Quasiexcitons in incompressible quantum liquids

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Photoluminescence (PL) has been used to study two-dimensional incompressible electron liquids in high magnetic fields for nearly two decades. However, some of the observed anomalies coincident with the fractional quantum Hall effect are still unexplained. We show that emission in these systems occurs from fractionally charged "quasiexciton" states formed from trions correlated with the surrounding electrons. Their binding and recombination depend on the state of both the electron liquid and the involved trion, predicting discontinuities in PL and sensitivity to sample parameters.

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The density of states of a two-dimensional electron gas (2DEG) in a high magnetic field B consists of discrete Landau levels (LLs). When B is so large that all electrons fall into the lowest LL, it is their mutual interaction that solely determines the ground state (GS) and low-energy excitations. Reminiscent of atomic physics except for the macroscopic LL degeneracy, this makes the 2DEG at high B an ideal laboratory of many-body physics in extended systems.

The incompressible quantum liquids (IQLs) (Ref. 1) were originally discovered in transport experiments[2] over two decades ago, but it took 15 years to demonstrate their hallmark fractionally charged quasiparticles (QPs) by shot-noise studies.[3] Photoluminescence (PL) was also used to probe IQLs, revealing anomalies at the LL fillings ν coincident with the fractional quantum Hall effect[4–8] (usually doublets at $\nu\approx\frac{1}{3}$ or $\frac{2}{3}$, but other features, too[9,10]). Other optical experiments include PL with acceptor-bound holes[11] and Raman scattering.[12]

The connection of PL anomalies with the microscopic properties of IQLs has been studied theoretically for over a decade. Depending on the strength and resolution of the Coulomb potential of photoinjected holes (controlled by h-2DEG separation[7]), the observed doublets were attributed either to the "bare exciton" and "magnetoroton assisted" emission[13,14] (efficient due to "gigantic suppression of the exciton dispersion by an IQL"[14]), or to recombination of different "anyon excitons"[15,16] consisting of several fractional IQL QPs bound to a hole. However, understanding of all reported anomalies is not yet complete and, e.g., discontinuities reported in Refs. 5 and 8 remain, to the best of our knowledge, unexplained.

To appreciate the complexity of the problem, one must recall that: (i) Even an unperturbed IQL has complicated dynamics whose understanding involves concepts of Laughlin correlations and fractionally charged QPs,[1] anyon statistics,[17] Haldane hierarchy,[18] or composite fermions (CFs).[19] (ii) Emergence of "multiplicative states" in e-h fluids with "hidden symmetry" (HS) (Ref. 20) greatly simplifies their optical response. (iii) Breaking of HS in real systems (due to finite layer widths w, charge separation, LL and valence band mixing, or disorder) restores the possibility of IQL-related anomalies in PL.

The HS is the exact particle-hole symmetry between conduction electrons and valence holes, requiring equal magnitudes of e-e, e-h, and h-h interactions. It allows mapping between e-h and two-pseudospin fluids, and leads to the conservation of an additional quantity related to the total pseudospin. The HS-related effects in real quantum wells are well known in the "dilute" regime ($\nu\ll 1$), in which PL is determined by the recombination of excitons ($X=e+h$) and trions ($X^{-}=2e+h$).[9,21] HS precludes radiative complexes larger than X, allowing for only one trion, the "dark triplet" X_t^-.[22] It is only due to the LL mixing that a "bright singlet" X_s^- occurs as well.[23,24]

In the "liquid" regime, few-body excitonic effects compete with many-body IQL dynamics, adding to each one's own complexity almost to guarantee fascinating physics. Different photoexcitations weakly coupled to the remaining IQL were proposed earlier. In the anyon exciton model[15,16] applicable for structures with strong charge separation (heterojunctions or wide asymmetric quantum wells), the holes repel positive quasiholes (QHs) and attract negative quasielectrons (QEs) of the IQL. The "dressed exciton" concept[13,14] introduced for narrower wells involves the Xs coupled to magnetorotons of the IQL. In another approach[16,25] the X^-s correlate with the surrounding electrons.

