Reversed-spin quasiparticles in fractional quantum Hall systems and their effect on photoluminescence

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Abstract

The energy, interaction, and optical properties of reversed-spin quasielectrons (QE_Rs) in fractional quantum Hall systems are studied. Based on the short range of the QE_R–QE_R repulsion, a partially unpolarized incompressible $v = \frac{4}{11}$ state is postulated within Haldane hierarchy scheme. To describe photoluminescence, a reversed-spin fractionally charged exciton hQE_R (QE_R bound to a valence hole h) is predicted. In contrast to its spin-polarized analog, hQE_R is strongly bound and radiative. © 2002 Elsevier Science B.V. All rights reserved.

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Laughlin quasielectrons (QEs) and quasiholes (QHs) of fractional quantum Hall (FQH) systems [1,2] can be thought of as empty (QH) or filled (QE) states of the lowest or the first excited composite fermion (CF) Landau level (LL) [3], respectively. The energies of these Laughlin quasiparticles (QPs) and their interactions with one another and with a valence hole (h) have been studied quite thoroughly by means of exact diagonalization of small systems. The influence of QPs on photoluminescence (PL) of the Laughlin electron fluid is important in those systems in which the hole is spatially separated from the electrons by a distance d that exceeds approximately one magnetic length (λ). In such systems, PL occurs from the radiative bound states of a hole and one or two QEs, called fractionally charged excitons (FCXs) [4]. Reversed-spin QEs (denoted by QE_Rs) [5,6] are

another type of elementary excitations of a Laughlin fluid, which can be thought of as particles in the reversed-spin lowest CF LL. As other QPs, the QE_Rs carry fractional charge and have finite size, energy, and angular momentum.

In this note, the single-particle properties of the QE_Rs, as well as the pseudopotentials defining their interaction with one another and with other QPs, are determined numerically and used to predict when the incompressible-fluid states with less than maximum polarization occur. The interaction of QE_Rs with valence holes is also studied as a function of the layer separation d. In analogy to the FCX states, stable reversed-spin FCXs (denoted by FCX_R) are predicted at a finite d and small Zeeman splitting. It is shown that the FCX_R optical selection rules are different from those of FCXs, because of the QE_R angular momentum and spin being different from those of a QE. For example, the ground state of one QE_R bound to a hole is optically active, in contrast to the nonradiative

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h–QE pair ground state. The stability of the radiative FCX and FCX$_R$ states, and thus also the PL spectrum of the Laughlin fluid, is shown to depend on the layer separation d, Zeeman splitting, and (critically) on the electron filling factor v.

In order to preserve the 2D translational symmetry of infinite systems, in a finite-size calculation we use Haldane spherical geometry [7] in which the LL degeneracy $g = 2S + 1$ is controlled by the strength $2S$ of the magnetic monopole placed in the center of the sphere of radius R. The monopole strength $2S$ is defined in the units of flux quantum $\phi_0 = hc/e$, so that $4\pi R^2 B = 2S\phi_0$ and $R^2 = S\lambda^2$. The many-body states on the sphere are labeled by the set of good quantum numbers: electron spin (J) and its projection (J_z), hole spin (σ_h), and the length (L) and projection (L_z) of the total angular momentum of the electron–hole system.

The ground state of the 2DEG in the lowest LL at the Laughlin filling factor $v = \frac{1}{3}$ is completely polarized in the absence of the Zeeman splitting, $E_Z = 0$. There are two types of elementary charge-neutral excitation of these ground states, carrying spin $\Sigma = 0$ or 1, which in literature are referred to as the charge- and spin-density waves, respectively. The most important features of their dispersion curves $\mathscr{E}_\Sigma(k)$ (k is the wave vector) are the magneto-roton minimum at the finite value of $k = 1.5\,\lambda^{-1}$ in $\mathscr{E}_0(k)$, the finite gap $\Delta_0 \approx 0.076e^2/\lambda$ at this minimum, and the vanishing of $\mathscr{E}_1$ in the $k \to 0$ limit. Equally important is the similarity of the charge- and spin-density waves in the $v = \frac{1}{3}$ state to those at $v = 1$. The latter can be understood by means of Jain CF picture [3] where the excitations of the $v = 1/3$ electron state correspond to promoting one CF from a completely filled lowest ($n = 0$) spin-$\downarrow$ CF LL either to the first excited ($n = 1$) CF LL of the same spin ($\downarrow$) or to the same CF LL ($n = 0$) but with the reversed spin ($\uparrow$).

