

QUASIPARTICLE INTERACTIONS IN FRACTIONAL QUANTUM HALL SYSTEMS

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A partially filled shell (Landau level) of Laughlin quasiparticles (QP's) gives rise to an incompressible daughter state if the QP's themselves are Laughlin correlated. This occurs only if the pseudopotential $V_{QP}(L')$ describing the interaction energy of a QP pair as a function of the total pair angular momentum L' satisfies special conditions. $V_{QP}(L')$ can be obtained quite accurately from numerical studies of small systems. It does not always satisfy these conditions (e.g. for quasielectrons of the Laughlin $\nu = 1/3$ state at their $\nu_{QE} = 1/3$ filling). In such cases, formation of pairs or larger clusters may explain the recently observed incompressible states (like $\nu = 4/11$).

Keywords: Fractional quantum Hall effect; quasiparticle interaction.

Laughlin incompressible fluid states¹ at filling factors $\nu = (2p + 1)^{-1}$ result from maximal avoidance of pair states with the largest pair angular momentum L' (or the smallest relative angular momentum $\mathcal{R} = 2l - L' = 1, 3, 5, \dots$, where l is the single particle orbital angular momentum²). This maximal avoidance of most strongly repulsive pair states is the definition of "Laughlin correlations". Jain's composite Fermion (CF) picture,³ which yields incompressible states at $\nu = n(2pn \pm 1)^{-1}$ where n and p are positive integers, implicitly assumes Laughlin correlations among the electrons. Jain states result from the mean field CF's filling an integral number of CF Landau levels (LL's). Quasiparticles (QP's) in partially filled CF shells can form incompressible states if their interactions support Laughlin correlations. This was assumed in Haldane's QP hierarchy scheme² and in CF hierarchies proposed by Jain and Goldman⁴ and Sitko *et al.*⁵ It was found, however, that certain CF hierarchy states did not occur,⁵ and the explanation why was given by Wojs and Quinn.⁶ The explanation involves the form of the pseudopotential describing the QP–QP interaction V_{QP} as a function of pair angular momentum L' (or $\mathcal{R} = 2l - L'$).

A recent observation^{6,7} of fractional quantum Hall effect at $\nu = \frac{3}{8}$ and $\frac{4}{11}$ renewed interest in correlations in partially filled CF LL's.^{8–11} While assumption of similar CF–CF and electron correlations would also imply incompressibility of these novel states, the nonmonotonic behavior of V_{QE} (QE denoting quasielectrons) precludes such interpretation.¹¹ The pseudopotentials for electrons in the lowest LL (a) and for QE's in the first excited CF LL (b) are compared in Fig. 1. The latter

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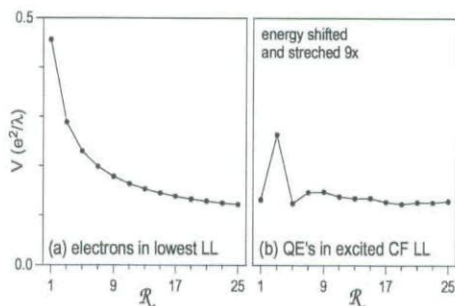


Fig. 1. Interaction pseudopotentials for the lowest electron LL (a) and the first excited CF LL (b). In (b), V calculated by Lee *et al.*⁸ is only known up to a constant. λ is the magnetic length.

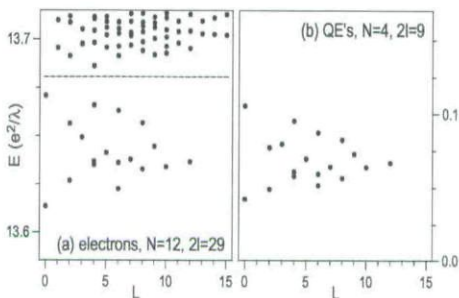


Fig. 2. Energy spectra for $N = 12$ electrons in the lowest LL with $2l = 29$, and for $N = 4$ QE's in the first excited CF LL with $2l = 9$. Energy scales are the same.

are determined from numerical diagonalization of small systems at values of the monopole strength $2Q$ corresponding to having two QE's in a Laughlin $\nu = 1/3$ incompressible state.^{6,8,12} These QE pseudopotentials are quite accurate up to an overall constant which has no effect on the electronic correlations.

In Fig. 2 the energy spectrum for $N = 12$ electrons at $2Q = 29$ (a), and the corresponding spectrum for $N_{\text{QE}} = 4$ QE's at $2l_{\text{QE}} = 9$ (b) are shown. The QE spectrum is obtained using $V_{\text{QE}}(\mathcal{R})$. The good qualitative agreement between the low energy states of (a) and (b) gives us some confidence in using $V_{\text{QE}}(\mathcal{R})$ to describe the interactions between the CF QE's. A function $\mathcal{G}_{L\alpha}(\mathcal{R})$, describing the probability that the α^{th} multiplet of total angular momentum L contains pair states with relative pair angular momentum \mathcal{R} , can be obtained from the numerical diagonalization. For Laughlin correlated states (e.g. the $N = 12$ electrons at $2l_e = 29$) $\mathcal{G}(\mathcal{R})$ has a minimum value at $\mathcal{R} = 1$ and a maximum value at $\mathcal{R} = 3$ corresponding to avoidance of pairs with largest repulsion. For the $N_{\text{QE}} = 4$ system at $2l = 9$, $\mathcal{G}(\mathcal{R})$ is large at $\mathcal{R} = 1$ and $\mathcal{R} = 5$, but takes on a minimum value at $\mathcal{R} = 3$. This corresponds to avoidance of the maximum repulsion between QE's at $\mathcal{R} = 3$. The increase in $\mathcal{G}(\mathcal{R} = 1)$ from its Laughlin correlated value can be interpreted as the formation of QE pairs or larger clusters.¹¹ The physics behind the clustering can be most easily understood by use of two sum rules⁶ satisfied by the pair probability

function $\mathcal{G}_{L\alpha}(\mathcal{R})$. They¹³ are $\sum_{L'} \mathcal{G}_{L\alpha}(L') = 1$ and

$$\frac{1}{2}N(N-1) \sum_{L'} L'(L'+1) \mathcal{G}_{L\alpha}(L') = L(L+1) + N(N-2)l(l+1). \quad (1)$$