In this paper we develop the idea of trions immersed in a Laughlin IQL and *predict discontinuity of the PL spectrum* at $\nu=\frac{1}{3}$.[5,8] We show that trions remain stable in realistic doped wells, but acquire effective charge $\mathcal{Q}$ of up to one "Laughlin quantum" $\varepsilon=\frac{1}{3}e$ due to partial screening by the IQL. In analogy with X and $X^{\pm}$, we find *neutral and charged "quasiexcitons"* (QXs): $\mathcal{X}$ and $\mathcal{X}^{\pm 1/3}$. They consist of a trion which is Laughlin-correlated with the IQL and binds 0, 1, or 2 QHs. The $\mathcal{X}^{\pm 1/3}$ binding energies are directly observable in PL, and their order-of-magnitude reduction from the $X^{\pm}$ is an *indication of the fractional charge* of their constituents. Combining information about the trion spectrum and its interaction with the 2DEG, the QX recombination allows for an *indirect optical probe of the IQL.*

For spin-polarized systems, we elucidate the earlier theory[13,14] by identifying the "dressed exciton" with $\mathcal{X}$, its suppressed dispersion with the $\mathcal{X}^{-1/3}$-QH pseudopotential of interaction among two Laughlin charge quanta, and the "magnetoroton-assisted emission" with the $\mathcal{X}^{-1/3}$ recombina-

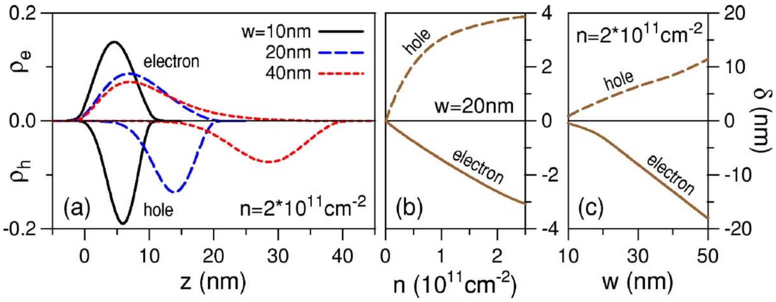

FIG. 1. (Color online) (a) Lowest-subband electron and heavy-hole charge-density profiles in the normal direction $\rho(z)$ for one-sided doped GaAs quantum wells. (b), (c) Displacements δ of the density maxima from the center of the quantum well as a function of electron concentration n and well width w.

tion. The PL discontinuity proposed here due to charged QXs is a different effect, requiring no thermal activation and no charge separation.

In unpolarized systems, we find a spin-flip $\mathcal{X}$ whose steep dispersion prevents charging and removes the PL discontinuity. Competition between X_s^- and X_t^- in realistic wells *predicts the dependence of PL anomalies on* w.

We use exact numerical diagonalization for $N \leq 10$ electrons and one valence hole on a Haldane sphere[18] (with radius R, magnetic monopole strength $2Q=4\pi R^2 Be/hc$, and magnetic length $\lambda=R/\sqrt{Q}$). The second-quantization Hamiltonian reads

$$H=\sum_i c_i^\dagger c_i U_i + \sum_{ijkl} c_i^\dagger c_j^\dagger c_k c_l V_{ijkl}. \quad (1)$$

Here, $c_i^\dagger$ and c_i are operators creating and annihilating an electron in the conduction band or a hole in the valence band, in the state labeled by a composite index i containing all relevant single-particle quantum numbers (band, subband, and LL indices, angular momentum, and spin). The single-particle energies U are counted from the ground states in conduction and valence bands, respectively. The Coulomb interaction matrix elements V were integrated in 3D by taking the actual electron and hole subband wave functions $\phi(z)$ calculated self-consistently[26] for $w=10$ and 20 nm GaAs quantum wells, doped on one side to $n=2\times 10^{11}$ cm^{-2} (yielding $\nu=\frac{1}{3}$ at $B=25$ T, the values used throughout the text). The diagonalization was carried out in configuration-interaction basis, $|i_1,\ldots,i_N;i_h\rangle = c_{i_1}^\dagger \ldots c_{i_N}^\dagger c_{i_h}^\dagger |\text{vac}\rangle$, where indices $i_1\cdots i_N$ denote the occupied electron states, and i_h describes the hole. Finite size and surface curvature errors were minimized by extrapolation to the $\lambda/R\to 0$ limit. The combination of closed geometry, used as an alternative to periodic boundary conditions for modeling in-plane dynamics, with exact treatment of the single-particle motion in the normal direction allowed for quantitative estimates of binding energies characterizing extended experimental systems.

