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Skyrmions in integral and fractional quantum Hall systems

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Abstract

Numerical results are presented for the spin excitations of a two-dimensional electron gas confined to a quantum well of width w. Spin waves and charged skyrmion excitations are studied for filling factors $\nu = 1, 3$, and 1/3. Phase diagrams for the occurrence of skyrmions of different size as a function of w and the Zeeman energy are calculated. For $\nu = 3$, skyrmions occur only if w is larger than about twice the magnetic length. A general necessary condition on the interaction pseudopotential for the occurrence of stable skyrmion states is proposed. © 2002 Published by Elsevier Science Ltd.

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1. Introduction

In a numerical study of reversed spin excitations of the spin polarized fractional quantum Hall (FQH) state near filling factor $\nu = 1/3$ [1], Rezayi [2] found that for charged excitations the lowest lying band of states had total angular momentum L equal to the total spin S when the Zeeman energy E_Z was taken to be zero. In this case, the minimum energy occurred at $L = 0$ and corresponded to a 'spin texture' containing $K = N/2$ spin flips for an N electron system. Sondhi et al. [3] and others [4–10] investigated the $\nu = 1$ state and found that for E_Z less than a critical value $\tilde{E}_Z$, the lowest charged excitation was a skyrmion [11] containing a number K of reversed spins that increased as E_Z decreased from $\tilde{E}_Z$ to zero. Skyrmions have been observed both in magnetization and transport studies for the $\nu = 1$ state [12–15] and, when E_Z was sufficiently decreased by application of hydrostatic pressure, for the $\nu = 1/3$ state [16]. The form of the Coulomb pseudopotential in higher Landau levels (LLs) suggested that skyrmions would not occur at $\nu = 3, 5, \ldots$ [17,18]. However, when allowance was made for the softening of the pseudopotential associated with finite well width w, skyrmions at $\nu = 3$ were predicted [19] and observed [20] in sufficiently wide quantum wells.

In this paper, we demonstrate the similarities between electron and composite Fermion (CF) [21] spin excitations in the integral and FQH systems. We present phase diagrams (in the w–E_Z plane) for the number of spin flips K in the lowest energy charged excitation of the $\nu = 1, 3$, and 1/3 fillings. In addition, we propose necessary conditions on the pseudopotentials (applicable both to integral and fractional filling) for low energy skyrmions at $E_Z = 0$.

2. Model

We perform numerical calculations for a system of N electrons confined to a spherical surface [22] of radius R. The radial magnetic field is produced by a Dirac monopole at the center, whose strength $2Q$ is given in units of the quantum of flux, $\phi_0 = hc/e$, so that $4\pi R^2 B = 2Q\phi_0$. The single particle states $|Q, l, m\rangle$, called monopole harmonics [23–26], are eigenfunctions of the orbital angular momentum l and its z-component m. They form degenerate LLs labeled by $n = l - Q$. The cyclotron energy $\hbar\omega_c \propto B$ is assumed to be much larger than the Coulomb energy scale $E_C = (e^2/\lambda) \propto \sqrt{B}$ (where $\lambda^2 = \hbar c/eB$ is the square of the magnetic length). However, the ratio $\eta = E_Z/E_C$ is taken as an arbitrary parameter. Finite well width enters the problem only through modifying the quasi-two-dimensional

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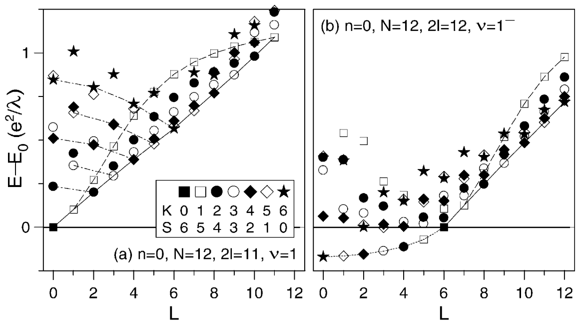

Fig. 1. The energy spectra of 12 electrons in the lowest LL calculated on Haldane sphere with $2l = 11$ (a) and 12 (b).

