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Photoluminescence of the Incompressible Laughlin Liquid: Excitons, Charged Excitons, and Fractionally Charged Excitons

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The photoluminescence (PL) of a two-dimensional electron gas (2DEG) in a high magnetic field is studied as a function of the filling factor and the separation d between the electron layer and the valence hole. Depending on the magnitude of d relative to the magnetic length λ, two distinct regimes in the response of the 2DEG to the valence hole occur, with different elementary emission processes contributing to the PL spectrum. At $d < \lambda$ ("strong coupling" regime), the hole binds one or two electrons to form an exciton (X) or one of three possible charged exciton (X^-) states, a spin-singlet or one of two spin-triplets. At $d > \lambda$ ("weak coupling" regime), the hole decouples or binds one or two Laughlin quasi-electrons to form fractionally charged excitons (FCXs). The binding energies as well as the emission energies and intensities of all X^- and FCX states are calculated.

1. Introduction

The magneto-optical properties of two-dimensional electron gas (2DEG) systems have been intensively studied experimentally [1–13] and theoretically [14–24]. It is known that in a dilute system confined in a so-called symmetric quantum well (QW), the photoluminescence (PL) spectrum is determined by a charged-exciton complex X^- (bound state of two electrons e and a valence hole h) and its interaction with the remaining electrons. The existence of X^- in bulk semiconductors was predicted by Lampert [25], but only in 2D quantum wells (QWs) does its binding energy turn out sufficiently large [17] for the experimental detection [4–13]. The observation of the X^- stimulated theoretical work [16–24], and it is now established that the only bound X^- state at zero magnetic field is the singlet state (X_s^-) with the total electron spin $J = 0$. Accordingly, the PL spectrum usually shows two peaks, due to the X (neutral exciton) and/or X_s^- recombination, split by the X_s^- binding energy Δ_s.

The situation is more complicated in a magnetic field B. At very high B, the optically active X states decouple from the electrons due to the "hidden symmetry" [14–16] and the X_s^- unbinds. Interestingly, a different X^- state binds in high fields [18]. It is a triplet (X_t^-) with $J = 1$ and finite total angular momentum. It has infinite radiative lifetime τ_t [19] because of (independently) the "hidden" and 2D translational symmetry, and thus will be further called X_{td}^- (d for "dark"). Although both symmetries are broken in experimental systems by mixing of Landau levels (LLs), valence band mixing effects, asymmetry of the QW, and disorder, the X_{td}^- recombination is expected to be weak and disappears at very large B.

The fact that X_{td}^- unbinds at $B = 0$ while X_s^- unbinds at high B implies a singlet–triplet crossing, which in a GaAs QW of width $w = 10$ nm was predicted at $B \approx 30$ T [20]. Surprisingly, the following PL experiment [10] showed no such transition up to

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$B \approx 50$ T. This puzzle was resolved by the identification of another bound state, a radiative excited spin-triplet X_{tb}^{-} (b for "bright") [23]. The emission spectrum from both triplets was eventually measured [13], and agrees well with the theoretical prediction [23].

The fact that the PL spectrum of a 2DEG in a symmetric QW only measures the X and/or X^- emission means that it is not a useful probe for electron correlations. The effect of surrounding 2DEG on the X^- emission is specially weak in dilute systems due to the Laughlin correlations effectively isolating an X^- from the electrons [21–23]. Indeed, the measured PL spectra are remarkably insensitive to the electron density [9, 13], and only at filling factors ν approaching 1/3 does the relative position of PL peaks change with density.

The PL spectra containing more information about the original (Laughlin) electron correlations of the 2DEG are obtained in asymmetrically doped wide QWs. In such structures, the spatial separation d of e and h layers weakens the e–h interaction [15], and the PL spectra show discontinuities [1–3] at the filling factor $\nu = 1/3$ at which Laughlin incompressible liquid state [26] is formed and the fractional quantum Hall (FQH) effect [27] is observed in transport experiments. The reason for these discontinuities is that at d exceeding the magnetic length λ, the effective Coulomb potential of the hole seen by the electrons is too weak and its resolution is too low, and a hole can no longer bind "whole" electrons to form X^- states. Instead, the hole interacts with charge excitations that are already present in the 2DEG and which near $\nu = 1/3$ happen to be a small number of Laughlin quasi-particles (QPs). At $\nu < 1/3$, the hole repels positively charged quasi-holes (QHs) and thus remains in the locally undisturbed incompressible electron state (such state can be called a "decoupled hole" and denoted by h) [38]. The situation is quite different at $\nu > 1/3$, when the hole attracts negatively charged quasi-electrons (QEs) and binds one or two of them to form a fractionally charged exciton (FCX), hQE or hQE_2 [38].