We begin with the calculation of X^- Coulomb binding energies Δ using $\phi(z)$, i.e., in the mean normal electric field due to a doping layer, but ignoring in-plane X^--IQL coupling. We included five LLs and two ϕ-subbands for both e and (heavy) h. The lowest-subband e and h density profiles for $w=10$, 20, and 40 nm are plotted in Fig. 1(a). The effect of charge separation in wider wells is evident. The shifts of the density maxima as a function of n and w are shown in Figs. 1(b) and 1(c). For the cyclotron energies ω_c (at $B=25$ T; after experiment[27]) and intersubband gaps Ω_s (from own calculations) we took $\omega_{ce}=44.5$ meV, $\omega_{ch}=7.7$ meV, $\Omega_{se}=29.6$ meV; $\Omega_{sh}=10.0$ meV (for $w=20$ nm), and $\omega_{ce}=44.5$ meV, $\omega_{ch}=8.1$ meV, $\Omega_{se}=89.8$ meV; $\Omega_{sh}=24.5$ meV (for $w=10$ nm). The valence subband mixing was neglected. The result for $w=10$ nm is $\Delta_s=2.3$ meV and $\Delta_t=1.5$ meV, in qualitative agreement with Refs. 23 and 24, which also predicted the X_s^- ground state (GS). For $w=20$ nm, neither symmetric-well nor lowest-subband approximation works well (e.g., the latter exaggerates charge separation in X/X^- which mostly affects the X_s^- and predicts its breakup at $B\geq 22$ T). Our best estimates are $\Delta_s=1.5$ meV and $\Delta_t=1.2$ meV. They are rather sensitive to the parameters, making prediction of the X^- GS in real samples difficult and somewhat pointless. However, we expect that the X_t^-s, additionally favored by the Zeeman energy, could at least coexist with the X_s^-s at finite temperatures.

Let us immerse a trion (either X_s^- or X_t^-, whichever state occurs at given w, n, and B) in an IQL. Effective e-X^- pseudopotentials are similar[24] to the e-e one.[28] In the lowest LL, this causes similar e-e and e-X^- correlations, described in a generalized two-component[25] CF picture[19] by attachment of $2p$ flux quanta to each e and X^-. At Laughlin/Jain fillings $\nu_{\text{IQL}}=s/(2ps+1)$, electrons converted to CF$_e$s fill the lowest s LLs in an effective magnetic field $B^*=B-2pn(hc/e)=B/(2ps+1)$. At $\nu\neq\nu_{\text{IQL}}$, QEs in the $(s+1)$st or QHs in the sth CF$_e$-LL occur, carrying effective charge $\varepsilon=\pm e/(2ps+1)$. We find that, similarly, an X^- which is Laughlin-correlated with surrounding electrons can be converted to a CF$_{X^-}$ with charge $\mathcal{Q}=-\varepsilon$.

This value can be obtained, e.g., by noting that when an X^- recombines, it leaves behind an (indistinguishable) electron which becomes a CF$_e$ that either fills a QH in the sth CF$_e$-LL or it appears as an additional QE in the $(s+1)$st CF$_e$-LL. More importantly, partial screening of the trion's charge is independent of either the particular X^- state or the filling factor, as long as correlations are described by the CF model. The same value $\mathcal{Q}=-\varepsilon$ results for any other distinguishable charge $-e$ immersed in an IQL, if it induces Laughlin correlations around itself (e.g., an impurity[30] or a reversed-spin electron).