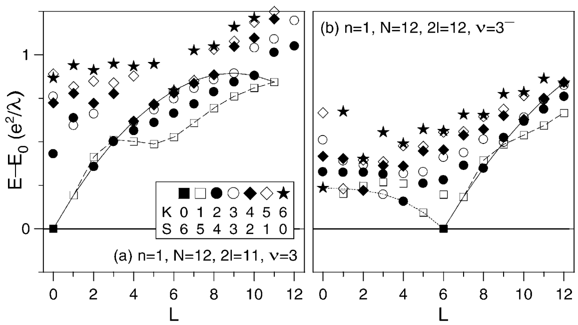

Fig. 2. Same as Fig. 1 but for the first excited LL.

interaction by replacing e^2/r (where r is the in-plane separation) by $V_\xi(r) = e^2 \int \mathrm{d}z\,\mathrm{d}z'\ \xi^2(z)\xi^2(z')/\sqrt{r^2 + (z - z')^2}$. Here $\xi(z)$ is the envelope function for the lowest sub-band of the quantum well. This change modifies the Coulomb pseudopotential [24–26] $V^{(n)}(\mathcal{R})$, defined as the interaction energy of a pair of electrons in the nth LL as a function of their relative angular momentum $\mathcal{R}$. There are four conserved quantum numbers: L, the total orbital angular momentum, S, the total spin, and their projections L_z and S_z. The eigenvalues depend only on L and S, and they are therefore $(2L + 1)(2S + 1)$-fold degenerate.

3. Integral filling

In Fig. 1(a) and (b) we present the low energy spectra of the $\nu = 1$ and 1^- (a single hole in $\nu = 1$) states, respectively. In this and all other spectra, only the lowest state at each L and S is shown, E_0 is the energy of the lowest maximally polarized state ($K = 0$), and the Zeeman energy E_Z is omitted.

The ferromagnetic ground state of Fig. 1(a) at $L = 0$ and $S = N/2 = 6$ results from the Coulomb interaction even when $E_Z = 0$. States with different values of S are indicated by the different symbols shown in the inset. The lowest excited state is a spin wave (SW) [27] consisting of a hole in the spin-↓ level and an electron in the spin-↑ level with $L = K = 1$. A dashed line marks the entire single SW band at $1 \le L \le 11$ (resulting from $\vec{L} = \vec{l}_e + \vec{l}_h$ with $l_e = l_h = l = 11/2$). The lowest energy excitation for a given value of either L or K occurs at $L = K$ where $K = (1/2)N - S$ is the number of spin flips away from the fully polarized ground state. The (near) linearity of $E(K)$ for this band of states (denoted by W_K) suggests that it consists of K SWs, each with $L = 1$, which are (nearly) non-interacting. As shown with the dot-dash lines connecting different states of the same number K of $L = 1$ SWs, only the $L = K$ state (in which the SWs have parallel angular momenta) is non-interacting, and all others (at $L < K$) are repulsive.

We have compared the linear W_K energy bands calculated for different electron numbers $N \le 14$, and found that they all have the same slope $u \approx 1.15e^2/\lambda$ when plotted as a function of the 'relative' spin polarization $\zeta = K/N$. The fact that $E - E_0 = u\zeta$ for the W_K band for every value of N has two noteworthy consequences in the $N \to \infty$ limit. (i) For any value of $E_Z \neq 0$, the interaction energy of each W_K state, $E - E_0 \propto K/N$, is negligible compared to its total Zeeman energy, KE_Z. (ii) The gap for spin excitations at $\nu = 1$ equals E_Z; if this gap can be closed (e.g. by applying hydrostatic pressure), the $\nu = 1$ ferromagnet becomes gapless and the density of states for the W_K excitations becomes continuous.

Because of the exact particle–hole symmetry in the lowest LL, the $\nu = 1^-$ state whose spectrum appears in Fig. 1(b) can be viewed as containing either one hole or one reversed spin electron in a $\nu = 1$ ground state. The band of states with $0 \le L \le 5$ and $S = L$ (dotted line) is the skyrmion band denoted by S_K. Its energy increases monotonically with S and L. For $6 \le L \le 12$, the single SW band (dashed line) and band of K SWs each with $L = 1$ (solid line) resemble similar bands in Fig. 1(a), except that their angular momenta are added to that of the hole which has $l_h = l = 6$.

