PHYSICAL REVIEW B **73**, 241302(R) (2006)

Quantum Hall skyrmions in a hole gas with a large spin gap

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(Received 10 January 2006; revised manuscript received 21 April 2006; published 7 June 2006)

Photoluminescence excitation spectra of a two-dimensional valence hole gas with large spin gap and strong disorder are studied in high magnetic fields. The characteristic field dependence of polarized emission associated with transitions from the lowest hole Landau level (LL) is the signature of quantum Hall ferromagnetism and small skyrmions around the LL filling factor $\nu=1$. This interpretation is supported by realistic numerical calculations of the skyrmion binding energy.

DOI: 10.1103/PhysRevB.73.241302 PACS number(s): 73.20.Mf, 73.21.Fg, 78.20.Ls, 78.67.De

Exchange interactions play a dominant role in determination of the ground state of two-dimensional electron systems (2DES's) in high magnetic fields B.[1] Even in the absence of Zeeman spin splitting $E_Z=g\mu_B B$ (g being the Landé factor), these "quantum Hall"[2] systems favor ferromagnetic order at the Landau level (LL) filling factor $\nu=1$.[3] However, addition (or removal) of even a single electron to (from) such a $\nu=1$ ferromagnet may result in complete depolarization[4,5] which occurs through formation of a "skyrmion"[6] excitation carrying a unit charge $\mp e$ and massive spin. Remarkably, this behavior is exactly opposite to the Nagaoka ferromagnetism[7] in a 2D Hubbard lattice (also induced by infinitesimal deviation from the exact half filling of a degenerate single-particle shell). Though formation of skyrmions in a 2DES has been reported in several experiments, the effect is believed to be relatively subtle, with observation requiring special experimental conditions: superior quality structures and vanishing Zeeman energy.

For $E_Z>0$ but still small compared to the characteristic Coulomb exchange energy $E_C=e^2/\lambda$ (where $\lambda=\sqrt{\hbar c/eB}$ is the magnetic length), skyrmions with finite spin ("topological size") K are predicted.[5,8] They cause continuous depolarization as a function of ν or B, at a rate of K spin flips per unit charge $-e$ or magnetic flux quantum $\phi_0=hc/e$ which is added to (or removed from) the $\nu=1$ ferromagnet. The occurrence of skyrmions instead of bare reversed-spin electrons and spin holes as the lowest charge excitations impacts both transport (by changing the activation gap and the carriers) and optical properties of the 2DES, allowing for various indirect experimental probes of these excitations.

In electron systems with small "Zeeman-to-Coulomb" ratio $\tilde{g}=E_Z/E_C$, the skyrmions were originally observed in nuclear magnetic resonance (NMR),[9] transport,[10] and optical[11] experiments, but also using such other probes as heat capacity.[12] Recently, they were found[13] in magnetoreflectivity spectra of a valence hole gas, in a unique quantum well in which the complex valence-band structure caused a near spin degeneracy of the lowest ($n=0$) hole LL at the value of B corresponding to $\nu=1$. However, the stability of skyrmions in systems with significant spin gap is still an open question. The condition for the smallest ($K=1$) skyrmion in an "ideal" 2DES is $\tilde{g}\leqslant 0.054$,[5] though even more severe (~ 0.02) estimates can be found in the literature. Indeed, the analogy between quantum Hall skyrmions and interband excitonic complexes[14] suggests that realistic effects such as finite width or LL mixing[15] and disorder may be important.

In this Rapid Communication experimental observation (see Fig. 1) combined with realistic theoretical calculation is presented, indicating that, in contrast to earlier expectations, spin depolarization due to emergence of skyrmions is a robust phenomenon characteristic of 2DES's even in the absence of perfect translational symmetry or spin degeneracy. We report low-temperature polarization-resolved photoluminescence excitation (PLE) studies of a narrow quantum well containing valence holes with a large spin gap and strong disorder. By detecting interband transition from the majority-spin lowest hole LL in the close vicinity of $\nu=1$, but not precisely at this filling, we find clear experimental evidence for quantum Hall ferromagnetism and "valence skyrmions" in this disordered system with relatively large E_Z. We also report detailed numerical calculations that correct the earlier K vs $\tilde{g}$ dependence and confirm the $K=2$ skyrmion size in our sample.

