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Second generation of incompressible quantum liquids in the fractional quantum Hall effect

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Abstract. We study "second generation" of fractional quantum Hall states which contain a partially filled composite fermion (CF) Landau level (LL). We explain the role of CF–CF interaction in the incompressibility of the underlying quantum electron liquid. In particular, using exact diagonalization on a Haldane sphere, we determine two- and three-body CF correlation functions for these liquids and show that the CFs form a paired state (instead of a Laughlin liquid) when filling $\nu = 1/3$ of their second LL. At $\nu = 1/2$ we show that the CFs group into even larger clusters rather than form a Moore–Read paired state. We also address the problem of spin polarization of interacting CFs and predict transition to the partially unpolarized ground state in sufficiently narrow quantum wells and/or weak magnetic fields.

1. Introduction

About two decades after the discovery of fractional quantum Hall (FQH) effect by Tsui, Störmer and Gossard [1] and Laughlin's idea of the incompressible electron liquid [2], and over a decade after introduction of Jain's composite fermion (CF) model [3], Pan *et al.* [4] discovered FQH effect in a spin polarized two-dimensional (2D) electron gas at a new, unexpected series of Landau level (LL) filling factors ν_e. These new fractions lied outside of the Jain sequence [3] of states at $\nu_e = n/(2pn \pm 1)$, which are defined by the complete filling of n lowest shells by the CFs each carrying an even number of magnetic flux quanta. In contrast, the most prominent of the new FQH states occur at $\nu_e = 4/11$ and $3/8$, corresponding to *fractional* fillings $\nu = 1/3$ and $1/2$ of the second CF LL. Evidently, partial filling of a degenerate CF shell in these electron liquids implies that their incompressibility must involve interactions and correlations among the CFs. Therefore, in contrast to the Laughlin and Jain states whose understanding within the CF model invoked only the emergence of a quasi-cyclotron gap in the single-CF spectrum, the new liquids have been called the "second generation" FQH states [5, 6, 7, 8].

Familiar values of $\nu = 1/3$ and $1/2$ suggested similarity between partially filled electron and CF LLs in the "first" and "second generation" FQH states [9]. In the first case, for $\nu_e = 4/11$, it revived Haldane's "quasiparticle hierarchy" [10]. Its CF formulation originally proposed by Sitko *et al.* [11] consists of the CF $\rightarrow$ electron mapping followed by reapplication of the CF picture in the second CF LL, leading to a "second generation" of CFs [5, 6, 7, 8]. However, this idea ignored the known requirement of a strong short-range repulsion between the particles to which a CF can be succesfully applied [12, 13, 14]. Indeed, this idea was later precluded by exact diagonalization studies [15], in which a different series of finite-size $\nu_e = 4/11$ liquids was

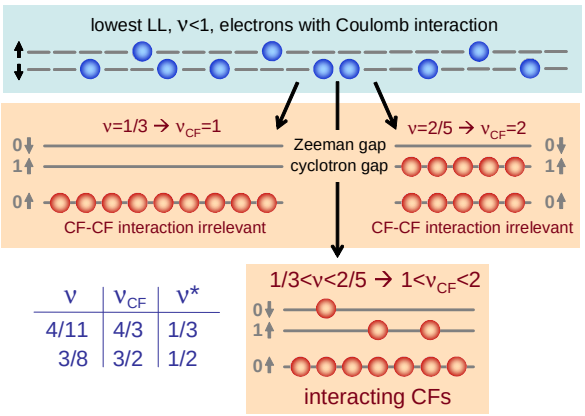

Figure 1. Schematic of the CF transformation applied to a partially filled lowest electron LL. At the electron filling factors $\nu = 1/3$ and $2/5$, the transformation yields completely filled one and two CF-LLs ($\nu_{\rm CF} = 1$ and 2), respectively, and their interaction is irrelevant for incompressibility. At $1/3 < \nu < 2/5$, the CFs partially fill CF-LL$_1$ ($1 < \nu_{\rm CF} < 2$), and the low-energy dynamics of the system is determined entirely by the CF–CF interaction.

identified with gaps which appear to persist in the thermodynamic limit. On the other hand, for $\nu_e = 3/8$, the Moore–Read liquid state [16, 17] of paired CFs in a half-filled shell was proposed [18]. However, it was eventually ruled out in favor of the anisotropic (stripe) order [19, 20].