Fig. 1 completely ignores the Zeeman energy. The total Zeeman energy measured from the fully polarized state is proportional to K. The total energy of the skyrmion band is $E(K) = E_S(K) + KE_Z$ and the lowest S_K state occurs when $E(K)$ has its minimum. If we very roughly approximate the skyrmion energy in a finite system by $E_S(K) \approx E_S(N/2) + \beta S^2$, where $\beta \ge 0$ is a constant, this minimum occurs at $K = 1/2(N - EZ/\beta)$ spin flips. This vanishes when $E_Z = \beta N$, defining the critical value, $\tilde{E}_Z$, and it reaches its maximum value $K = N/2$ (or complete depolarization) when $E_Z = 0$. At such E_Z the ground state at $\nu = 1^\pm$ is a finite size skyrmion, its gap for spin excitations ('internal' spin excitations introduced by Fertig et al. [10]) is much smaller than (and largely independent of) E_Z. This is in contrast to the exact $\nu = 1$ filling and allows spin coupling of the electron system to the magnetic ions, nuclei, or charged excitons.

In Fig. 2 we show the numerical results analogous to those in Fig. 1 but for the $n = 1$ LL. Features clearly apparent in the lowest LL are now absent. For example, the W_K band in Fig. 2(a) departs noticeably from linearity, and it

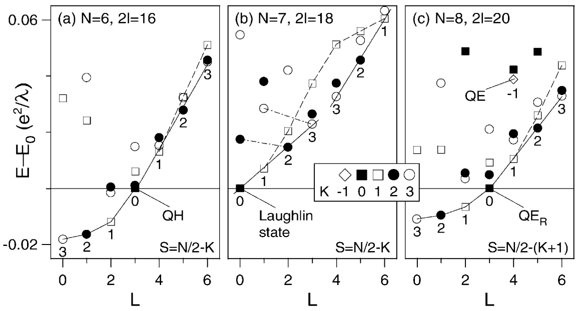

Fig. 3. The energy spectra of $N = 6$–8 electrons calculated on Haldane sphere at the values of $2l$ corresponding to $\nu = (1/3)^-$ (a), $\nu = 1/3$ (b), and $\nu = (1/3)^+$ (c).

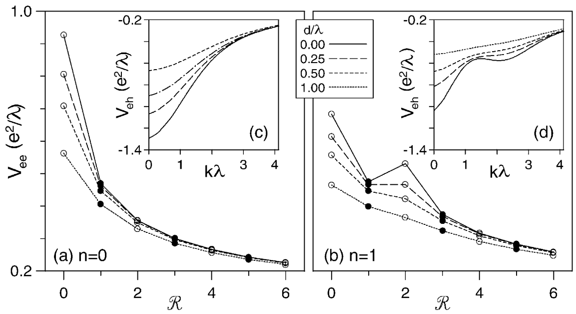

Fig. 4. The e–e interaction pseudopotentials in the lowest (a) and first excited (b) LL, calculated for the potential $V_d(r)$. Only data for $\mathscr{R} \le 6$ is shown, and open and closed circles distinguish between singlet and triplet states. Insets: corresponding e–h pseudopotentials; k is the e–h wave vector.

does not generally lie below the single SW band (dashed line). More striking is the fact that the S_K band of Fig. 2(b) goes above the single hole state at $L = 6$, in contrast to the behavior in Fig. 1(b). Therefore, skyrmions are not the lowest energy charged excitations in excited LLs even when $E_Z = 0$. This effect was first predicted by Jain and Wu [17,18].

The only difference between the filling factors $\nu = 3, 5, \ldots$ and 1 is that the monopole harmonics $|Q, l = Q + n, m\rangle$ correspond to the excited LL instead of the lowest. Matrix elements of the Coulomb interaction e^2/r between these higher monopole harmonics give a different pseudopotential $V^{(n)}(\mathscr{R})$ from that for $n = 0$. Though one might expect skyrmions to be the lowest energy charged excitations in this case, the change in the pseudopotential from $V^{(0)}(\mathscr{R})$ to $V^{(n)}(\mathscr{R})$ with $n \ge 1$ causes the charged spin flip excitations to have higher energy than the single hole or reversed spin electron.

4. Fractional filling

Since the CF picture [21] describes the FQH effect in terms of integral filling of effective CF levels, it is interesting to ask [28,29] if spin excitations analogous to the SWs and skyrmions occur at Laughlin fractional fillings $\nu = (2p + 1)^{-1}$ (where $p = 1, 2, \ldots$). In Fig. 3 we display numerical results for $\nu \approx 1/3$.

