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Spin Transition in a Correlated Liquid of Composite Fermions

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We study spin polarization of the $\nu_{\mathrm{e}} = 4/11$ fractional quantum Hall state corresponding to the $\nu = 1/3$ filling of the second composite fermion Landau level, and predict a spin phase transition in realistic systems.

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1. Introduction

The quantum Hall effect occurs in a quasi-two-dimensional (2D) electron system cooled down to a very low temperature and placed in a strong perpendicular magnetic field B. It consists of precise quantization of the Hall resistance and the simultaneous vanishing of longitudinal resistance at a universal series of the filling factor ν_{e}, defined as the occupation fraction of the macroscopically degenerate Landau levels (LLs) or, equivalently, as dimensionless 2D electron concentration $\nu_{\mathrm{e}} = 2\pi\varrho\lambda^2$ (where ϱ is the areal density and $\lambda = \sqrt{\hbar c/eB}$ is the magnetic length).

Integral quantum Hall effect which occurs at $\nu_{\mathrm{e}} = 1, 2, \ldots$, is a consequence of a single-particle cyclotron gap in a many-electron spectrum whenever a number of LLs are completely filled. In contrast, *fractional* quantum Hall (FQH) effect [1] which occurs at fractional LL fillings is a many-body phenomenon [2]. The FQH states in the lowest LL occur at $\nu_{\mathrm{e}} = n/(2np \pm 1)$, where n and p are a pair of integers; with the most prominent Laughlin state at $\nu_{\mathrm{e}} = 1/3$. FQH effect occurs also in higher LLs ($\nu_{\mathrm{e}} > 1$), where a partially filled excited LL is nearly decoupled from a number of completely filled, inert lowest levels. The key condition necessary for the FQH effect is the formation of an incompressible quantum liquid by the interacting electrons which partially fill a degenerate LL. Hence, understanding of the FQH effect involves explanation of the emergence of a nondegenerate ground state with an excitation gap, solely due to the Coulomb interaction and exclusively at the special values of ν_{e}, independent of the material or geometry of the sample. This makes FQH effect a nice example of an emergent, non-perturbative, many-body phenomenon in condensed matter physics.

In the lowest LL, the experimentally observed FQH states are elegantly explained by Jain's composite fermion (CF) model [3]. The CF transformation involving attachment of an even number $2p$ of magnetic flux quanta $\phi_0 = hc/e$ to each electron converts a partially filled LL of electrons with strong (Coulomb) interaction into a system of nearly noninteracting CFs in a weaker residual magnetic field $B^* = B - 2p\phi_0\varrho$. The sequence $\nu_{\mathrm{e}} = n/(2np \pm 1)$ results from the condition of a complete filling of a number n of effective LLs by the CFs (the connection of electron and CF filling factors being $\nu_{\mathrm{CF}}^{-1} = \nu_{\mathrm{e}}^{-1} - 2p$). In filled CF shells, the weak residual CF–CF interactions play no role and the incompressibility of $\nu_{\mathrm{e}} = n/(2np \pm 1)$ states is explained as a consequence of the single-particle cyclotron gap of the CFs in reduced field B^*.

Recently, Pan et al. [4] observed the FQH effect at previously unexpected filling factors between two neighboring Jain states, i.e. at $1/3 < \nu_{\mathrm{e}} < 2/5$ ($p = 1$ and $n = 1$ or 2, respectively) and thus corresponding to the *fractional* CF filling factors, $1 < \nu_{\mathrm{CF}} < 2$. In particular, the incompressible states observed at $\nu_{\mathrm{e}} = 4/11$, 3/8, 5/13 correspond to $\nu_{\mathrm{CF}} = 4/3$, 3/2, and 5/3, respectively. Incompressibility of these states must depend on the CF–CF interaction within a partially filled second CF LL. Hence, they have been called "second generation" states, in contrast to the "first generation" (Laughlin and Jain) states whose incompressibility can be explained within a simplified picture of essentially noninteracting CFs.