This paper contains a review of the main ideas published in a series of our earlier papers dealing with the problem of CF–CF interaction [14, 15, 21, 22, 23]. It is organized as follows. In Sec. 2 we recall the CF picture of the $\nu_e = 4/11$ and $3/8$ states. In Sec. 3 we explain how the CF wave functions and CF–CF Haldane interaction pseudopotentials [24] are extracted from the N_e-electron exact-diagonalization calculations [14], and justify the use of such effective CF–CF pseudopotentials for the description of correlated many-electron states at $\nu = 1/3$ or $1/2$. In Sec. 4 we discuss the role of anharmonic contributions to the short-range pseudopotential in determining short-range correlations. In Sec. 5 we move to the discussion of numerical calculations for N interacting CFs, in which we identified the series of finite-size nondegenerate ground states with excitation gaps which extrapolate to $\nu = 1/3$ or $1/2$ in the limit of large N [15]. In Sec. 6 we analyze the wave functions of these N-CF ground states and, in particular, calculate their two- and three-body correlation functions, from which we find that the $\nu_e = 4/11$ state is a paired state of CFs [21, 22]. Although the precise form of the correlation between the CF pairs is not known, it is demonstrated that they are certainly not of a Laughlin form defined by the maximum avoidance of the relative two-pair angular momentum [25]. Finally, in Sec. 7 we consider spin-flip excitations of the "second generation" states, discuss the possibility of partially unpolarized ground states, and construct the spin phase diagram at $\nu_e = 4/11$ (from which we predict a spin transition under realistic realistic conditions) [23].

2. Composite fermion picture of the "second-generation" incompressible liquids

In the CF model, electrons filling a fraction ν_e of the lowest LL capture part of the external magnetic field B in form of quantized flux tubes of strength $2p\phi_0$ (here, $\phi_0 = hc/e$ is the flux quantum and p is an integer). By binding magnetic flux tubes, electrons are converted into CFs, which experience reduced magnetic field B^*. The (real) electron and (effective) CF filling factors are related to one another through $\nu_{\rm CF}^{-1} = \nu_e^{-1} - 2p$. For $1/3 < \nu_e < 2/5$ the choice of $2p = 2$ yields $1 < \nu_{\rm CF} < 2$ and a fractional filling $\nu \equiv \nu_{\rm CF} - 1 < 1$ of the second CF LL.

In particular, let us look at the CF picture of the $\nu_e = 4/11$ FQH state of electrons, illustrated in Fig. 1. Assuming complete spin polarization, the CFs fill their entire lowest LL (CF-LL$_0$) and a fraction $\nu_{\rm QE} \equiv \nu = 1/3$ of their second LL (CF-LL$_1$). Similarly, the electron filling factor $\nu_e = 3/8$ translates into the $\nu = 1/2$ filling of CF-LL$_1$. The CFs in the partially filled CF-LL$_1$ represent quasielectrons (QEs) of the underlying incompressible $\nu_e = 1/3$ Laughlin liquid [2] (the liquid itself represented by the completely filled CF-LL$_0$). This CF↔QE equivalence is exact at $\nu \ll 1$ but, remarkably, it appears valid at higher values of ν as well.

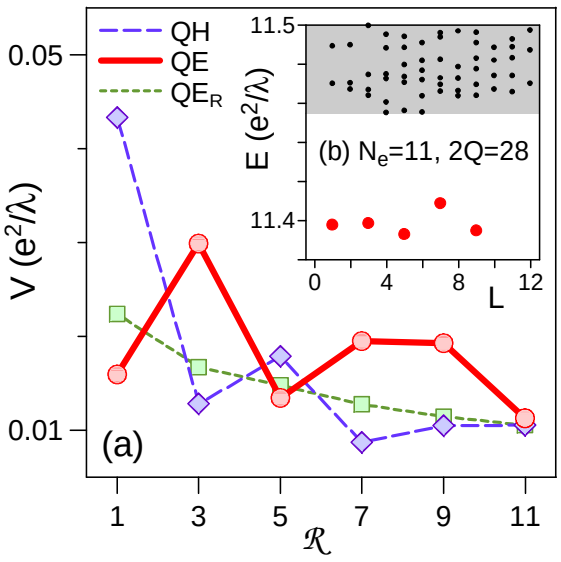

Figure 2. (a) Haldane interaction pseudopotentials of different CFs (QE, QH, and QE_R). (b) Example of 11-electron spectrum used to obtain V_{QE}. λ is the magnetic length.