A trion coupled to an IQL and carrying reduced charge is a many-body excitation. To distinguish it from an isolated $2e+h$ state, we call it a charged *quasiexciton* (QX) and denote it by $\mathcal{X}^-\equiv\mathcal{X}^{-\varepsilon}$. Being negatively charged, an $\mathcal{X}^-$ interacts with IQL QPs. At $\nu<\nu_{\text{IQL}}$, the $\mathcal{X}^-$ binds to a QH to become a neutral $\mathcal{X}^-$QH$=\mathcal{X}$, with a binding energy called Δ^0. Depending on sample parameters and spin of the trion, $\mathcal{X}$

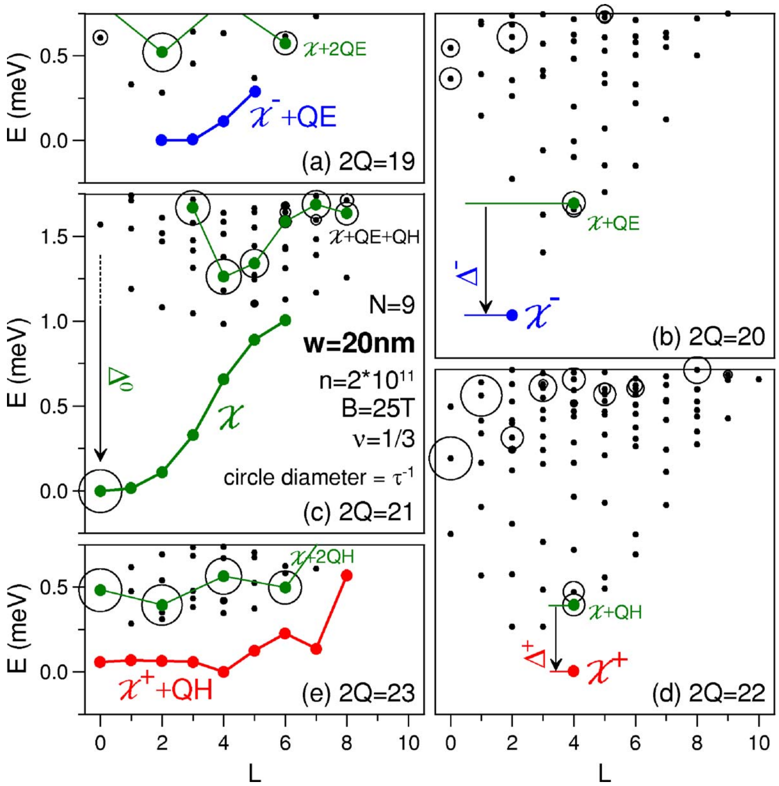

FIG. 2. (Color online) Excitation energy spectra (energy E as a function of total angular momentum L) of $9e+h$ systems on a sphere, with up to two QEs or QHs in Laughlin $\nu=\frac{1}{3}$ IQL. Oscillator strengths τ^{-1} are indicated by open circles.

may bind an additional QH to form a positively charged $\mathcal{X}^{-}\mathrm{QH}_2=\mathcal{X}^{+}$, with binding energy Δ^{+}. At $\nu>\nu_{\mathrm{IQL}}$, the $\mathcal{X}^{+}$ attracts and annihilates a QE: $\mathcal{X}^{+}+\mathrm{QE}\rightarrow\mathcal{X}$; this process releases energy $\Delta_{\mathrm{IQL}}-\Delta^{+}$ (where $\Delta_{\mathrm{IQL}}=\mathcal{E}_{\mathrm{QE}}+\mathcal{E}_{\mathrm{QH}}$ is the IQL gap). The $\mathcal{X}$ may annihilate another QE: $\mathcal{X}+\mathrm{QE}\rightarrow\mathcal{X}^{-}$, with energy gain

$$\Delta^{-}=\Delta_{\mathrm{IQL}}-\Delta^{0} \tag{2}$$

that can be interpreted as $\mathcal{X}^{-}$ binding energy.

The $\mathcal{X}$ and $\mathcal{X}^{\pm}$ are different states in which a hole can exist in an IQL. If $\Delta^{\pm}>0$, then depending on ν, either $\mathcal{X}^{-}$ or $\mathcal{X}^{+}$ is the most strongly bound state. If $\Delta^{-}\neq\Delta^{+}$, the PL spectrum will be discontinuous at ν_{IQL}. For long-lived $\mathcal{X}^{\pm}$ (made of a dark X_t^{-}), recombination of the $\mathcal{X}$ is also possible, especially at $\nu\approx\nu_{\mathrm{IQL}}$ (within a Hall plateau), when QP localization impedes $\mathcal{X}^{\pm}$ formation.

