- (with $l^*_{e^-} = 5/2$) have opposite charges and attract one another; what results in their excitonic dispersion. We have checked that the present interpretation remains valid at inter-layer separations d up the order of λ, when X^-'s unbind (detailed analysis of spatially separated systems will be presented elsewhere).

VIII. PHOTOLUMINESCENCE

A single X^- cannot emit a photon by $e - h$ recombination and leave behind a free electron. In the sim-

plest terms, this is because the luminescence operator conserves total angular momentum, and an X^- has $l_{X^-} = S - 1$, while the electron has $l_{e^-} = S$. For separated electron and hole planes, the hidden symmetry theorem does not hold, and it is possible to have weak luminescence from an X^- interacting with other charged particles. However, the luminescence intensity is much weaker than the fundamental luminescence line due to a neutral X^0. The existence of free X_k^- complexes appears to act as a trap that inhibits a strong luminescence intensity from X^0's. Observation of a strong X^- luminescence signal seems likely to be associated with excitons bound to an impurity and/or mixing of higher Landau levels. This might break the selection rule that forbids luminescence for a free X^-.

IX. SPECULATION

The generalized CF picture will be of value if it can make predictions for systems which at the moment are too large to evaluate numerically. An example that we have not been able to study numerically is that of $N_e = 14$ and $N_h = 5$. The configuration with the largest binding energy is (viii) $4e^- + 5X^-$, but the configuration (ix) $5e^- + 3X^- + X_2^-$ is only slightly smaller in binding energy. Which of these configurations has the lowest energy at a given value of $2S$ will depend on both the binding energy and the interparticle interactions. For $2S = 36$, we can choose $m_{e^- - e^-} = m_{X^- - X^-} = 5$ and $m_{e^- - X^-} = m_{e^- - X_2^-} = m_{X^- - X_2^-} = 4$. This choice satisfies all the requirements imposed by the Pauli principle and by the hard cores of the different pseudopotentials. From Eq. (2), we find $2S_{e^-}^* = 2S_{X^-}^* = 2S_{X_2^-}^* = 4$ so that $2l_{e^-}^* = 4$, $2l_{X^-}^* = 2$, and $2l_{X_2^-}^* = 0$. This leads to an $L = 0$ state of configuration (ix). If it is lower in energy than the lowest state of configuration (viii), it is very probably a Laughlin incompressible state. For configuration (viii), we find that there is a quasihole in the electron shell of angular momentum $l_{e^-}^* = 2$ and a pair of quasiparticles in the X^- shell of angular momentum $l_{X^-}^* = 2$. This gives $L_{e^-} = 2$, $L_{X^-} = 1, 3$, and thus $L = 1, 2^2, 3^2$, and 4. It seems likely that the quasiparticle energy in configuration (viii) more than compensates its slightly higher binding energy and that configuration (ix) is an incompressible quantum fluid state. It is unlikely that one will be able to diagonalize the nineteen particle electron-hole system at $2S = 36$, but the nine particle systems ($4e^- + 5X^-$ and $5e^- + 3X^- + X_2^-$) might be possible.

X. SUMMARY

Charged excitons and excitonic complexes play an important role in determining the low energy spectra of electron-hole systems in a strong magnetic field. We have introduced general Laughlin type correlations into the wavefunctions, and proposed a generalized CF picture to elucidate the angular momentum multiplets forming the lowest energy bands for different charge configurations occurring in the electron-hole system. We have found Laughlin incompressible fluid states of multi-component plasmas at particular values of the magnetic field, and the lowest bands of multiplets for various charge configurations at any value of the magnetic field. It is noteworthy that the fictitious Chern–Simons fluxes and charges of different types or colors are needed in the generalized CF model. This strongly suggests that the effective magnetic field seen by the CF's does not physically exist and that the CF picture should be regarded as a mathematical convenience rather than physical reality.

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