In this paper, we discuss the results of numerical calculations of various radiative e–h complexes that contribute to the PL spectrum of a 2DEG depending on d and ν. The energy and PL spectra are obtained from exact diagonalization in Haldane spherical geometry [28, 29]. The presented unified description of the PL from the 2DEG at high B given in terms of emission from well defined bound states (X, X^-, and hQE_n) reduces the problem of a very complicated interaction between a hole and the 2DEG to a simpler one, of determining the single-particle properties of the bound "quasi-particles". In particular, the optical selection rules for these states are formulated, following from the translational invariance.

2. Model

Preserving the 2D translational symmetry of an infinite 2DEG in a finite-size calculation turns out to be essential for the identification of the optical selection rules of bound e–h states and the degeneracies in their energy spectrum. For any system of electrons and/or holes, this symmetry causes conservation of two orbital quantum numbers. For systems with total electric charge, $\mathcal{Q} \neq 0$ (such as X^-), they are the total angular momentum $\mathcal{M}$ and an additional angular momentum quantum number $\mathcal{K}$ associated with a partial decoupling of the CM motion in a homogeneous magnetic field [30, 31]. The energy levels of a charged system fall into degenerate LLs labeled by $\mathcal{L} = \mathcal{M} + \mathcal{K}$, and the states within each LL have $\mathcal{K} = 0, 1, 2, \ldots$. Since both $\mathcal{M}$ and $\mathcal{K}$

commute with the PL operator $\mathcal{P}$, the following selection rule governs the recombination of isolated charged e–h complexes: $\Delta\mathcal{M} = \Delta\mathcal{K} = 0$.

In order to confine electrons to a finite area (in order to achieve finite degeneracy of electron and hole LLs) without breaking the 2D symmetry, Haldane [28] proposed to put electrons and holes on a sphere of radius R. The magnetic field B perpendicular to the surface is due to a monopole placed in the center. The monopole strength $2S$ is defined in the units of elementary flux $\phi_0 = hc/e$, so that $4\pi R^2 B = 2S\phi_0$ and the magnetic length is $\lambda = R/\sqrt{S}$. The single-particle states are the eigenstates of angular momentum l and its projection m and are called monopole harmonics [28, 29]. The energies ε fall into $(2l+1)$-fold degenerate angular momentum shells separated by the cyclotron energy $\hbar\omega_c$. The n-th ($n \geq 0$) shell (LL) has $l = S + n$ and thus $2S$ is a measure of the system size through the LL degeneracy. Due to the spin degeneracy, each shell is further split by the Zeeman gap, E_Z.

As a result of 2D rotational invariance, a many-body e–h system on a sphere has two good quantum numbers, length L and projection L_z of the total angular momentum **L**. The mapping between quantum numbers $\mathcal{M}$ and $\mathcal{K}$ on a plane and the 2D algebra of **L** on a sphere allows conversion of the results between the two geometries [29]. In particular, LLs of a charged e–h complex are represented on a sphere by L multiplets, states within each LL are labeled by L_z, and the optical selection rule is $\Delta L = \Delta L_z = 0$. The price paid for closing the many-body Hilbert space without breaking the 2D symmetry is the surface curvature affecting interaction matrix elements. However, if the correlations modeled have short range ξ, the effects of curvature (scaled by a small parameter ξ/R) can be eliminated by extrapolation to $R \to \infty$.

Using a composite index $i = [nm\sigma]$ (σ is the spin projection), the e–h Hamiltonian can be written as $H = \sum c^\dagger_{i\alpha} c_{i\alpha} \varepsilon_{i\alpha} + \sum c^\dagger_{i\alpha} c^\dagger_{j\beta} c_{k\beta} c_{l\alpha} V^{\alpha\beta}_{ijkl}$, where $c^\dagger_{i\alpha}$ and $c_{i\alpha}$ create and annihilate particle α (e or h) in state i, and $V^{\alpha\beta}_{ijkl}$ are the Coulomb matrix elements. The Hamiltonian H is diagonalized numerically in the basis of Ne–1h Slater determinants. Small density of holes is assumed which allows ignoring h–h interaction effects and inclusion of only one valence hole in the basis. The Ne–1h eigenstates are labeled by L, L_z, and J.

The most important factors that determine the dimension of the Hamiltonian matrix that must be diagonalized are the number of electrons N, the LL degeneracy controlled by $2S$, the number of excited LLs included, and the inclusion of spin-unpolarized states. Using modified Lanczos algorithms we diagonalized Hamiltonians of dimensions up to about 5×10^6. In the X^- problem, the number of particles is small and the inclusion of reversed spins and LLs is possible. The numerical results for X^- states agree well with recent experiments [10, 12, 13]. However, when studying FCX, as many electrons as possible need be included in the calculation, and the model must be simplified by assuming maximum spin polarization ($J = N/2$), neglecting excited LLs, and setting zero QW width, $w = 0$. This allowed for calculation for $N \leq 9$ at $\nu \sim 1/3$, but the results are largely qualitative.