The QX's resemble normal excitons in n- or p-type systems, except that the concentration of their constituent QPs can be varied (in the same sample) by a magnetic field. Also, their kinetics ($\mathcal{X}\leftrightarrow\mathcal{X}^{\pm}$) is more complicated because of the involved QE-QH annihilation.

We have tested the QX idea numerically for Laughlin $\nu=\frac{1}{3}$ IQL. First, we calculated spin-polarized $Ne+h$ energy spectra for $w=20$ nm, in search of the QX$_t$s. The X_t^{-} has 94% squared projection onto the lowest LL, so we ignored LL mixing in the $Ne+h$ calculation (direct tests confirmed that it is negligible). The low-lying states in Fig. 2 are understood using the CF picture[19,25] and addition rules for angular momentum. On a sphere, the CF transformation introduces an effective monopole strength $2Q^{*}=2Q-2(K-1)$, where $K=N-1$ is the total number of free electrons and X^{-}s. The angular momenta of constituent QPs are $l_{\mathrm{QH}}=Q^{*}$, $l_{\mathrm{QE}}=Q^{*}+1$, and $l_{\mathcal{X}^{-}}=Q^{*}-1$. The $\mathcal{X}^{-}$ is a dark GS in (b) at $L=l_{\mathcal{X}^{-}}=2$, and $\mathcal{X}^{+}$ is found in (d) at $L=l_{\mathcal{X}^{+}}=|(2l_{\mathrm{QH}}-1)-l_{\mathcal{X}^{-}}|=4$. Bands of $\mathcal{X}^{-}$-QE and $\mathcal{X}^{+}$-QH pairs are marked in (a) and (e). In (c) the radiative $L=0$ GS is a multiplicative state, opening a $\mathcal{X}=\mathcal{X}^{-}$-QH band,[16] earlier called a "dressed exciton" and identified[13,14] as responsible for the doublet structure in PL.

The continuous $\mathcal{X}$ dispersion shown in Fig. 3(a) results[13,14] from the in-plane dipole moment being proportional to the wave vector $k=l/R$. Here we find why it is suppressed (compared to X): because of the reduced charge of the $\mathcal{X}$s constituents, $\mathcal{X}_t^{-}$ and QH. In the absence of an IQL

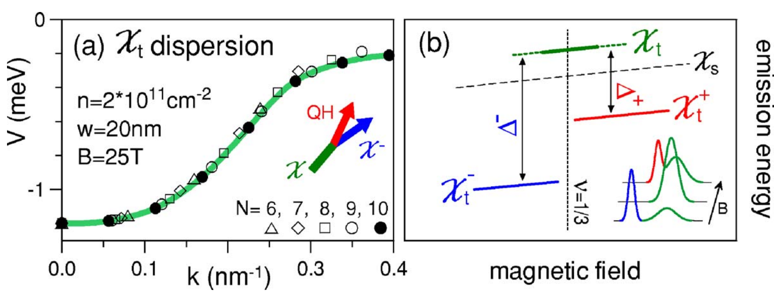

FIG. 3. (Color online) (a) Dispersion of neutral quasiexciton $\mathcal{X}_t$ in Laughlin $\nu=\frac{1}{3}$ IQL; $\mathcal{X}_t$ splits into $\mathcal{X}_t^{-}$ and QH at $k>0$. (b) Schematic of PL discontinuity due to $\mathcal{X}_t^{\pm}$ emission.