Together with the expression for the energy of the state $|l^N; L\alpha\rangle$, viz

$$E_\alpha(L) = \frac{1}{2}N(N-1) \sum_{L'} \mathcal{G}_{L\alpha}(L') V(L'), \quad (2)$$

the sum rules make it clear that a harmonic pseudopotential $V_H(L') = A + BL'(L'+1)$, where A and B are constants, introduces no correlations. For the harmonic pseudopotential, every multiplet with the same total angular momentum has the same energy. This makes it apparent that a model pseudopotential with $V(L') = V_H(L') + \Delta \delta(L', 2l-1)$, will have its lowest energy state for any given value of the total angular momentum L when $\mathcal{G}_{L\alpha}(L')$ maximally avoids $L' = 2l-1$ or $\mathcal{R} = 1$. However, for a model pseudopotential with $V(L') - V_H(L') = (1-\beta)\delta_{\mathcal{R},1} + \frac{1}{2}\beta\delta_{\mathcal{R},3}$, where $0 \leq \beta \leq 1$, it can be demonstrated numerically that Laughlin correlation [$\mathcal{G}(\mathcal{R}=3) \gg \mathcal{G}(\mathcal{R}=1)$] occur only if the parameter β is small compared to the value $\frac{1}{2}$. "Anti-Laughlin correlations" [$\mathcal{G}(\mathcal{R}=3) \ll \mathcal{G}(\mathcal{R}=1)$] occur when β is large compared to $\frac{1}{2}$. At $\beta \approx \frac{1}{2}$, $\mathcal{G}(\mathcal{R}=3)$ is approximately equal to $\mathcal{G}(\mathcal{R}=1)$, and the ground state apparently well-described by the Moore-Read wave function.¹⁴

To test the picture of pairing (or formation of larger clusters), we have performed numerical calculations for systems containing up to 18 QE's using the $V_{\text{QE}}(\mathcal{R})$ shown in Fig. 1. $L = 0$ condensed states were found in four families with $2l = 3N - 7$, $2l = \frac{3}{2}N + 2$, $2l = 2N - 3$, and $2l = 2N + 1$. These correspond to $\nu_{\text{QE}} = \frac{1}{3}$, $\nu_{\text{QE}} = \frac{2}{3}$, and the electron-hole conjugate states corresponding to $\nu_{\text{QE}} = \frac{1}{2}$. The $\nu_{\text{QE}} = \frac{1}{3}$ state appeared for all values of N [we calculated for $5 \leq N \leq 12$] and the $\nu_{\text{QE}} = \frac{2}{3}$ state for all even values of N with $4 \leq N \leq 18$. The $\nu_{\text{QE}} = \frac{1}{2}$ states occurred at $2l = 2N - 3$ but only for odd values of $N/2$, the number of QE pairs. A simple pairing model based on Halperin's idea¹⁵ of formation of pairs or larger clusters¹⁶ having Laughlin type correlations between one another leads to incompressible ground states at $\nu_{\text{QE}} = \frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{3}$, and at $\nu_{\text{QH}} = \frac{1}{5}$, $\frac{1}{4}$, and $\frac{2}{7}$. In the CF hierarchy picture,⁵ this leads to novel incompressible states at $\nu = \frac{5}{13}$, $\frac{3}{8}$, $\frac{4}{11}$, and at $\nu = \frac{5}{17}$, $\frac{3}{10}$, and $\frac{4}{13}$. These states would contain pairs of QE's, with Laughlin correlations between pairs. All of these states have been observed experimentally.⁷ However, a simple complete pairing model of QE's has several shortcomings. It cannot explain odd values of N , and it would predict that Laughlin correlations among the pairs of CF QE's occur at $2l = 3N - 5$ instead of the observed value of $3N - 7$, and at $2l = \frac{3}{2}N + 1$ instead of the observed value of $2l = \frac{3}{2}N + 2$.¹¹ It is noteworthy that the formation of clusters of k Fermions (when the clusters are treated as Fermions) which have Laughlin correlations between the clusters would occur at $2l = mN - [(m-1)k + 1]$. This would predict correlated pairs at $2l = 2N - 3$ and correlated triplets at $2l = 3N - 7$ as we find in our numerical

results. Another difficulty is that paired states at $2l = 2N - 3$ and $2N + 1$ do not occur in the “numerical experiments” at every even value of N .

Clearly we know why the individual QE’s cannot support Laughlin correlations. We do not know the effective pseudopotential between pairs (or larger clusters) with great accuracy, so it is not possible to use the cluster–cluster pseudopotential and the sum rules (which can be generalized to 3, 4, . . . body pseudopotentials) to predict a priori the exact nature of the correlations. The numerical results for finite systems contain information about the correlations that has not yet been fully exploited. It may lead us to a complete understanding of the various potential hierarchies of fractional quantum Hall states that are actually realized in experimental systems.

Support from grants DE-FG 02-97ER45657 of Materials Research Program of Basic Energy Sciences – US Dept. Energy (JJQ), 2P03B02424 and PBZ-Min-008/P03/03 of Polish Ministry of Scientific Research Information Technology (AW), and R14-2002-029-01002-0 of ABRL and PNU (KSY) is acknowledged.

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