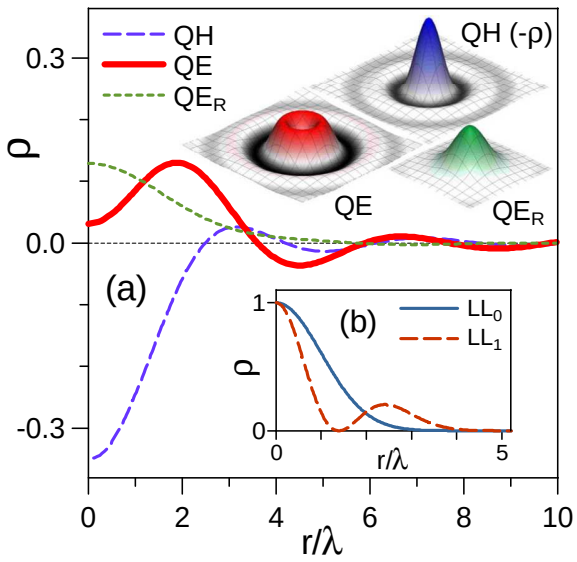

Figure 3. a (a) Radial charge distributions of different CFs (QE, QH, and QE_R) obtained from the 10-electron exact diagonalization. (b) Same for electrons in two lowest LLs.

3. Haldane pseudopotential for interaction of Laughlin quasielectrons (QEs)

In order to study the QE–QE correlations at $\nu_e = 4/11$ or 3/8 (i.e., correlations in the partially filled CF-LL_1), we must first determine the form of QE–QE interaction. Within an isolated LL, interaction hamiltonian can be conveniently defined by its Haldane pseudopotential $V(\mathcal{R})$ [24], i.e., by the dependence of the pair interaction energy V on the relative angular momentum $\mathcal{R}$ (for identical fermions, $\mathcal{R}$ must be an odd integer). The pseudopotential can be extracted from exact diagonalization of N_e electrons (with the Coulomb interaction) on a Haldane sphere [10], with the angular momentum of the LL shell chosen equal to $l = 3(N_e - 1)/2 - 1$ (on a sphere, $2l + 1$ is the LL degeneracy), such as that for $N_e = 11$ in Fig. 2(b). In such spectra, the lowest energy band contains the states of two QEs, and the dependence of the N_e-electron energy E on the total angular momentum L is (up to a constant) the QE–QE pseudopotential.

The QE–QE pseudopotential obtained in this way is plotted in Fig. 2(a), together with the pseudopotentials calculated analogously for the other Laughlin quasiparticles: quasiholes (QHs) and reversed-spin quasielectrons (QE_R). The easily noticed and essential feature of the QE–QE interaction is the relatively weak repulsion at the smallest allowed relative angular momentum, $\mathcal{R} = 1$. This is strikingly different from the pseudopotentials in other CF-LLs (e.g., from V_{QH} or V_{QER}) or from the known electron pseudopotentials in LL_0 or LL_1 (not shown).

The reason for weak short-range QE–QE repulsion is the ring-like charge distribution profile $\varrho(r)$ of the QEs, very different from other CFs or from electrons in other LLs. The curves of $\varrho(r)$ shown in Fig. 3(a) were calculated from the wave functions of the appropriate N_e-electron ground states on a sphere (e.g., at $l = 3N_e/2 - 2$ for the QE).

Before moving on to analysis of many-QE systems, let us pause for a moment to repeat and stress what follows: (i) The weak short-range QE–QE repulsion is not an assumption. It is evident from exact diagonalization [14, 22] but also from an independent Monte Carlo calculation [19], both of which involve the Coulomb interaction between quasi-2D electrons. Therefore, it does not depend on any assumptions on the nature of QEs themselves, and it ought to be considered a fact implied by "numerical experiments" carried out on a well-defined model. (ii)

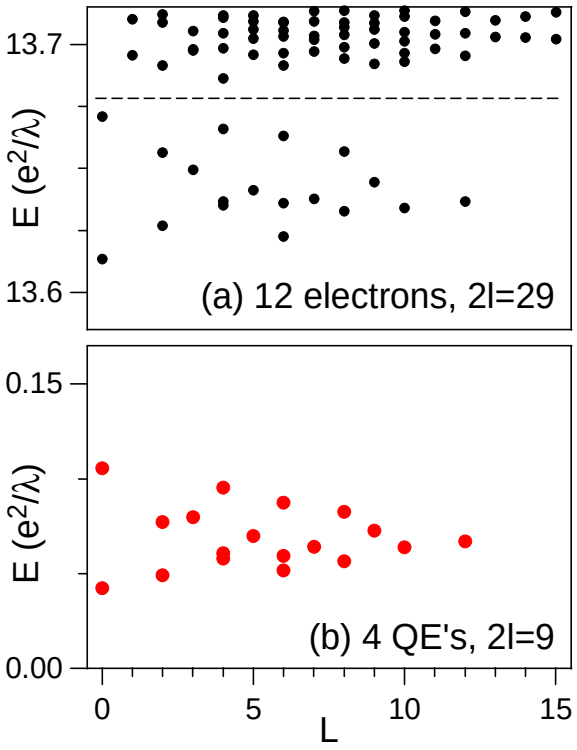

Figure 4. Energy spectra (energy E versus angular momentum L) for 12 electrons in LL_0 with $2l = 29$ (a) and for 4 QEs interacting through V_{QE} in CF-LL_1 with $2l = 9$ (b).