One can define three types of QPs (elementary excitations) of the Laughlin $v = \frac{1}{3}$ fluid that constitute the charge- and spin-waves: Laughlin QHs and QEs and Rezayi QE$_R$s. Each of the QPs is characterized by such single-particle quantities as electric charge ($\mathscr{Q}_{QH} = +\frac{1}{3}e$ and $\mathscr{Q}_{QE} = \mathscr{Q}_{QE_R} = -\frac{1}{3}e$), degeneracy g_{QP} of the single-particle Hilbert space, and energy ε_{QP}. The charge-neutral excitations of Laughlin ground states are composed of a pair of QH and either QE ($\Sigma = 0$) or QE$_R$ ($\Sigma = 1$).

In order to estimate the energies ε_{QP} needed to create an isolated QP of each type, we applied the exact diagonalization procedure to systems of $N \leqslant 11$ electrons. By extrapolating the results to $N \to \infty$, we found the following values appropriate for an infinite system: $\varepsilon_{QE} = 0.0664e^2/\lambda$ and $\varepsilon_{QE_R} = 0.0383e^2/\lambda$. Our estimation of the so-called "proper" QP energies (obtained by adding the term $\mathscr{Q}_{QP}^2/2R$ to ε_{QP}) are $\tilde{\varepsilon}_{QE} = 0.0737e^2/\lambda$, $\tilde{\varepsilon}_{QE_R} = 0.0457e^2/\lambda$, and $\tilde{\varepsilon}_{QH} = 0.0258e^2/\lambda$. Consequently, the energies of spatially separated QE–QH and QE$_R$–QH pairs are equal $\mathscr{E}_0 = \tilde{\varepsilon}_{QE} + \tilde{\varepsilon}_{QH} = 0.0995e^2/\lambda$ and $\mathscr{E}_1 = \tilde{\varepsilon}_{QE_R} + \tilde{\varepsilon}_{QH} = 0.0715e^2/\lambda$ (these are activation energies in transport experiments).

Which of the two negatively charged QPs (QE or QE$_R$) occur at low energy in the particular system depends on the Zeeman term which is influenced not only by magnetic field, but also by many material parameters. Once the QPs content is established, the correlations in the system can be understood by studying the appropriate interaction pseudopotentials defined as the dependence of pair interaction energy V on the relative pair angular momentum $\mathscr{R}$ (larger $\mathscr{R}$ corresponds to larger separation) [8]. Here, $\mathscr{R}_{QE–QE_R} = l_{QE} + l_{QE_R} - L$ and $\mathscr{R}_{QE–QE} = l_{QE} + l_{QE} - L$, where l_{QP} and L denote the one- and two-QP angular momenta, respectively. The QH–QH, QE–QE, QE–QH, and QE$_R$–QH pseudopotentials can be found elsewhere [8,9], and therefore we will limit this discussion to $V_{QE_R–QE_R}$ and $V_{QE–QE_R}$ only.

Two QE$_R$s can be formed in a N-electron system with at least two reversed spins at $2S = 3(N - 1) - 2$. An example of such spectrum is shown in Fig. 1(a) for $N = 8$ with $J = 2, 3$, and 4 corresponding to two, one and zero reversed spins, respectively. It is clear that the maximally spin-polarized system ($J = \frac{1}{2}N$) is unstable at filling factors not equal to the Laughlin value $v = \frac{1}{3}$.