The FQH experiments reveal incompressibility at a given ν, but not its microscopic origin. Even such basic property as magnetization of the "second generation" states is not probed directly, but only through the FQH measurements in tilted magnetic fields (allowing relatively independent control of the Zeeman spin splitting, which affects polarized and unpolarized states in a different way). Therefore, experimental suggestion [4] of full spin polarization of the $\nu_e = 4/11$ state (and only partial polarization of the other new states) is not completely convincing [5]. In this paper we: (i) study two competing spin states at $\nu_e = 4/11$, (ii) show that, although having qualitatively different many-body correlations (and nature of elementary excitations), they both can lead to FQH effect, and (iii) predict a transition between them in realistic experimental conditions.

2. Energies and interactions of composite fermions

The familiar values of second-CF-LL fillings, $\nu \equiv \nu_{CF} - 1 = 1/3$, $1/2$, and $2/3$ suggested analogy to electron states at the same fillings of either the lowest or the second electron LL. However, this analogy relies on the (not obvious and not generally true) similarity between the short-range CF–CF and e–e pseudopotentials (interaction energy V as a function of pair angular momentum $\mathcal{R}$).

Let us denote consecutive CF LLs by CF-LL$_n$ ($n = 0, 1, \dots$) and also indicate spin of a CF by an arrow $\sigma = \uparrow$ or $\downarrow$. To interpret the $\nu_e = 4/11$ state within the CF model (i.e., assuming a complete filled, inert CF-LL$_0$ $\uparrow$) we must hence resolve whether the partially filled ($\nu = 1/3$) second CF shell is CF-LL$_1$ $\uparrow$ or CF-LL$_0$ $\downarrow$. This clearly depends on the effective cyclotron and Zeeman energies of the CFs, and on the correlation energies in the two competing CF-LLs.

A single CF in CF-LL$_1$ $\uparrow$ is equivalent to a Laughlin quasielectron (QE) — a fractionally charged quasiparticle of a Laughlin liquid [2], and, similarly, a single CF in CF-LL$_0$ $\downarrow$ is equivalent to a Rezayi reversed-spin quasielectron (QE$_\mathrm{R}$) [6]. We therefore adopt the (simplified) notation and restate the problem as follows: Is the experimentally observed $\nu_e = 4/11$ state a $\nu = 1/3$ state of QEs or QE$_\mathrm{R}$s? This depends on the comparison of QE and QE$_\mathrm{R}$ creation and correlation energies.

The numerical (exact-diagonalization) calculations were carried out in the Haldane geometry [7]. In this model, N particles (electrons or CFs) are confined to a sphere, with the normal magnetic field B yielding the desired LL degeneracy $g = 2Q + 1$ produced by a Dirac monopole of strength $2Q\phi_0$ in the center.

The QE/QE$_\mathrm{R}$ creation energies ε were obtained from the comparison of ($N \leq 12$)-electron ground states at exactly $\nu = 1/3$ (i.e., at $2Q = 3N - 3$) and at $\nu = 1/3$ minus one flux quantum (i.e., at $2Q = 3N - 4$), and with total spin projection $S_z = N/2$ or $N/2 - 1$ for the QE and QE$_\mathrm{R}$, respectively [8, 9]. Figure 1a shows the result as a function of the layer width w. Similarly, the short-range behavior of QE and QE$_\mathrm{R}$ interaction pseudopotentials $V(\mathcal{R})$ were obtained from the lowest bands in the N-electron spectra at $\nu = 1/3$ minus two flux quanta (i.e., at $2Q = 3N - 5$), and with $S_z = N/2$ or $N/2 - 2$ [8, 9]. The long-range behavior of $V(\mathcal{R})$ results from the known $-e/3$ electric charge of QEs and QE$_\mathrm{R}$s. Matching the two limits, we obtain the result shown in Fig. 1b for $w = 0$.