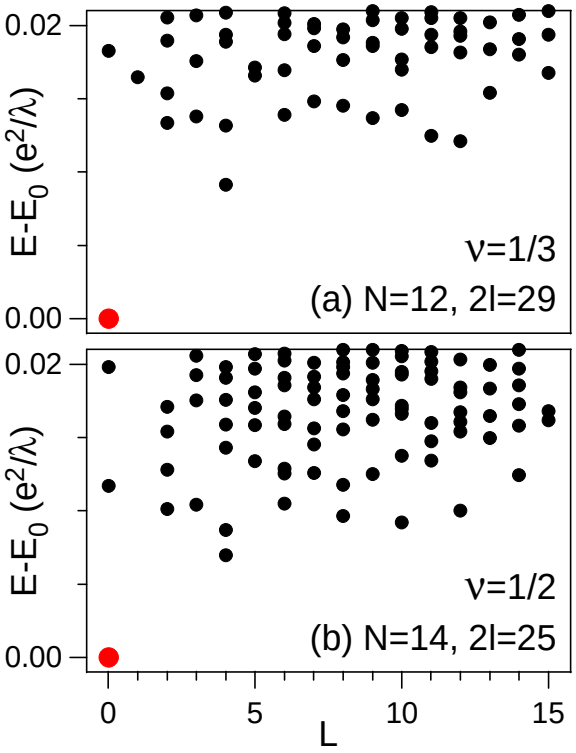

Figure 5. Sample N-QE excitation spectra (energy E versus angular momentum L; E_0 is ground state energy) corresponding to fractional fillings $\nu = 1/3$ and $1/2$ of CF-LL_1.

This peculiar short-range behavior of $V_{QE}(\mathcal{R})$ invalidates analogy between electron and QE systems at the same ν. In particular, it precludes Laughlin correlations among QEs at $\nu = 1/3$ or Moore–Read pairing of CFs at $\nu = 1/2$.

4. Role of harmonic pseudopotential in determining pair correlations

Let us now move to the argument that $V_{QE}(\mathcal{R})$ cannot support Laughlin correlations among the QEs. This results from the property of the "harmonic" pseudopotential $V_H(\mathcal{R})$, defined as proportional to the average squared distance $\langle r^2 \rangle$. Its general form on a sphere is $V_H(\mathcal{R}) = \alpha + \beta \cdot L(L+1)$, with constant α and β, and $\mathcal{R} = 2l - L$. For large $2l$ (or on a plane) this is equivalent to $V_H \propto \mathcal{R}$ at $\mathcal{R} \ll 2l$ at short range. The following operator identity, $\sum_{ij} \hat{L}_{ij}^2 = \hat{L}^2 + N(N-2)\,\hat{l}^2$ [13], links the total angular momentum L of N single-particle angular momenta l with the pair angular momenta L_{ij}. It implies that V_H induces no correlations, in a sense that all many-body multiplets at the same L have the same energy. In other words, the total interaction energy of a many-body system with interaction V_H deoes not depend on the relative occupation of different pair eigenstates. This is obviously not the case when V is not harmonic. Any positive anharmonic contribution to V yields avoidance of the corresponding pair state in the low-energy many-body states. In particular, the dominant anharmonic repulsion at $\mathcal{R} = 1$ leads to the Laughlin correlated ground state of electrons at $\nu = 1/3$. In comparison to V_H the pseudopotentials in LL_0, LL_1, and CF-LL_1 are all qualitatively different: strongly superharmonic, roughly harmonic, and strongly subharmonic at short range, respectively. Consequently, the correlations in those partially filled shells must also be different.