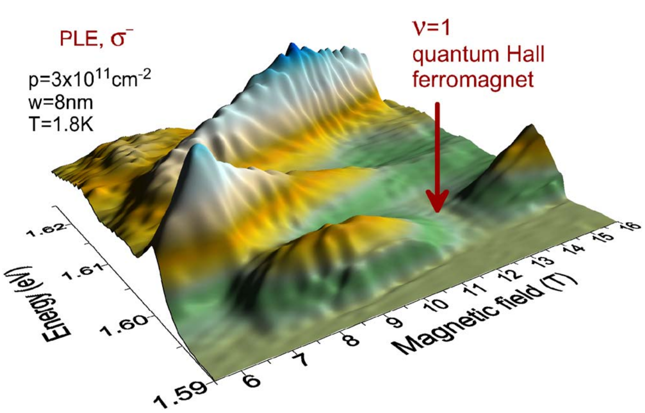

FIG. 1. (Color online) Polarized PLE spectrum of the 2D valence hole gas as a function of the magnetic field. Red (dark gray) arrow points to the forbidden absorption region attributed to quantum Hall ferromagnetism. Emergence of intensity at lower and higher magnetic fields is the signature of skyrmions.

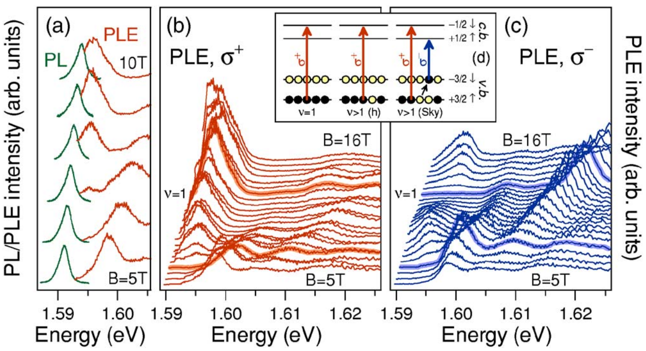

FIG. 2. (Color online) Comparison of σ^{+}-polarized PL and PLE spectra at several magnetic fields (a), and quasicontinuous (with step $\Delta B = 0.5$ T) field evolution of PLE spectra in both polarizations (b), (c). Thick lines, $\nu = 1$ and 2. Inset: Zeeman energy diagram and the allowed transitions.

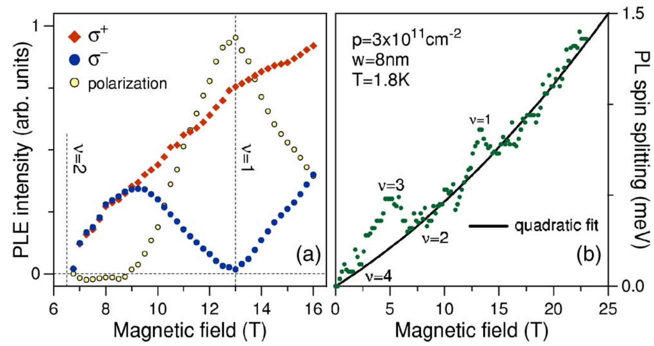

FIG. 3. (Color online) Intensity I of lowest $\sigma^{+/-}$ PLE lines associated with lowest spin-$\uparrow/\downarrow$ hole LL's (a) and spin splitting of PL energy (b) plotted as a function of magnetic field. PLE polarization in (a) is defined as $(I^{+}-I^{-})(I^{+}+I^{-})^{-1}$.

We studied a $w = 8$ nm $GaAs/Al_{0.3}Ga_{0.7}As$ quantum well grown by molecular beam epitaxy on a (001) semi-insulating GaAs substrate and modulation C-doped in the barrier on one side. The sheet concentration and mobility of the holes measured at low temperature were $p = 3\times10^{11}$ cm^{-2} and $\mu = 3.3\times10^{3}$ cm^{2}/V s. The measurements were carried out at low temperatures ($T = 1.8$ K), in magnetic field $B \leqslant 23$ T. The spectra were recorded in Faraday configuration, for different σ^{+} and σ^{-} helicities of both the excited and emitted light.

In PL for both $\sigma^{\pm}$ polarizations, only one relatively broad line due to the recombination of photocreated conduction electrons with the 2D holes was observed in the entire range of B. The PL spectra in the σ^{+} polarization are shifted to higher energies compared to σ^{-}, and this displacement increases gradually with increasing B.