The $V_{QE_R–QE_R}$ is shown in Fig. 1(b). Although the obtained values depend on the system size (the repulsive character of interaction is restored only in the $N \to \infty$ limit with $V_{QE_R–QE_R}(1) \approx 0.01e^2/\lambda$), the monotonicity of $V_{QE_R–QE_R}$ seems to be independent of N. Moreover, the super-linear shape of the curve indicates Laughlin correlations and thus incompressibility at $v_{QE_R} = \frac{1}{3}, \frac{1}{5}, \ldots$ (in analogy to Haldane hierarchy picture of completely spin-polarized states [7,8]). For example, Laughlin $v_{QE_R} = \frac{1}{3}$ state occurs at the elec-

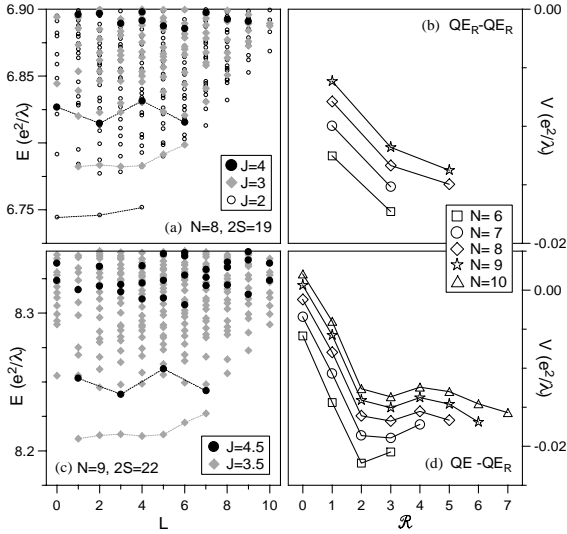Fig. 1. (a), (c) The N-electron energy spectra calculated on Haldane sphere at different values of $2S$. The pseudopotentials of the QE_R–QE_R (b) and QE–QE_R (d) interactions calculated for $N \leqslant 10$.

tron filling factor of $v = \frac{4}{11}$ and corresponds to the 75% spin polarization (this state has also recently been proposed in the CF model [10]). In view of the fact that the spin-polarized $v_{QE} = \frac{1}{3}$ state is compressible [8], the experimental observation [11] of the FQH effect at $v = \frac{4}{11}$ confirms the formation of QE_Rs in the $v = \frac{1}{3}$ state.

An QE–QE_R pair can be formed in the system with at least one reversed spin. Another example of such spectrum is shown in Fig. 1(c) for $N = 9$ at $2S = 3(N-1) - 2 = 22$. The lowest energy states in the two considered subspaces, $J = \frac{1}{2}N = \frac{9}{2}$ and $J = \frac{1}{2}N - 1 = \frac{7}{2}$, contain a QE–QE and QE–QE_R pair, respectively. The pseudopotential $V_{QE–QE_R}(\mathcal{R})$ was calculated for $N \leqslant 10$ and the results are presented in Fig. 1(d). The values of the pseudopotential depend on N and in the limit $N \to \infty$ we found $V_{QE–QE_R}(0) \to 0.015 e^2/\lambda$ and $V_{QE–QE_R}(1) \to 0.01 e^2/\lambda$.

The behavior of $V_{QE–QE_R}(\mathcal{R})$ is qualitatively different from that of $V_{QE_R–QE_R}(\mathcal{R})$ and $V_{QE–QE}(\mathcal{R})$. The most significant feature of the $V_{QE–QE_R}(\mathcal{R})$ function is that it is super-linear in $L(L+1)$ only at $1 \leqslant \mathcal{R} \leqslant 3$ and sub-linear at $0 \leqslant \mathcal{R} \leqslant 2$ and at larger $\mathcal{R}$. As a result, $m = 2$ is the only possible Jastrow exponent in

the many-body wave function that describes Laughlin correlations and thus yields incompressibility of the system.