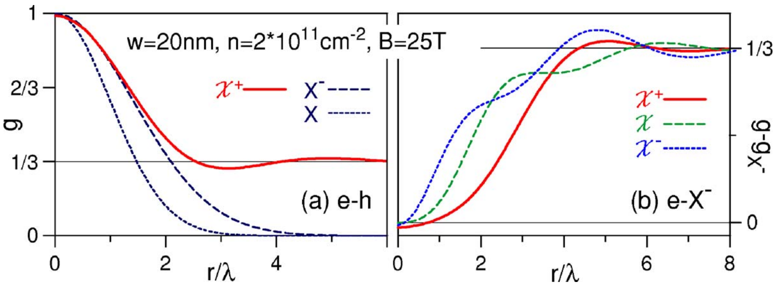

FIG. 4. (Color online) (a) The e-h pair-distribution functions (PDFs) of quasiexciton $\mathcal{X}_t^+$ and isolated X_t^- and X, normalized to measure the local filling factor. (b) The e-X^- PDFs for different QXs; curve for $\mathcal{X}^+$ resembles e-e PDF of Laughlin liquid; shoulders for $\mathcal{X}$ and $\mathcal{X}^+$ reflect additional charge quanta pushed onto the hole.

(in an empty well), an isolated, stationary ($k=0$) X has no dipole moment ($d=0$). When the X moves (i.e., has $k>0$), Lorentz force acts on its constituents inducing charge separation ($d\propto k$), and the X splits into e and h, each carrying a charge $\pm e$. In an IQL, charge quantum is reduced to ε. This has no consequence at $k=0$, and the $\mathcal{X}$ is equivalent to an X decoupled from the remaining electrons. But a moving $\mathcal{X}$ acquires dipole moment in a different way than X, by splitting into $\mathcal{X}^-$ and QH, each carrying only one small quantum $\pm\varepsilon$. Indeed, the $\mathcal{X}$ and X dispersions become similar when energy and length scales are rescaled in account of the $e\to\varepsilon$ charge reduction. Note that we also explain the emission from $\mathcal{X}$ at $k\lambda\sim1.5$, proposed[13,14] for the lower peak in PL, as the $\mathcal{X}^-\to$QE recombination assisted by QH scattering. However, a small dV/dk and a large τ^{-1} at $k\lambda\sim1.5$ needed for this emission requires significant well widths, $w>20$ nm.

By identifying the multiplicative states containing an $\mathcal{X}$ with $k=0$, one can estimate $\Delta^{\pm/0}$ as marked in Figs. 2(b)–2(d). More accurate values were obtained by comparing the appropriate energies identified in the spectra obtained at different values of $2Q$, in which either $\mathcal{X}^\pm$, $\mathcal{X}$, or QP is alone in the IQL, followed by extrapolation to $N\to\infty$. Our best estimates, whose reasonable accuracy of under 0.05 meV is confirmed by Eq. (2), are $\mathcal{E}_{\rm QH}=0.73$ meV, $\mathcal{E}_{\rm QE}=1.05$ meV, $\Delta^0=1.20$ meV, $\Delta^-=0.52$ meV, and $\Delta^+=0.27$ meV. Depending on $\mathcal{X}^0/\mathcal{X}^\pm$ kinetics, either $\Delta^+\neq\Delta^-$ or $\Delta^0\neq\Delta^\pm$ asymmetry will make PL energy jump at $\nu=\frac{1}{3}$, as sketched in Fig. 3(b). Similar behavior has been observed.[5,8]

The $\mathcal{X}^\pm$ discontinuity is different from that due to anyon excitons[15,16] anticipated in much wider wells (e.g., for $w\geq40$ nm at $n=2\times10^{11}$ cm^{-2}). The two effects can be distinguished by different magnitude ($\sim\Delta_{\rm IQL}$ vs $\Delta^\pm$) and opposite direction of the jump of emission energy when passing through $\nu=\frac{1}{3}$. In the present case, the small ratio of $\mathcal{X}^\pm$ and $X^\pm$ binding energies is the *signature of the fractional charge* of the IQL excitations—directly observable as splittings in PL.

The QX's are defined through a sequence of gedanken processes: (i) trion binding: $2e+h\to X^-$, (ii) Laughlin correlation: $X^-\to\mathcal{X}^-$, (iii) QH capture: $\mathcal{X}^-\to\mathcal{X}/\mathcal{X}^+$. Hence, $\mathcal{X}$ and $\mathcal{X}^\pm$ are in fact the same X^-, only differently separated from the surrounding electrons.