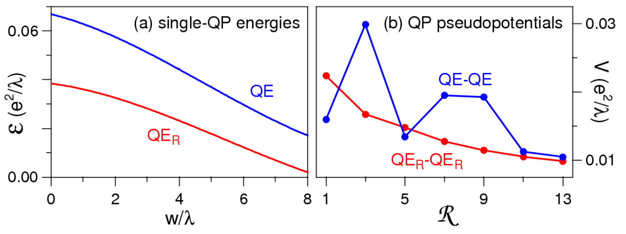

Fig. 1. (a) Dependence of the QE and QE$_\mathrm{R}$ energies ε on electron layer width w. (b) QEs and QE$_\mathrm{R}$s interaction pseudopotentials $V(\mathcal{R})$ in an ideal 2D layer ($w = 0$).

3. Incompressible states of composite fermions

Knowing the relevant pseudopotentials, the $\nu = 1/3$ incompressible ground states of QEs and QE$_\mathrm{R}$s can be identified in exact-diagonalization calculation. Since $V_{\mathrm{QER}}(\mathcal{R})$ is similar to the electron pseudopotential in the lowest LL, the QE$_\mathrm{R}$ form a familiar Laughlin state at $2l = 3N - 3$ (here, N is the number of QE$_\mathrm{R}$s and l is the angular momentum of their LL shell on a sphere). However, $V_{\mathrm{QE}}(\mathcal{R})$ is quite different having stronger repulsion at $\mathcal{R} = 3$ than at $\mathcal{R} = 1$. In consequence, the incompressible N-QE ground states are different and (on a sphere) occur at different values of $2l$.

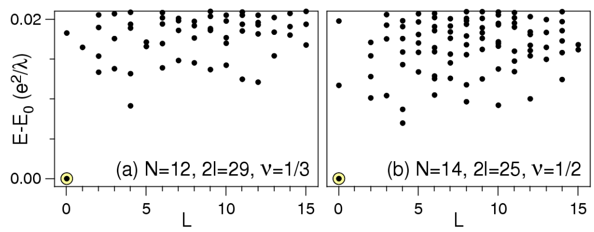

Fig. 2. Sample N-QE excitation spectra (energy E vs. angular momentum L; E_0 is ground state energy) corresponding to fractional fillings $\nu = 1/3$ and $1/2$ of CF-LL$_1$.

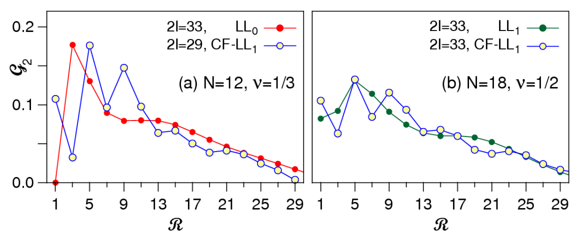

Fig. 3. Discrete pair-correlation functions (Haldane pair amplitude $\mathcal{G}_2$ vs. relative pair angular momentum $\mathcal{R}$) at fractional fillings $\nu = 1/3$ and $1/2$ of CF-LL$_1$, compared to the same fillings of LL$_0$ (equivalent to CF-LL$_0$) or LL$_1$.

From the calculations for different N and $2l$ (two sample spectra are shown in Fig. 2) we indeed have iden-

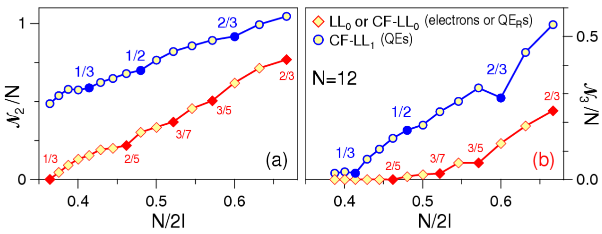

Fig. 4. Number of pairs $\mathcal{N}_2$ (a) and triplets $\mathcal{N}_3$ (b) with the minimum relative angular momentum, calculated for N electrons in LL_0 (equivalent to QE_R's in CF-LL_0) or QEs in CF-LL_1, as a function of $N/2l \approx \nu$. Incompressible states are labeled by ν.