The values of N and $2l$ in frames (b), (a), and (c) correspond to a Laughlin $\nu \approx 1/3$ condensed state, Laughlin quasihole (QH), and Laughlin quasielectron (QE) or reversed spin quasielectron (QE$_R$), respectively. For each of these cases, the lowest CF LL has a degeneracy of seven. Clearly the single SW dispersion (dashed line) and the linear W_K band (solid line) both appear in Fig. 3(b). The S_K bands beginning at $L = 0$ lie below the single QH state (a) and below the single QE$_R$ state (c). The solid and dashed lines at $3 \le L \le 6$ in Fig. 3(a) and (c) are completely analogous to those in Fig. 1(b), and correspond to the single SW band and the W_K band, except that their angular momenta are added to $l_{QH} = 3$ or $l_{QE_R} = 3$. What is clearly different from the $\nu = 1$ case is the smaller energy scale, and a noticeable difference between the $\nu = (1/3)^-$ (QH) and $\nu = (1/3)^+$ (QE$_R$) spectra. Since the QH–QH and QE$_R$–QE$_R$ interactions are known to be different [30], this lack of QH–QE$_R$ symmetry is not unexpected. It implies a lack of symmetry between the CF skyrmion (QE$_R$ + K SW) and CF antiskyrmion (QH + K SW) states in contrast to the skyrmion–antiskyrmion symmetry of $\nu = 1$. Because the CF skyrmion energy scale is so much smaller than E_C at $\nu = 1$, the critical E_Z at which skyrmions are stable is correspondingly smaller [16].

5. Effect of finite well width

As suggested by Cooper [19] and confirmed experimentally by Song et al. [20], skyrmions become the lowest energy charged excitations in higher LLs if the quantum well is sufficiently wide. The finite well width w can be accounted for by using effective potential $V_\xi(r)$ and selecting a sub-band wave function $\xi(z)$ appropriate to the depth and width of the quantum well. In Fig. 4 we show the e–e and e–h pseudopotentials for the lowest (ac) and excited (bd) LL as a function of a parameter d which is proportional to w. In the calculation we have used a potential $V_d(r) = e^2/\sqrt{r^2 + d^2}$ as done earlier by He et al. [31]. Comparing the resulting pseudopotentials with those obtained using an envelope function $\xi_0(z) \propto \cos(\pi z/w)$ appropriate to the lowest sub-band of an infinitely deep quantum well gives $w \approx 5d$ (w is slightly larger than the actual width of a finite depth well). It is clear that finite width must have the largest effect on those pseudopotential coefficients corresponding to the smallest average e–e or e–h separation. As a result, increasing w causes suppression of the maxima of $V_{ee}(R)$ and of the minima of $V_{eh}(k)$ characteristic of the excited LLs. This makes the $n = 1$ pseudopotentials (and, in consequence, the many-body spectra of Fig. 2) more similar to those of the lowest LL.

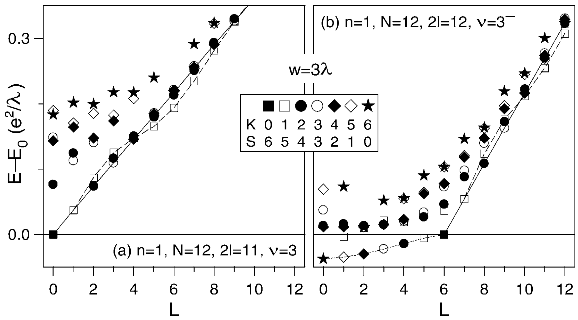

Fig. 5. Same as Fig. 2 but for finite well width $w = 3\lambda$.

In Fig. 5 we show the same energy spectra as given in Fig. 2, but for the Coulomb pseudopotential in the $n = 1$ LL replaced by one appropriate for $w = 3\lambda$. The W_K band in Fig. 5(a) is now much closer to linear with K, and the S_K band in Fig. 5(b) now has $E < E_0$. We have done similar calculations for the $n = 2$ LL with similar results. These results show that skyrmions are the lowest excitations in higher LLs if w is sufficiently large.