3. Coulomb Potential of the Hole as a Perturbation: Strong and Weak Coupling Regimes

The potential $V_{UD}(r)$ seen by the electrons due to the positive charge of the optically injected valence hole h can be described by two effective parameters, strength U and

spatial resolution D^{-1}. The response of the 2DEG to the perturbation $V_{UD}(r)$ depends on the relation between U and D and the characteristic energies and lengths of the unperturbed system. The pair of perturbation parameters U and D^{-1} in principle can be varied independently, but in experimental samples they are predominantly controlled through the effective spatial separation d of the hole from the 2DEG layer. The magnitude of d depends on the electric field oriented across the 2DEG plane and resulting from asymmetric doping. In symmetrically doped QWs, the hole moves in the same physical layer as the electrons ($d = 0$). However, in the asymmetrically (one-sided) doped QWs the displacement d can be a non-negligible fraction of the QW width w.

Clearly, at $d = 0$ both U and D^{-1} are the largest, and the strongest response of the 2DEG to the hole can be expected. The actual magnitudes of U and D^{-1} at $d = 0$ depend on the details of hole wave function, which to some extent depend on the magnetic field. In the extreme case of $B = \infty$ (lowest LL approximation), the e and h wave functions are identical and the interaction Hamiltonian is particle–hole symmetric [14–16]. This "hidden" symmetry causes vanishing of all electric moments in the optically active ground state of a bound e–h pair (X), which in turn causes the decoupling of X states from the 2DEG. As a result, the X is the most strongly bound optically active e–h state that can form in the 2DEG. In the opposite case, the (infinitely heavy) hole acts as a point charge and its attraction to an electron is larger by a factor of $\sim \sqrt{2}$. In an intermediate situation, the enhanced e–h attraction always breaks the hidden symmetry and causes coupling of an X to electrons. As a result, radiative X^- states occur, and the X^- rather than X should be regarded as the most stable bound state formed by a hole injected into the 2DEG [23]. This can be rephrased as that at finite B and $d = 0$ the 2DEG responds to $V_{UD}(r)$ by binding two electrons to the hole to form an X^- and screen the positive charge of the hole.

In the other limit of large d, both U and D^{-1} are too small to cause a strong response of an incompressible electron liquid. A hole is no longer able to pick out and bind a single electron from the 2DEG, which can be understood by noticing that the elementary charge excitations of an unperturbed 2DEG (QEs and QHs) have finite energy ($\varepsilon_{\mathrm{QE}}$ and $\varepsilon_{\mathrm{QH}}$), and that the electron wave function in the vicinity of the hole that corresponds to an X^- can be expanded in the basis of these QE and QH excitations. On the other hand, too large d implies too large D and the size of an isolated X^- (2e–1h ground state) that would exceed the characteristic e–e distance $\sqrt{\varrho}$ (ϱ means density) and make such an X^- unstable when inserted into the 2DEG (independently of the preceding energetic argument). The only allowed response of the 2DEG to the hole in this ("weak") coupling regime is to screen the hole positive charges with the negatively charged QEs that are already present in the 2DEG. The fact that only the existing QEs can bind to the hole makes the response critically depend on the filling factor ν. For example, near the Laughlin filling of $\nu = 1/3$, the hole repels the QHs and moves in the locally incompressible $\nu = 1/3$ liquid, causing no local response of the 2DEG. Conversely, the binding of one or more QEs to the hole is expected at $\nu > 1/3$, and the resulting bound FCX states hQE_n are the most stable bound states at large d. Since the "uncoupled" state h has different emission energy $\hbar\omega$ and oscillator strength τ^{-1} than the hQE_n states, discontinuity in the PL spectrum is expected at $\nu = 1/3$, in agreement with the experimental PL data for asymmetric structures [1–3].

4. Strong Coupling Regime: Neutral and Charged Excitons

4.1 Isolated charged exciton: energy spectrum

An isolated neutral (X) or charged (X^-) exciton consists of only two or three particles. In the numerical calculation of the energy and PL spectra of this relatively simple quantum mechanical system we were able to include the effects of the finite magnetic field B that causes LL mixing (by including up to five electron and hole LLs, $n \leq 4$, with the lowest LL degeneracy of $2S+1=21$) and of the finite QW width w (by using the values of the two-body Coulomb matrix elements appropriate for the lowest subband of the QW rather than for an ideal 2D system). It should be stressed that due to different electron and hole effective masses and due to a different height of the electron and hole confinement potential at the well/barrier interface, the electron and hole wave functions in the z-direction are different even in the symmetric structures with $d=0$. To a first approximation, this is included by introducing a pair of effective widths of the electron and hole layers, w_e^* and w_h^*, obtained by fitting the actual density profile $\varrho(z)$ of each carrier with $\cos^2(\pi z/w^*)$. For a $w=10$ nm $GaAs/Al_{0.33}Ga_{0.67}As$ QW the effective widths are $w_e^* = w + 3.3$ nm and $w_h^* = w + 1.5$ nm. Following Cole et al. [34], we have also took into account the nonlinear dependence of the hole cyclotron mass on both B and w.