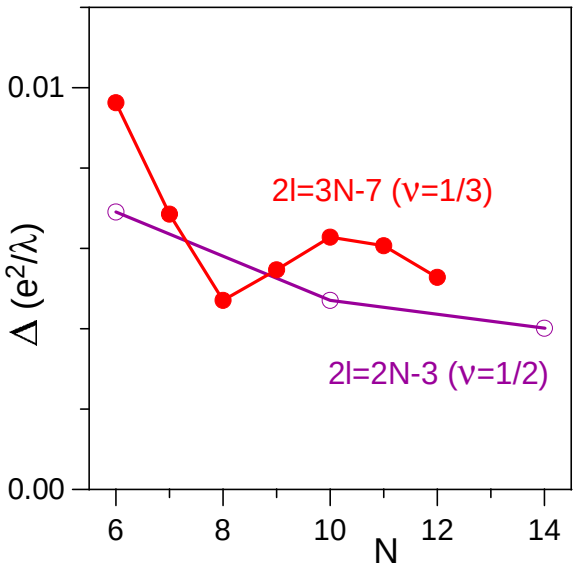

Figure 6. Excitation energy gaps Δ for the series of ground states of N QEs on a sphere, at $2l = 3N-7$ and $2N-3$ ($\nu = 1/3$ and $1/2$), as a function N.

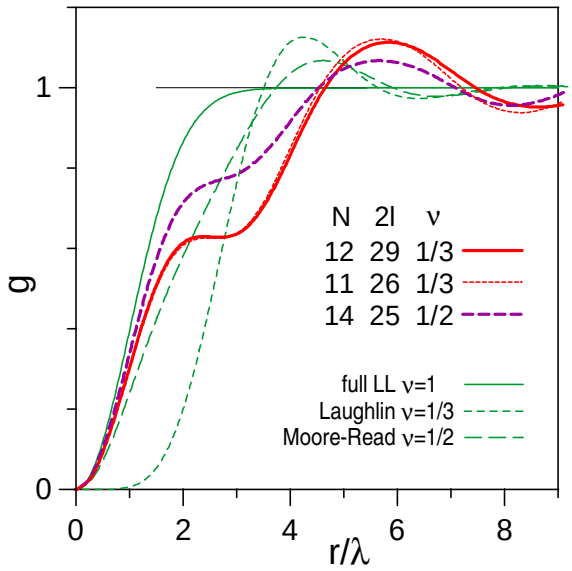

Figure 7. QE pair-distribution functions $g(r)$ of N-QE ground states at $2l = 3N-7$ and $2N-3$ ($\nu = 1/3$ and $1/2$) compared to known states of electrons.

The above discussion raises several questions regarding the nature of the "second-generation" liquids: (i) Can $V_{\rm QE}(\mathcal{R})$ yield incompressibility? (ii) At what filling factors? (iii) What are the QE–QE correlations? and (iv) What is the many-QE wave function at $\nu = 1/3$ and $1/2$?

5. Evidence for incompressibility from exact-diagonalization of N interacting QEs

Armed with the knowledge of $V_{\rm QE}(\mathcal{R})$ we are ready to address the problem of QE incompressibility by exact diagonalization of the N-QE interaction hamiltonians at the values of N and $2l$ corresponding to a given ν. This approach rests on the following nontrivial assumptions or additional properties: (i) The two-body pseudopotential is sufficient to describe interaction among many QEs. (ii) The pseudopotential determined at $\nu \ll 1$ can also be used at $\nu = 1/3$ or $1/2$. (iii) The QE–QH excitations are negligible.

In place of a rigorous analysis of the above issues let us demonstrate directly the accuracy of the mapping of an N_e electron system with Coulomb interaction at $1/3 < \nu_e < 2/5$ onto a corresponding N-QE system with $V_{\rm QE}(\mathcal{R})$. In Fig. 4 we compare the 12-electron spectrum at $2l = 29$ with the corresponding 4-QE spectrum at $2l = 9$. Remarkably, the QE spectrum reproduces the bottom of the electron spectrum rather well (with higher many-electron states involving additional QE–QH pairs), justifying the CF mapping.

Two examples of N-QE spectra of $V_{\rm QE}(\mathcal{R})$ and showing non-degenerate ($L = 0$) ground states with a gap are shown in Fig. 5. By looking at different combinations of $(N, 2l)$, we have identified a whole series of such ground states at $2l = 3N-7$. This is different from Laughlin's relation $2l = 3N-3$, even though it extrapolates to the same filling factor $\nu = 1/3$. We also found a gapped ground state for $(N, 2l) = (14, 25)$, coincident with the $2l = 2N-3$ series of the Moore–Read electron liquid in LL_1, but the assignment of $\nu = 1/2$ to this state is less certain. At other $(N, 2l)$ either the ground state is degenerate ($L \neq 0$) or the excitation gap is marginal.