Earlier successful techniques for studying skyrmions involved investigation of the spin polarization via NMR (Ref. 9) and interband absorption or reflectance experiments.[11,13] In our studies we used polarization-resolved PLE (polarized emission intensity at a fixed energy ω_0 recorded as a function of excitation energy $\omega > \omega_0$), which is an indirect measure of the absorption of circularly polarized light ($\sigma^{\pm}$). Simple spin and polarization selection rules [cf. Fig. 2(d)] resulting from angular momentum conservation make polarized PLE a probe of interband excitation of polarized electrons. Due to the spin selectivity combined with Pauli phase space blocking, polarized PLE is therefore sensitive to the occupation of LL's by the electrons with a given spin, i.e., to the spin polarization of the electron (or hole) system.

We used a charge-coupled device camera as a detector, so that the whole PL spectra could be recorded together with a laser line for each applied laser excitation wave length. The advantage of this technique compared to typical PLE measurements with detection fixed at one energy is the possibility to study evolution of a whole PL spectrum. A special optical-axis geometry was constructed to reduce the intensity of the laser line. For sufficiently high precision, experiments were performed with a small magnetic field step $\Delta B = 0.25$ T.

In Fig. 2(a) the PL and PLE spectra for σ^{+} polarization are displayed for several magnetic fields. The PLE spectra were obtained with detection set on the low-energy side of the PL emission (at $\frac{1}{3}$ of the maximum intensity). A detailed analysis showed that the shape and energy position of the PLE lines did not depend on the detection energy in the PL spectra. At low magnetic fields, PL can only be excited from the excited LL's of the 2D holes, while the excitation from the lowest LL appears in the PLE spectra at $B \geqslant 6$ T.

The field evolution of the PLE spectra in σ^{+} and σ^{-} polarization is compared in Figs. 2(b) and 2(c), revealing completely different behavior. The most noticeable distinction is the field dependence of the lowest-energy PLE signals, corresponding to the interband optical excitation from the highest LL in the valence band (lowest heavy-hole LL) to the lowest electron LL (i.e., to the $n = 0 \rightarrow 0$ transitions). The "$0 \rightarrow 0$" line appears in the σ^{+} PLE spectra at $B = 6.5$ T and then gradually gains intensity with the increase of magnetic field. The same feature in the σ^{-} PLE also appears around $B = 6.5$ T, but it gains intensity with increasing field only up to $B = 9$ T. When the field grows further, the σ^{-} line first gradually weakens to disappear completely at $B = 13$ T [cf. red (dark gray) arrow in Fig. 1], and then reappears and regains intensity.

The comparison of intensities of these lowest-energy σ^{+} and σ^{-} PLE lines in a growing magnetic field is best visible in Fig. 3(a). These curves are consistent with the skyrmion picture. From the definition of the filling factor $\nu = 2\pi p \lambda^{2}$, the complete filling of the spin-degenerate lowest hole LL (corresponding to $\nu = 2$) occurs at $B \approx 6.5$ T. At this filling, the $0 \rightarrow 0$ interband transition is not possible for either spin for the lack of electrons, and the intensity of the lowest PLE line vanishes. The complete spin polarization of the holes is achieved at $B \approx 13$ T corresponding to $\nu = 1$. In this case, the σ^{-} transition is forbidden for the lack of electrons in the $n = 0$ spin-$\downarrow$ hole LL [cf. Fig. 2(d)], while the high σ^{+} intensity reflects complete depletion of the $n = 0$ spin-$\uparrow$ hole LL (and its filling by the electrons).

When B is *decreased* from $\nu = 1$, each consecutive hole forced to reverse its spin due to shrinking LL degeneracy invokes additional K spin flips to become a skyrmion. This not only fills the spin-$\uparrow$ LL with holes, but also (uniquely for the skyrmion scenario) puts electrons in the spin-$\downarrow$ LL, allowing their interband excitation observed in the σ^{-} PLE [cf. Fig. 2(d)]. When B *increases* from $\nu=1$, each consecutive vacancy (electron) in the spin-$\downarrow$ hole LL becomes an (anti)skyrmion, and the additional spin flips give rise to additional enhancement of the σ^- PLE.