If QEs and QE_Rs could coexist in the $v = \frac{1}{3}$ "parent" state (unlikely due to their sensitivity to Zeeman energy), one could apply a generalized CF picture [12] to predict the allowed combinations of Jastrow exponents $[m_{QE–QE}, m_{QE_R–QE_R}, m_{QE–QE_R}]$ describing the incompressible states of such two-component plasma. Based on the behavior of the three involved QP pseudopotentials for different values of $\mathcal{R}$, we reduced the number of possible exponent combinations to a few, from which only $[1, 1, 2]$ satisfies the incompressibility condition. Such hypothetical mixed QE/QE_R state corresponds to the $v = \frac{5}{13}$ state with 80% spin polarization.

The PL spectra of a spin-polarized 2DEG can be understood in terms of QEs and their interaction with one another and with a valence hole (h), where electron and hole layers are separated by a finite distance d (of the order of λ). It was shown [4] that in the regime where the electron–electron repulsion is weak compared to the electron–hole attraction, a hole can bind one or two QEs to create FCX (hQE or hQE$_2$). The optical selection rules following from the 2D translational symmetry leave hQE* (the asterisk denotes an excited state) and hQE$_2$ to be the only optically active ("bright") states. By similar consideration, in the partially unpolarized systems we expect the positively charged h to bind one or two QE_R forming states denoted here by FCX$_R$. However, the radiative recombination of FCX$_R$ will be governed by different selection rules due to different angular momenta of QE and QE_R and thus different angular momenta of initial (bound) states ($l_{FCX_R} \neq l_{FCX}$).

Because of strong QE_R–QE_R repulsion, binding of more than one QE_R by the valence hole h should be more difficult. Similarly to the QE case, the simplest FCX$_R$, hQE$_R$, is expected to occur in a system containing free QE_Rs at the values of d at which the binding energies of hQE and hQE$_R$ are smaller than the Laughlin gap to create additional QE–QH pairs. We have studied a good number of numerical spectra for different values of d. As expected, we found that both hQE$_R$ and hQE ground states develop at d larger than λ.

An example of such spectrum for a 7e–h system (with up to one reversed electron spin) at $d = 4\lambda$ and
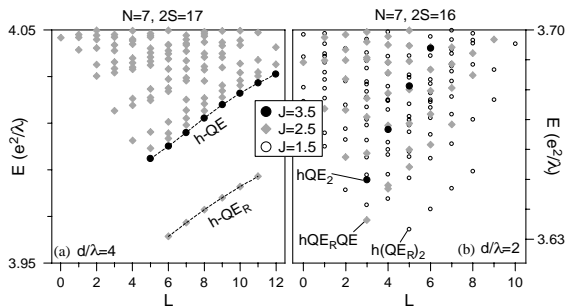
Fig. 2. The energy spectra of a system of seven electrons and one hole on Haldane sphere at $d/\lambda = 4$ with $2S = 17$ (a) and at $d/\lambda = 2$ with $2S = 16$ (b).

$2S = 17$ is shown in Fig. 2(a). In the CF picture this configuration corresponds to six CFs filling the first CF LL and the seventh CF lying either in the second CF LL with parallel spin ($J = \frac{1}{2}N = \frac{7}{2}$) or in the first CF LL with reversed spin ($J = \frac{1}{2}N - 1 = \frac{5}{2}$). As shown in Fig. 2(a), at sufficiently large d the lowest energy states contain well-defined h–QE or h–QE_R pairs with all possible values of L resulting from the rules of addition of angular momenta. Applying the appropriate orbital selection rule for radiative recombination [4,9] we determined that, unlike the dark hQE, the reversed-spin hQE$_R$ ground state is radiative. As shown in Fig. 2(b) for $d = 2\lambda$, the numerical spectra contain also larger FCX$_R$ complexes, $h(QE_R)_2$ and hQE$_R$QE. However, due to their weaker binding and

because $h(QE_R)_2$ turns out dark and hQE$_R$QE is very sensitive to Zeeman energy, the emission of hQE$_R$ is expected to dominate the PL spectrum of a partially unpolarized system at $v \approx \frac{1}{3}$.

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