To verify that the $2l = 3N-7$ (and the more problematic $2l = 2N-3$) series of finite-size states indeed represent the extended $\nu = 1/3$ (and $\nu = 1/2$) incompressible QE liquids, in Fig. 6 we plot the energy gap Δ as a function of N. Indeed, it seems plausible that the gap will not

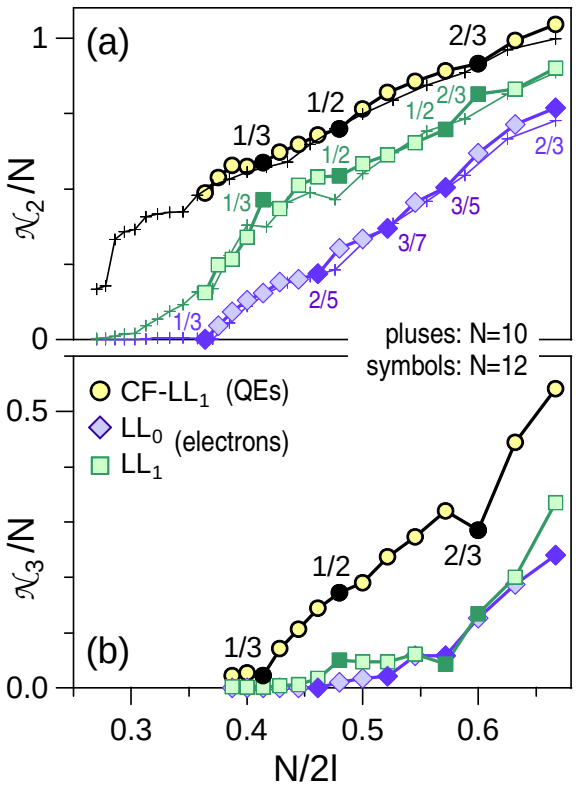

Figure 8. Average number of pairs $\mathcal{N}_2$ (a) and triplets $\mathcal{N}_3$ (b) with the minimum relative angular momentum, calculated for N electrons (in LL_0 or LL_1) or QEs (in CF-LL_1) as a function of $N/2l \approx \nu$. Incompressible states are labeled by ν.

close in large systems, and thus that it is due to QE–QE interaction rather than merely due to an artificial boundary. Moreover, Fig. 7 makes it clear that the pair correlation functions $g(r)$ for the $N = 11$ and 12 states of the $2l = 3N - 7$ series are essentially identical, at the same time being very different from both the curve for 14 QEs at $2l = 25$, and several other curves obtained for known FQH states of electrons [21].

6. Evidence for pairs and clusters of QEs from 2- and 3-body correlation functions

The nature of gapped N-QE ground states identified in the previous section from the exact diagonalization can be elucidated by their pair and triplet Haldane amplitudes [22]. These amplitudes, $\mathcal{G}_2(\mathcal{R})$ and $\mathcal{G}_3(\mathcal{T})$, are the discrete two- and three-body correlation functions, which count the fraction of pairs or triplets as a function of two- and three-body relative angular momentum, respectively. They are easily calculated as the expectation values of the model, short-range two- and three-body interaction pseudopotentials, $V_{\mathcal{R}}(\mathcal{R}') = \delta(\mathcal{R}, \mathcal{R}')$ and $W_{\mathcal{T}}(\mathcal{T}') = \delta(\mathcal{T}, \mathcal{T}')$. Two-body matrix elements $\langle i, j|V_{\mathcal{R}}|k, l\rangle = \langle i, j|L\rangle \langle L|k, l\rangle \delta(L, 2l - \mathcal{R})$ are simply the products of the appropriate Clebsh-Gordan coefficients. Analogously, the three-body matrix elements $\langle i, j, k|W_{\mathcal{T}}|l, m, n\rangle = \langle i, j, k|L\rangle \langle L|l, m, n\rangle \delta(L, 3l - \mathcal{T})$ involve expansion parameters related to the Racah coefficients.

Up to the normalization factors, the amplitudes corresponding to the minimum allowed values of $\mathcal{R}_{\min} = 1$ and $\mathcal{T}_{\min} = 3$ have a simple interpretation of the average number of "compact" pairs or triplets, $\mathcal{N}_2 = \binom{N}{2} \mathcal{G}_2(\mathcal{R}_{\min})$ and $\mathcal{N}_3 = \binom{N}{3} \mathcal{G}_3(\mathcal{T}_{\min})$. In Fig. 8(a) we plot $\mathcal{N}_2/N$ as a function of $N/2l$ for the ground states of N particles at different values of $2l$. The data for $N = 10$ and 12 nearly overlap, suggesting a genuine, size independent effect, while the difference between QEs in CF-LL_1 and electrons in LL_0 or LL_1 strikes out immediately. While $N/2l$ is only an approximate measure of ν in finite system, the exact values of ν assigned to the particular 12-particle incompressible ground states are indicated next to the filled symbols. The defining