The Zeeman energy $E_Z = g^*\mu_B B$ (g^* is the effective gyromagnetic factor and μ_B is the Bohr magneton) enters the problem of an X^- in two ways. (i) Because of the complete spin polarization of the 2DEG, binding of an X^- state may or may not require a spin flip, depending on the total spin J of two electrons in the bound X^- state. Therefore, the binding energy Δ of each spin triplet ($J=1$) X^- state with the minimum

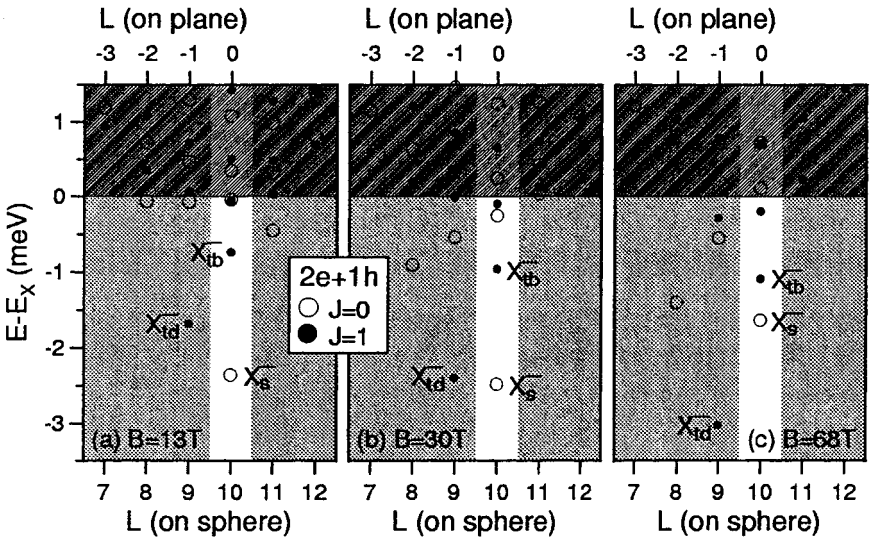

Fig. 1. Energy spectra (energy E vs. angular momentum L) of two electrons and one valence hole confined in a symmetric GaAs quantum well of width $w=11.5$ nm at magnetic fields a) $B=13$, b) 30, and c) 68 T, calculated on a Haldane sphere with the Landau level degeneracy $2S+1=21$. E_X is the exciton energy

Zeeman energy ($J_z = J$) is insensitive to the Zeeman energy, but for spin singlet ($J = 0$) states Δ is reduced by E_{Ze} compared to the pure Coulomb binding energy. For GaAs, E_{Ze} is roughly a linear function of energy through both cyclotron energy $\hbar\omega_{\mathrm{c}} \propto B$ and confinement energy $\propto 1/w^2$. After Snelling et al. [32], for $w \sim 10$ nm at $B = 0$, we have $g_{\mathrm{e}}^* = (w_0/w)^2 + g_0$ with $w_0 = 9.4$ nm and $g_0 = -0.29$. After Seck et al. [33] we find $\mathrm{d}g_{\mathrm{e}}^*/\mathrm{d}B = 0.0052\ T^{-1}$. (ii) The recombination of an X or X^- state can occur through relaxation of a conduction electron with either spin to the valence band. Since the angular momentum of the photon (± 1) depends on the spin of the relaxing electron, and the emission energy $\hbar\omega$ includes the Zeeman energy of the recombining e–h pair, $E_{\mathrm{Ze}} + E_{\mathrm{Zh}}$, each PL peak associated with the emission from an X or X^- state is split into two circularly polarized peaks by $E_{\mathrm{Ze}} + E_{\mathrm{Zh}}$. The splitting is most evident for X_{s}^- because of the large population of both electron spin levels even at a very low temperature.