tified the $\nu = 1/3$ sequence of gapped ground states at $2l = 3N - 7$ (for $\nu = 1/2$ the series is $2l = 2N - 3$) [10]. As seen from the Haldane amplitudes $\mathcal{G}_2(\mathcal{R})$ shown in Fig. 3, these $\nu = 1/3$ and $1/2$ ground states of QEs have quite different short-range correlations from the known "first generation" electron states at the same ν in the relevant LLs. Moreover, Fig. 4 showing the number of pairs ($\mathcal{N}_2$) and triplets ($\mathcal{N}_3$) with maximum relative angular momentum (the quantity proportional to the Haldane two- and three-body amplitudes $\mathcal{G}$), makes it evident that there are no such triplets at $\nu = 1/3$ of QEs ($\mathcal{N}_3 = 0$), i.e., that this is a paired state of the QEs [11]. This is in contrast to a known property of a Laughlin $\nu = 1/3$ state (e.g., of the QE_Rs), in which $\mathcal{N}_2 = 0$.

4. Spin phase diagram of composite fermions

Knowing the $\nu = 1/3$ ground states of both QEs and QE_Rs, let us now compare their correlation energies (per particle), $u = (E + U_\text{bckg})/N$. Here E is the N-quasiparticle interaction energy and U_bckg is a correction due to the charge-compensating background. In

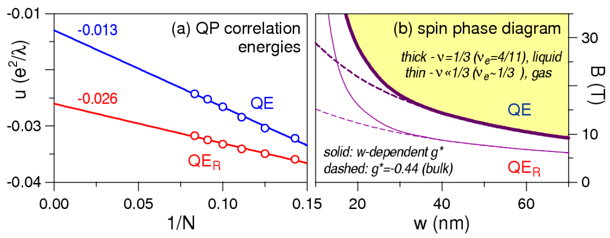

Fig. 5. (a) Correlation energy u in the $\nu = 1/3$ state of QEs or QE_Rs as a function of their inverse number N^{-1}, in an ideal 2D layer ($w = 0$). (b) Phase diagram (critical layer width w vs. magnetic field B) for the QE–QE_R spin transition at $\nu_\text{e} = 4/11$, assuming the effective (width dependent) electron Landé g^*-factor for GaAs. For comparison, the thick dashed line is for a constant (bulk) value $g^* = -0.44$. Two thin lines additionally ignore the correlation energy u (adequately for $\nu \ll 1$).

Fig. 5a we plot $u(N)$ obtained for $w = 0$ from diagonalization for $N \le 12$. The results of extrapolation are $u_\text{QER} = -0.026\, e^2/\lambda$ (very close to the value for an electron Laughlin state when the charge difference $e \to e/3$ is taken into account) and $u_\text{QE} = -0.013\, e^2/\lambda$. The difference $\Delta u = u_\text{QE} - u_\text{QER}$ was recalculated in a similar way for various widths w.

Whether QEs or QE_R's will form a $\nu = 1/3$ state at $\nu_\text{e} = 4/11$ depends on the Coulomb and Zeeman energies. The condition for the QE $\leftrightarrow$ QE_R transition is $\Delta\varepsilon + \Delta u = E_Z$. Figure 5b shows the spin phase diagram calculated assuming width dependence of the Landé factor g^* appropriate for GaAs wells. Clearly, the spin transition in narrower wells shifts to higher B ($\propto \varrho_\text{e}$). Also, comparison of boundaries dividing correlated QE/QE_R liquids and non-interacting QE/QE_R gases demonstrates the role of interactions in stabilizing the QE_R phase.

Acknowledgments

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