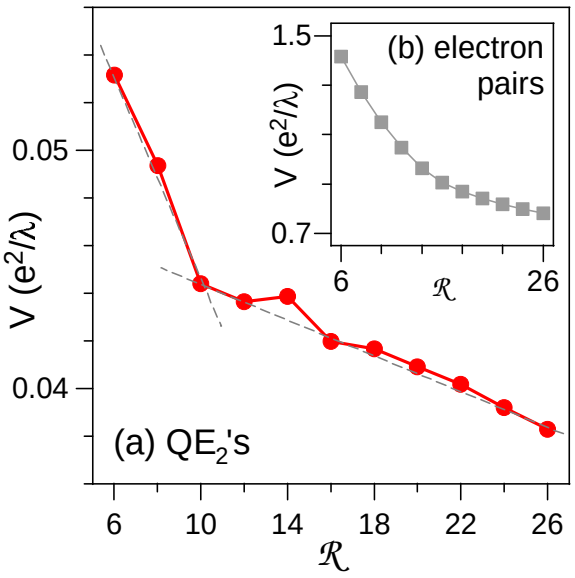

Figure 9. Pseudopotentials for interaction between pairs of (a) CFs in second LL (i.e., between QE_2s) and (b) electrons in lowest LL.

property of the Laughlin state – the complete avoidance of the $\mathcal{R} = 1$ pair state – is clearly visible in Fig. 8(a) as the vanishing of $\mathcal{N}_2$ at the $\nu = 1/3$ filling of LL_0. Evidently, the $\nu = 1/3$ state of QEs shows a different behavior.

More telling of the QE correlations is Fig. 8(b), in which we show a matching plot of $\mathcal{N}_3/N$. The most striking result is the vanishing of $\mathcal{N}_3$ of the QEs at the $\nu = 1/3$ filling of CF-LL_1. Combined with the value of $\mathcal{N}_2 \approx N/2$, this strongly suggests pairing (in real space) of the QEs at $\nu = 1/3$. On the other hand, $\mathcal{N}_3/N \sim 0.4$ for the QE filling $\nu = 1/2$ indicates formation of some triplets (or larger clusters) in this state. This is in obvious contrast to the known vanishing of $\mathcal{N}_3$ in the Moore–Read state describing the half-filled electron LL_1.

The question of correlations among the CF pairs (QE_2 molecules) that leads to incompressibility and the FQH effect at $\nu_e = 4/11$ remains open. However, the pseudopotential of the QE_2–QE_2 interaction has been calculated [15]. Fig. 9 shows that it is quite different than the pseudopotential for electron molecules in LL_0. In particular, it is roughly linear at small $\mathcal{R}$. This, however, does not make the situation similar to the electrons forming a Moore–Read state in LL_1 due to different QE_2 and e statistics.

7. Spin phase diagram and depolarization in an unpolarized correlated QE liquid

Let us now address the question of spin of CFs, completely ignored in the preceding sections. It is long known [26] that QE_R (the reversed-spin quasielectron; represented by a spin-flip CF in CF-LL_0) has lower Coulomb energy ε than QE. Hence, the latter remains the lowest negatively charged excitation of the Laughlin liquid only when it is additionally favored by a sufficient Zeeman energy, E_Z. The comparison of ε_{QE} and ε_{QER} as a function of width w of the quasi-2D electron layer is shown in Fig. 10(b).

Whether QEs or QE_R (or their combination) will occur in a gas depends only on the competition of ε_{QE} and ε_{QER}. However, in a liquid (e.g., at $\nu = 1/3$), their correlation energies per particle u must also be compared [27]. They are defined as $u = (E + U_{bckg})/N$ where $U_{bckg} = (Ne/3)^2/2R$ accounts for the charge-compensating background (R is the radius of the Haldane sphere). For the QE_Rs, many-body interaction energy E is calculated similarly as described for the QEs, only using a different pseudopotential V_{QER} shown in Fig. 2(a). The finite-size estimates of u_{QE} and u_{QER} are compared in Fig. 10(a).