In Fig. 6 we sketch the phase diagrams (in the w–E_Z plane) for charged excitations at the integral filling of the lowest and excited LLs ($\nu = 1$ and 3), and at the fractional filling $\nu = 1/3$. For $n = 0$ ($\nu = 1$ or 1/3), skyrmions are the lowest charged excitations below a critical value of E_Z which is relatively insensitive to the width w. As E_Z is decreased, larger skyrmions (with increasing K) become the lowest energy states. For $n = 1$, no skyrmions occur unless $w \leq 2\lambda$ (we have checked that this value remains correct for small skyrmions in the $N \to \infty$ limit).

6. Pseudopotentials and skyrmion stability

Only the leading pseudopotential coefficients $V(0)$, $V(1)$, $V(2)$, corresponding to small average in-plane e–e separations, are strongly influenced by finite w. In fact, the change in the energy $E_S(K) - E_0$ of the skyrmion band from positive to negative occurred in the $n = 1$ LL only when the pseudopotential coefficient $V(2)$ was quite strongly affected by the increase in w. For this reason, we investigate $E_S(K)$ for a simple model pseudopotential with the following properties: (i) $V(0)$ is sufficiently large to cause decoupling of the many-body states that avoid all $\mathcal{R} = 0$ pairs (i.e. the skyrmion states) from all other states that contain some $\mathcal{R} = 0$ pairs; (ii) behavior of $V(\mathcal{R})$ between $\mathcal{R} = 1$ and 3 can be varied similarly to how $V(1)$ varies with increasing w. We choose the simplest possible model pseudopotential with these properties by defining $U_x(\mathcal{R})$ as follows: $U_x(0) = \infty$, $U_x(1) = 1$, $U_x(2) = x$, and $U_x(\mathcal{R}) = 0$ for $\mathcal{R} > 2$. This choice of U_x guarantees that skyrmions are its only finite energy eigenstates, and their energy depends on one free parameter x. A simple relation between the fractional grandparentage coefficients [24–26] $\mathcal{G}_K(\mathcal{R})$ at $\mathcal{R} = 0, 1$, and 2

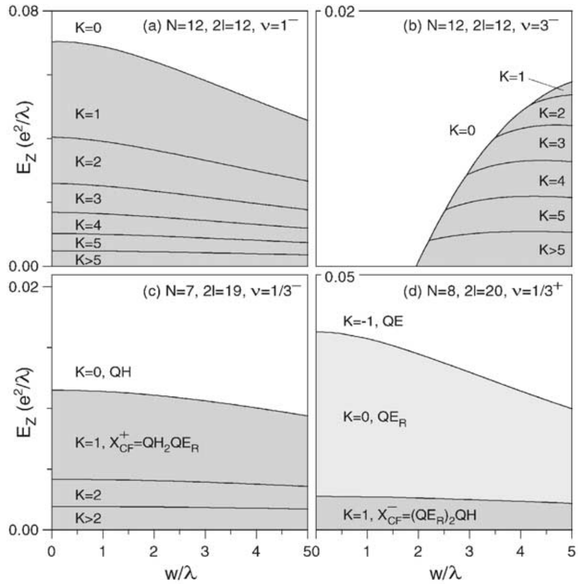

Fig. 6. Phase diagrams for the occurrence of skyrmions with different K as a function of well width w and Zeeman energy E_Z, calculated at $\nu = 1^-$ (a), 3^- (b), $(1/3)^-$ (c), and $(1/3)^+$ (d). Numbers in top left corners give the upper bounds of the vertical axes (the lower bound are zero in all frames).

yields $E_S(K) - E_0 = \mathcal{G}_K(2)(x - \alpha)$, where $\mathcal{G}_K(2)$ is a positive constant and $\alpha^{-1} = 2 - (N-1)^{-1}$. Since for every value of K, $E_S(K) - E_0$ changes sign at $x = \alpha$, and $\alpha \to 1/2$ in large systems, we conclude that skyrmions are the lowest charged excitations when $U_x(2)$ drops below half of $U_x(1)$, and U_x becomes superlinear between $\mathcal{R} = 1$ and 3. Therefore, we suggest that for both integral and FQH states the stability of skyrmion states requires an effective pseudopotential for which: (i) $V(0)$ is larger enough to cause Laughlin correlations; (ii) $V(2)$ is less than or equal to half of $V(1) + V(3)$.

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