In Fig. 1 we show the 2e–1h energy spectra calculated for a $w = 11.5$ nm GaAs QW at $B = 13$, 30, and 68 T. In each frame, the energy E is measured from the exciton energy E_{X}, so that for the bound X^- states it is opposite to the binding energy Δ. Open and full symbols denote singlet and triplet electron spin configurations, respectively, and only the state with the lowest Zeeman energy ($J_z = J$) is marked for each triplet. Similarly, each state with $L > 0$ represents a degenerate multiplet with $|L_z| \leq L$. The angular momentum L calculated in the spherical geometry and given on the horizontal axes under each graph translates into the angular momentum on a plane $\mathcal{L}$ in such way [30, 31] that the $L = S$ and $S - 1$ multiplets correspond to the planar LLs with $\mathcal{L} = 0$ and -1, respectively.

4.2 Isolated charged exciton: radiative recombination

Due to the conservation of L in the PL process, only those 2e–1h states from the $L = S$ channel are radiative. This is because [16, 21–23] an annihilated e–h pair has $l_{\mathrm{X}} = 0$, and the electron left over in the lowest LL has $l_{\mathrm{e}} = S$. Recombination of other, non-radiative states requires breaking the rotational symmetry (e.g., by interaction with other charges). This result is independent of chosen spherical geometry; on a plane the 2D translational symmetry leads to the conservation of both $\mathcal{M}$ and $\mathcal{K}$, and the corresponding PL selection rule for 2e–1h states is $\mathcal{L} = 0$ [31].

Three X^- states in Fig. 1 are of particular importance. The X_{s}^- and X_{tb}^-, the lowest singlet and triplet states at $L = S$, are the only strongly bound radiative states, while X_{td}^- has by far the lowest energy of all non-radiative ($L \neq S$) states. In agreement with earlier predictions [20], the transition from X_{s}^- to X_{td}^- ground state is found at $B \approx 30$ T, and a new, radiative excited triplet state X_{tb}^- is identified in all frames. The binding energy Δ of each of these three X^- states, extrapolated to the $R/\lambda = \sqrt{S} = \infty$) limit, is plotted as a function of B in Fig. 2a. Clearly, Δ_{td} increases most quickly of all curves with increasing B, Δ_{s} remains almost constant (especially its Coulomb part drawn with the thin line), and Δ_{tb} remains smaller than both Δ_{td} and Δ_{s} at any value of B.

The extrapolated values of the oscillator strength τ^{-1} of the X and two radiative X^- states are shown in Fig. 2b. The ratio $\tau_{\mathrm{tb}}^{-1} \approx 2\tau_{\mathrm{s}}^{-1}$ remains almost independent of B, and the resulting three PL peaks (X, X_{s}^-, and X_{tb}^-) are precisely those observed in experiments [6–13]. Above, we assumed that both electrons and holes are completely spin-polarized ($J_z = J$), but typically, all electron spins and only a fraction of hole spins χ_{h} are aligned with the field. As a result, the X_{tb}^- PL has definite circular polarization (σ_+)

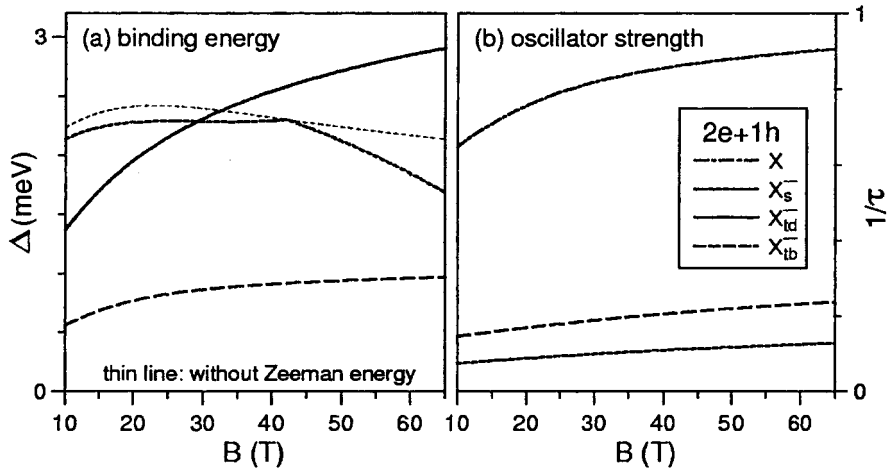

Fig. 2. a) Binding energies Δ and b) oscillator strengths τ^{-1} of different charged exciton states X^- in a symmetric GaAs quantum well of width $w = 11.5$ nm, plotted as a function of the magnetic field B

and its intensity is reduced by χ_h, while the X_s^- PL peak splits into a $\sigma_\pm$ doublet (separated by the appropriate Zeeman energy) with the intensity of the two transitions weighted by χ_h and $1 - \chi_h$. An increase of χ_h from 1/2 to 1 with increasing B may explain the observed [10] increase of τ_{tb}^{-1} by up to a factor of two, while τ_s^{-1} remains constant.