This is evident in the e-h pair-distribution functions $g(r)$ shown in Fig. 4(a) and normalized so as to measure electron concentration near the hole in the units of ν. The $\mathcal{X}^+$ curve calculated for $N=10$ is compared with $g_{X^-}(r)=\exp(-r^2/4)$ which accurately describes an X_t^-. The similarity at short range proves that the $\mathcal{X}^+$ is an X^- well separated from the 2DEG. In Fig. 4(b) we plotted $\delta g=g-g_{X^-}$ which measures the e-X^- correlations in different QX states. Clearly, $\delta g_{\mathcal{X}^+}$ resembles the e-e pair-distribution function of a Laughlin $\nu=\frac{1}{3}$ liquid, while shoulders in $\delta g_{\mathcal{X}}$ and $\delta g_{\mathcal{X}^-}$ reflect additional charge quanta pushed onto the hole in $\mathcal{X}$ and $\mathcal{X}^-$. Let us add that integration of $\left[g(r)-\frac{1}{3}\right]$ directly confirms fractional electron charge of $-\frac{4}{3}e$, $-e$, and $-\frac{2}{3}e$ bound to the hole in the $\mathcal{X}^-$, $\mathcal{X}$, and $\mathcal{X}^+$ states.

The accuracy of the lowest-LL approximation is demonstrated in Fig. 5, in which we compare the excitation energy spectra similar to Figs. 2(a) and 2(d), but calculated for the $7e+h$ systems, with and without inclusion of one higher e and h LL. Evidently, neither the $\mathcal{X}$ dispersion nor the $\mathcal{X}^+$ binding energy appear sensitive to the LL mixing. This is in contrast to the behavior of X or X^-, and the difference obviously reflects weaker interactions among the fractional QX constituents (compared to the same cyclotron energy scale).

Let us now turn to spin-unpolarized spectra, in search of QXs formed from the X_s^-. Its binding depends on LL mixing, so we used the following approximation. In the calculation of Coulomb matrix elements, the highest Haldane e-e pseudopotential, V_0, was reduced by 10%. This only affects interactions within the trion, induces binding of X_s^- *in the lowest LL*, and draws its energy below X_t^-. The X_s^- constructed in this way has an 85% overlap with the actual X_s^- calculated including LL mixing. Importantly, it retains correct pair correlations g_{eh} and g_{ee}, which determine its cou-

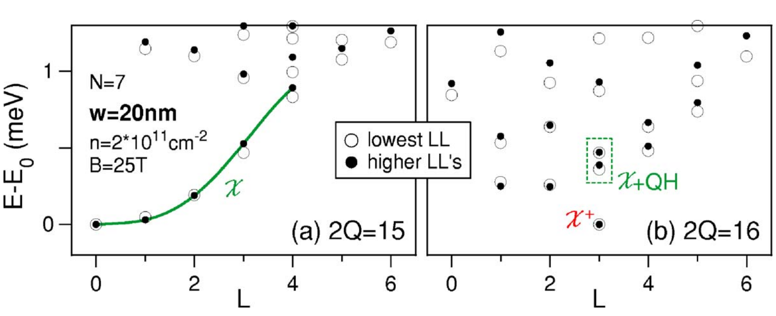

FIG. 5. (Color online) Excitation spectra similar to Fig. 2, but for the $7e+h$ systems with and without Landau level mixing.