It is remarkable that skyrmions occur in our sample despite strong disorder (mean free path shorter by 2–3 orders of magnitude than in high-quality electron systems). This is explained by the local nature of the PL or PLE probe, sensitive to the presence of skyrmions regardless of their localization (in contrast to a transport experiment requiring mobile skyrmions acting as charge carriers).

With increasing ν from 1 to 2, the number of skyrmions grows. Being charged, they begin to interact with one another similarly to electrons in a partially filled polarized LL, and possibly form a Laughlin-correlated liquid proposed earlier for trions.[16] In the dilute regime (close to $\nu=1$), the size K of each skyrmion is truly a single-skyrmion property. However, above a certain ν, this size must shrink to $K=1$ to accommodate all skyrmions in the limited space of a LL (regardless of the equilibrium size of an isolated skyrmion at the same $\tilde{g}$). At some larger critical ν, there are enough (minimum-size) $K=1$ skyrmions to cause complete spin depolarization.

This indeed seems to occur in our experiment: Fig. 3(a) shows the merger of σ^+ and σ^- intensities or, equivalently, the vanishing of PLE polarization below $B=9$ T. This magnetic field corresponds to $\nu\approx1.4$ (a fairly close value to $\nu=\frac{4}{3}$ obtained from a simple single-electron phase space filling argument). Clearly, the field increase from $\nu=2$ beyond $\nu=1$ drives the hole system through a sequence of phases with different polarizations: (i) a simple "filled LL" $\nu=2$ paramagnet, (ii) a correlated paramagnet at $2>\nu\geq1.4$, (iii) a high-density liquid of small skyrmions at $1.4>\nu>1^+$, (iv) a dilute gas of larger (equilibrium-size) skyrmions at $\nu=1^+$, (v) a quantum Hall $\nu=1$ ferromagnet, and (vi) a dilute gas of equilibrium-size antiskyrmions at $\nu=1^-$, etc.

Having established the stability of skyrmions (at least in the dilute regime of $\nu\approx1$), let us now estimate the hole Zeeman spin gap E_Z. From the comparison of σ^+ and σ^- PL spectra we read the spin splitting of the electron-hole pair recombination energy Δ_{PL}, plotted in Fig. 3(b) as a function of B. It contains the (known) electron and (unknown for arbitrary w and B) hole Zeeman gaps, as well as an additional splitting due to the spin-asymmetric exchange interaction of the recombining hole with the hole gas of finite spin polarization. Although the hole g factor is expected to show some field dependence,[18] both Zeeman terms are rather regular functions of B, in contrast to the exchange term which follows the oscillatory spin polarization of the holes.[17] However, the latter term vanishes whenever the holes are paramagnetic [e.g., at $6.5\leq B\leq9$ T—see Fig. 3(a), but also in the vicinity of $\nu=4$] and weakens in the dilute regime (at small ν). Indeed, strong positive deviation from an otherwise nearly monotonic field dependence occurs in Fig. 3(b) only near $\nu=1$ and 3 (high hole density combined with strong spin polarization). Except for those two regions, Δ_{PL} can be well approximated by the quadratic curve drawn with the solid line. Using this fit and taking $g_e=-0.15$ for the electron Landé factor,[18] we obtain for the holes $E_Z=0.55$ meV and $\tilde{g}=0.034$ at $B=13$ T.

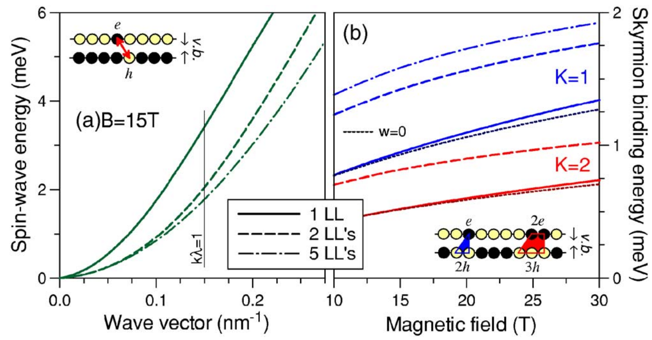

FIG. 4. (Color online) Valence spin-wave energy dispersion (a) and magnetic-field dependence of Coulomb binding energies of small skyrmions (b) calculated with and without inclusion of higher LL's ($w=0$ is zero-width approximation). Schematic LL diagrams similar to Fig. 2(d) define long spin waves and small skyrmions in terms of their e and h constituents.