4.3 Interaction of charged excitons with electrons

Even in dilute systems, recombination of bound e–h complexes can in principle be affected by their interaction with each other or with excess electrons. Especially the recombination of a X_{td}^- might become allowed in a collision assisted process in which the translational symmetry of an isolated X_{td}^- is broken. The critical question is if the $e–X^-$ correlations are of the Laughlin type [22, 26], meaning that the many-body $e–X^-$ wave function contains a Jastrow prefactor $\prod(x_i - y_j)^\mu$ (where x and y are complex coordinates of e and X^-, and μ is an integer). If it is so, then a number μ of the highest energy $e–X^-$ pair eigenstates are avoided in the low-energy many-body states [35, 36] (just as the p leading e–e pair eigenstates are avoided in the Laughlin $\nu = (2p+1)^{-1}$ state of electrons). This means lack of high-energy $e–X^-$ collisions, and thus an effective isolation of the X^- states from the 2DEG and insensitivity of the X^- binding or recombination to the electron density. In particular, Laughlin $e–X_{td}^-$ correlations would eliminate the possibility of collision-assisted recombination of the X_{td}^-.

To determine if the Laughlin correlations occur in a mixed $e–X^-$ liquid, one must calculate the $e–X^-$ interaction pseudopotential $V(L)$, defined as the dependence of the pair interaction energy V on the pair angular momentum L [35]. The general criterion for the occurrence of Laughlin correlations in a many-body system confined to a degenerate LL and interacting through $V(L)$ is that V must have short range, i.e. decrease sufficiently quickly with increasing L [36]. On a sphere, V must be a superlinear function of $L(L+1)$. It turns out that this criterion is satisfied by the $e–X^-$ repulsion in narrow QWs [23]. This implies a simple connection between ν and the maximum al-

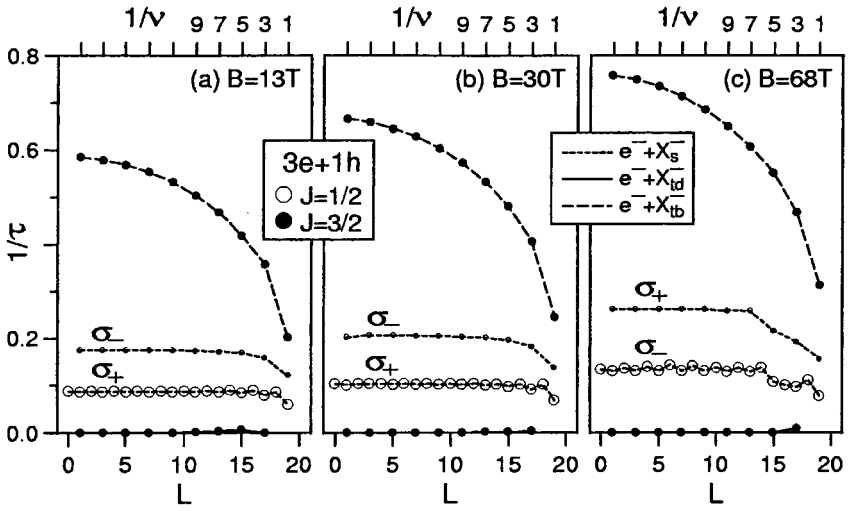

Fig. 3. Oscillator strengths τ^{-1} of different charged excitons X^- interacting with an electron in a symmetric GaAs quantum well of width $w = 11.5$ nm at magnetic fields a) $B = 13$ T, b) 30 T, and c) 68 T, calculated on a Haldane sphere with the Landau level degeneracy $2S + 1 = 21$, and plotted as a function of the $e–X^-$ pair angular momentum L

lowed L (i.e., minimum average separation) for a $e–X^-$ pair: $L + \mu \leq l_e + l_{X^-}$ for $\nu \leq \mu^{-1}$, and allows the calculation of the effect of the $e–X^-$ interaction on the X^- recombination as a function of ν.

In Figs. 3 and 4 we plot the PL oscillator strength τ^{-1} and energy $\hbar\omega$ (measured from the exciton energy E_X) for the 3e–1h eigenstates corresponding to an X^- interacting with another electron, respectively. We assume that the Zeeman energy will polarize all electron spins prior to recombination, except for those two in the X_s^-, and concentrate