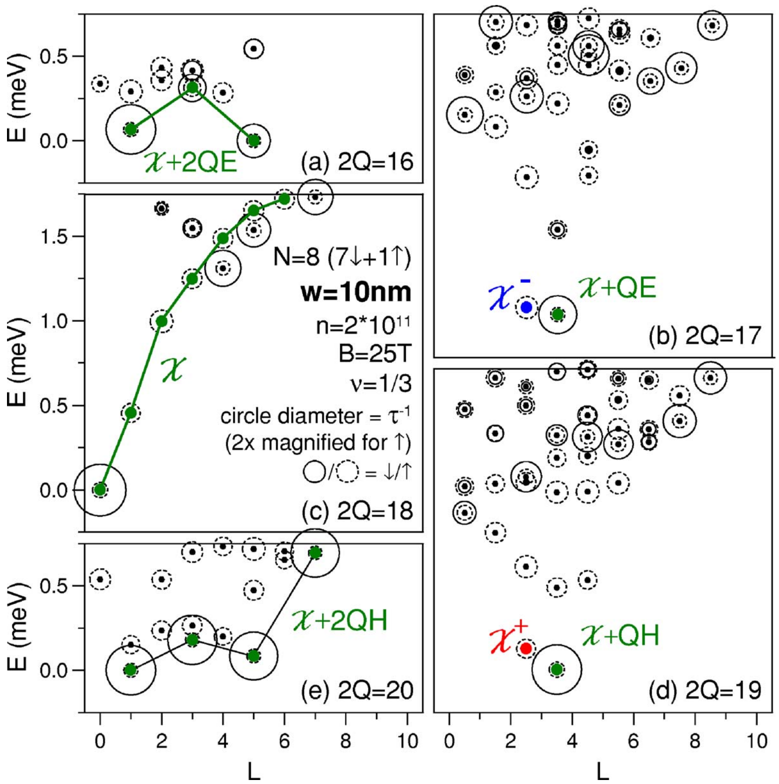

FIG. 6. (Color online) Excitation spectra similar to Fig. 2, but for the $8e+h$ systems with one reversed-spin electron and $w=10$ nm. τ^{-1} is separate for spin-↓ and ↑ recombination.

pling to the 2DEG. Therefore, the $Ne+h$ spectra calculated with one reversed-spin electron and using the reduced V_0 capture the essential difference in QX dynamics caused by the replacement of X_t^- by X_s^- in the $w=10$ nm well.

From the analysis of τ^{-1} and L we found $\mathcal{X}_s$ and $\mathcal{X}_s^{\pm}$ in the spectra in Fig. 6. In contrast with Fig. 2, charged QX_ss are the excited states at $2Q=17$ and 19. Also at $2Q=16$ and 20, the multiplicative states with an $\mathcal{X}_s$ and two QPs lie below the $\mathcal{X}_s^-$-QE and $\mathcal{X}_s^+$-QH pairs.

This opposite behavior results from the X_s^- having different charge distribution than the X_t^-. It has little effect on its Laughlin correlation with the electrons, but affects its interaction with the QPs. Indeed, the $\mathcal{X}_s$ dispersion in Fig. 6(c) indicates stronger $\mathcal{X}_s^-$-QH attraction.

We compare $\Delta^0 \sim 2$ meV with $\Delta_{\rm IQL}=2.02$ meV using Eq. (2) to find that Δ^- is very small or even negative, as in Fig. 6(b). Hence, even in the absence of free QHs, the $\mathcal{X}_s^-$ is unstable toward creation of a QE-QH pair. Similarly, negative Δ^+ in Fig. 6(d) implies instability of the $\mathcal{X}_s^+$. As a result, the neutral $\mathcal{X}_s$ is the most strongly bound state regardless of the presence of QEs or QHs.

This may add a continuous peak for the $w=20$ nm well [see Fig. 3(b)], but precludes PL discontinuity in narrow wells with a strong X_s^- GS. The $\mathcal{X}_s$ peak splits into a $\sigma_\pm$ doublet due to spin-↓ and ↑ recombination involving either QEs or "reversed-spin" QE_Rs,[29] but temperature-activated emission at $k>0$ is not expected.

The QX idea can be extended to other IQLs (e.g., $\nu=\frac{2}{3}$ or $\frac{2}{5}$). However, different behavior of QX_ts and QX_ss at $\nu=\frac{1}{3}$ is an example that PL discontinuity is not guaranteed. Via Eq. (2), it is governed by sample and ν-dependent $\Delta_{\rm IQL}$ and Δ^0 which must be recalculated.

In summary, we have studied anomalies in the PL of the IQLs in the regime of small charge separation. The emission spectrum is due to recombination of QXs formed from trions immersed in a 2DEG with Laughlin correlations. In spin-polarized systems, the neutral QX is equivalent to a nearly decoupled exciton at $k=0$, and its suppressed dispersion results from reduced charge of the constituents. The positive and negative spin-polarized QXs have fractional charge of one IQL QP. A spin-flip QX formed from a singlet trion was also found, with a steeper dispersion that prevents its charging by the QPs. Featureless PL for $w=10$ nm and anomalies predicted for $w=20$ nm agree qualitatively with experiments.

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