Transition between the QE and QE_R liquids at $\nu = 1/3$ occurs when $\varepsilon_{QE} + u_{QE} =$

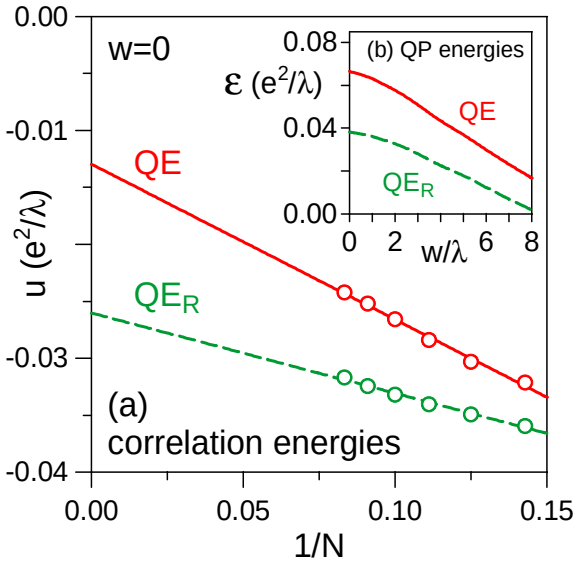

Figure 10. (a) Correlation energies u in the $\nu = 1/3$ liquids of QEs or QE_Rs, as a function of their inverse number N^{-1}. (b) QE and QE_R Coulomb energies ε as a function of electron layer width w.

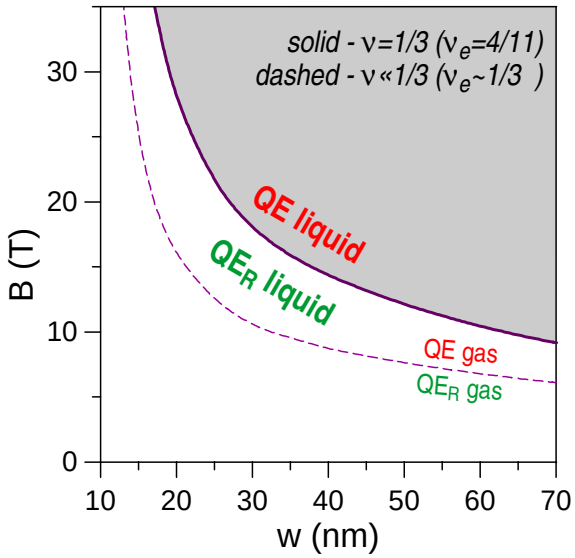

Figure 11. Phase diagram (the critical layer width w vs magnetic field B) for QE–QE_R spin transition at $\nu = 1/3$ (i.e., at $\nu_e = 4/11$). Thin dashed line is for a gas of uncorrelated QEs or QE_Rs at $\nu \ll 1/3$ (i.e., at $\nu_e \approx 1/3$).

$\varepsilon_{QER} + u_{QER} + E_Z$. By combining the calculated values of ε and u (to reduce the finite size error we use the values extrapolated to large N) and the width dependence of electron Landé g-factor [28] we have obtained [23] the spin phase diagram displayed in Fig. 11.

Clearly, larger widths and stronger magnetic fields both favor spin polarization, and the transition to a partially unpolarized phase (note that pure liquids of QEs and QE_Rs correspond to complete and intermediate spin polarizations of the whole electron liquid, $P = 100\%$ and 50%, respectively) requires the opposite conditions. The role of CF–CF interactions in stabilizing the QE_R liquid is also evident from comparison with the phase boundary calculated neglecting $u_{QE} - u_{QER}$ (i.e., for a CF gas). It is noteworthy that all FQH experiments so far [4] were done either deep inside the QE phase or close to the predicted phase boundary.

8. Conclusion

We used a combination of the CF theory and exact numerical diagonalization to study "second-generation" incompressible quantum liquids corresponding to the fractional filling of CF-LL_1. We have explained that the low-energy electron dynamics in these states can be mapped onto the problem of a smaller number of Laughlin QEs interacting through an effective pair pseudopotential. Short-range behavior of this QE–QE pseudopotential is very different from that of electrons in LL_0 or LL_1. This leads to the particular form of QE–QE correlations in CF-LL_1, which is very different from electron correlations in a partially filled LL_0 or LL_1. In particular, we have demonstrated that the $\nu_e = 4/11$ state is not a Laughlin state of QEs, despite having a familiar value of $\nu_{QE} = 1/3$. Instead, we have shown that this state involves QE pairing (similar to the Moore–Read state describing electrons at the half-filling of LL_1). On the other hand, we have shown that the $\nu_e = 3/8$ state is not a paired Moore–Read state of QEs despite having the same $\nu_{QE} = 1/2$. Instead, it seems to involve formation of larger QE clusters. We have also looked at the possible spin transition at $\nu_e = 4/11$, corresponding to the crossover between a paired QE state and a Laughlin state of QE_Rs.

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