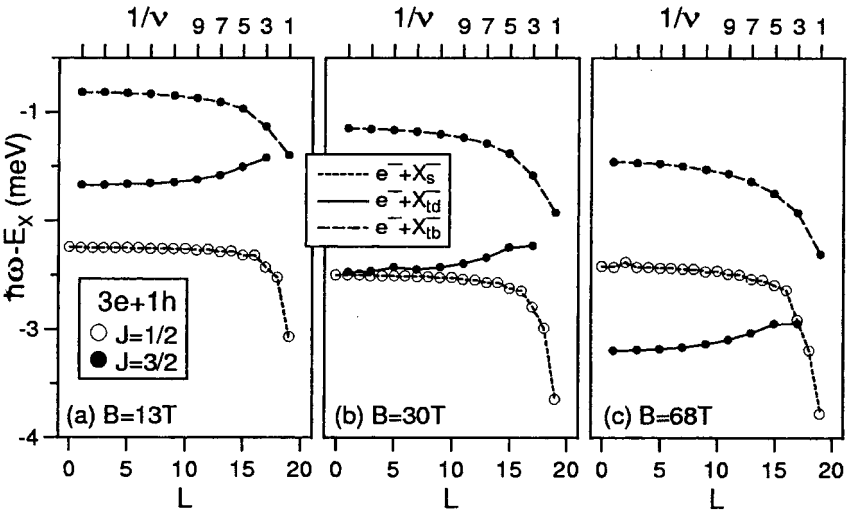

Fig. 4. Same as in Fig. 3 but the graphs show the recombination energy $\hbar\omega$ as a function of L. E_X is the exciton energy

on the following three initial configurations: e–X_s^- with $J_z = J = 1/2$ and e–X_{tb}^- and e–X_{td}^- with $J_z = J = 3/2$. For each of the three configurations, τ^{-1} and E are plotted as a function of L (i.e. of ν).

For X_{tb}^- and X_{td}^-, only an e↑–h↓ pair can be annihilated, and an emitted photon has a definite circular polarization σ_+. Two indistinguishable electrons left in the final state have $J = 1$, so their L must be odd ($2l_e$ minus an odd integer). For X_s^-, both σ_+ and σ_- PL are possible, with the energy of the latter transition shifted by $E_{Ze} + E_{Zh}$. For σ_+, the two electrons in the final state can have either $J = 0$ and L even, or $J = 1$ and L odd; while for σ_+ they can only have $J = 1$ and L must be odd.

As expected, for $L \to 0$ (i.e., $\nu \to 0$) both $\hbar\omega$ and τ^{-1} converge to the values appropriate for single X^- states plotted in Fig. 2 (the energies shown in Fig. 4 correspond to the σ_+ transitions given by E_X plus Coulomb binding energy; if present in the PL spectrum, additional σ_- transitions will appear at the energy higher by $E_{Ze} + E_{Zh}$). There is no significant effect of the e–X^- interactions on the X^- recombination at small L. This justifies a simple picture of PL in dilute e–h plasmas, according to which, recombination occurs from a single isolated bound complex and hence is virtually insensitive [9] to ν. Somewhat surprisingly, the Laughlin correlations prevent increase of the X_{td}^- oscillator strength through interaction with other charges (τ_{td}^{-1} remains ten times longer than τ_s even at $\nu = 1/3$). This explains the absence of an X_{td}^- peak even in the PL spectra [6–10] showing strong recombination of a higher-energy triplet state X_{tb}^- (except at very low temperatures [13]). An interesting feature in Fig. 4 is the merging of $\hbar\omega_{tb}$ and $\hbar\omega_{td}$ which actually has been observed [13].

5. Weak Coupling Regime: Fractionally Charged Excitons

The fractionally charged excitons (FCXs) hQE_n formed in strongly asymmetric QW structures ($d > \lambda$) consist of n QEs bound to a valence hole h. Since QEs are collective excitations of a many-electron system, as many electrons as possible must be included in the computation. This was only possible by a severe limitation of the single-particle Hilbert space to the lowest LL. Although the inter-LL e–h scattering was quite important at $d = 0$ (where it caused binding of X_s^- and X_{tb}^- states), it is greatly reduced at larger d and, unlike at $d = 0$, the lowest-LL approximation is expected to be more justified. Nevertheless, the model studied is necessarily a very ideal one and, consequently, such realistic elements as the finite QW width have also been excluded. Therefore, the numerical results obtained in this section are not meant to describe experiments as accurately as those for X^- states, although the fact of the FCX binding or the optical selection rules for different FCX states are expected to be valid for experimental systems.

The binding of FCX states relies on the attractive interaction between the (oppositely charged) h and QE, and the weak QE–QE interaction at short range [37]. On the other hand, the stability of the FCX states at sufficiently large d against the formation of X or X^- excitons results from the h–QE attraction being sufficiently small compared to the Laughlin gap $\varepsilon_L = \varepsilon_{QE} + \varepsilon_{QH}$. The comparison of the h–QE attraction energy (the largest h–QE pseudopotential parameter) to ε_L shows that the FCXs rather than X or X^- states should be the most stable bound states formed by a hole injected into the 2DEG at d larger than about a magnetic length λ [38].