What is the skyrmion size K at $\nu\sim1$ for $\tilde{g}=0.034$? The smallest ($K=1$) valence skyrmion is formed when a reversed-spin valence hole h induces and binds a spin wave (SW), which at small wave vector k is equivalent to an e-h pair (e denotes a vacancy in the spin-$\downarrow$ valence level; see schematic LL diagrams in Fig. 4). This happens when the h-SW binding exceeds the SW creation energy, and the stability condition can be expressed in terms of the $K=1$ "skyrmion binding energy" $\mathcal{E}_1=V(he)-V(h_2e)$, which must be larger than E_Z. Here, $V(\cdots)$ means the Coulomb energy of a given bound state. Analogously, skyrmions with $K>1$ are formed from the smaller ones when $\mathcal{E}_K=V(he)+V(h_Ke_{K-1})-V(h_{K+1}e_K)>E_Z$.

The binding of a K skyrmion depends on the attraction between the charge $+e$ (of the "$K-1$" skyrmion) and the dipole moment d (of the neutral SW). Therefore, while they involve nontrivial exchange and correlation effects among all $2K+1$ of the h and e constituents, the magnitudes of $\mathcal{E}_K$ generally decrease with increasing K (radius of the charged particle) and, since d is proportional to k, also with increasing SW energy dispersion $V(k)$ at k of the order of the inverse skyrmion radius, λ^{-1}.

We have carried out realistic exact-diagonalization calculations of $\mathcal{E}_K$ for small ($K\leq2$) skyrmions in a $w=8$ nm GaAs well. The model Hamiltonians included h-h, e-e, and e-h interactions and the LL mixing. The heavy-hole wave functions used to calculate Coulomb matrix elements corresponded to the lowest subband in the normal direction, and the cyclotron gap was taken after Ref. 19. The calculations were done on a Haldane sphere[20] for several values of the LL degeneracy $\Gamma=2Q+1\leq31$ (where $2Q$ is the total magnetic flux piercing the sphere, $R^2=Q\lambda^2$, and R is the sphere radius), and the results were extrapolated to the $Q=\infty$ limit.

The main result is a significant enhancement of $\mathcal{E}_K$ compared to the best earlier estimates $\mathcal{E}_1=0.0545E_C$ (Ref. 5) and $\mathcal{E}_2=0.03E_C$ (Ref. 14) when the LL mixing is included. It is caused predominantly by a significant reduction of the SW dispersion at $k\lambda<1$ (i.e., at $k<0.15$ nm^{-1}), that is about a two times enhancement of its effective mass. This is clear in Fig. 4(a), showing also that only excitations to the first excited LL are efficient at small k.

The plots of $\mathcal{E}_1$ and $\mathcal{E}_2$ as a function of B are shown in Fig. 4(b). The lowest LL approximation for $w=8$ nm nearly coincides with the earlier prediction for $w=0$.[14] The binding enhancement through scattering to the $n=1$ LL is significant, from $\mathcal{E}_1=0.89$ to 1.35 meV (by 50%) and from $\mathcal{E}_2=0.49$ to 0.78 meV (by 60%) at $B=13$ T. Additional ~ 0.1 meV enhancement may result from inclusion of $n>1$ LL's (cf. dash-dotted line for $K=1$), but an accurate estimate would require a more realistic treatment of the excited hole energy levels. These values, together with $E_Z=0.55$ meV estimated from Fig. 3(b), lead to the prediction of $K=2$ for the skyrmion size at $\nu \sim 1$ in our experiment (this corresponds to about 50 nm diameter, compared to about 25 nm for $K=1$).

In conclusion, in the PLE spectra of the 2D hole gas we found evidence for quantum Hall ferromagnetism and small skyrmions at $\nu \approx 1$. The skyrmions occur despite large Zeeman spin gap E_Z (too large, according to earlier theoretical estimates) and significant disorder (probably causing their localization). Realistic calculations confirm this interpretation, predicting skyrmion size $K=2$.

The authors thank L. Gravier for his input in the initial stage of this work and J. Misiewicz for helpful discussions. The structures used in the experiment have been kindly supplied to us by A. Fisher and K. Ploog. The authors were partially supported by Grants No. 2P03B02424 (Polish MENiS) and No. RITA-CT-2003-505474 (EC).

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