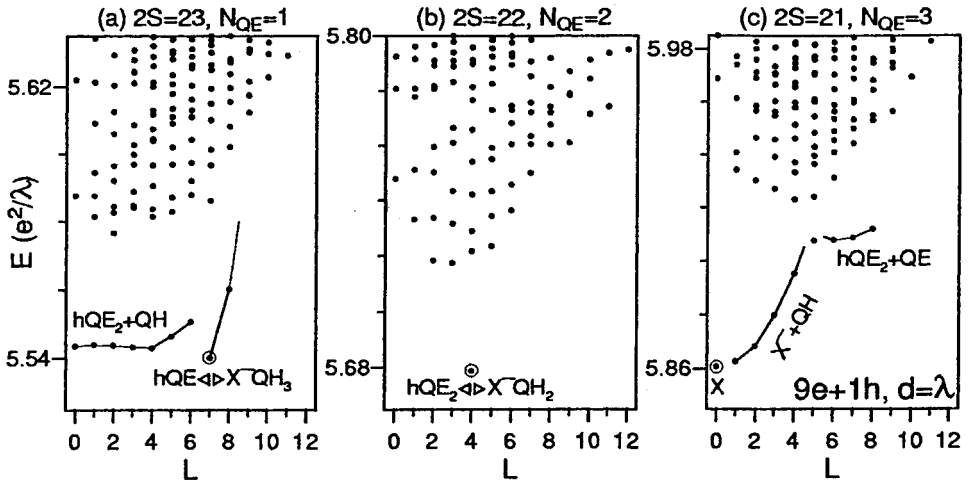

Fig. 5. Energy spectra (energy E vs. angular momentum L) of nine electrons and one valence hole on a Haldane sphere with the Landau level degeneracy $2S+1 = 24$, 23, and 22, corresponding to $N_{\mathrm{QE}} = 1$, 2, and 3 quasi-electrons in the Laughlin $\nu = 1/3$ state of nine electrons. The separation between electron and hole planes is $d = \lambda$. λ is the magnetic length

5.1 Numerical energy spectra

The series of 9e–1h energy spectra calculated for $d = \lambda$ and 2λ are presented in Figs. 5 and 6, respectively. The three spectra shown for each d are obtained for different values of $2S$, corresponding to $N_{\mathrm{QE}} = 1$, 2, and 3 QEs in the Laughlin $\nu = 1/3$ state of 9e system (without interaction with the valence hole). In Fig. 5 ($d = \lambda$, intermediate-coupling regime), new low-energy bands of states emerge in addition to those characteristic of strong coupling and containing an X or X^-. These new states contain various FCXs interacting with the remaining QPs of the 9e liquid. In some cases the FCX states occur

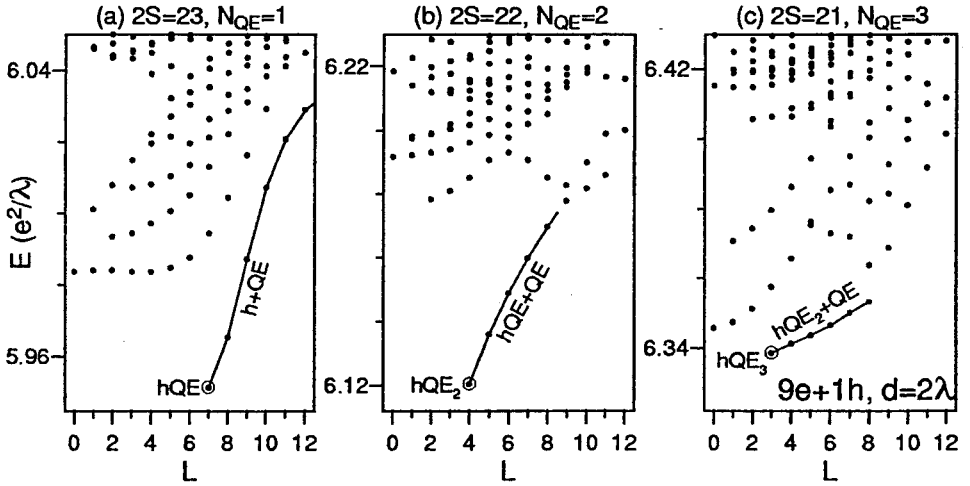

Fig. 6. The same as in Fig. 5 but for $d = 2\lambda$

in the same spectrum with the X or X^- states. For example, the hQE_2–QE band in Fig. 5c coexists with the X state (meaning X weakly coupled to the Laughlin $\nu = 1/3$ state of the remaining 8e) and the X^-–QH band (meaning X_{td}^- Laughlin-correlated with the remaining 7e; QH denotes a quasi-hole in the two-component, e–X^- Laughlin liquid [22]). In other cases, low-lying X or X^- states of the strong-coupling regime occur at the same L as a low-lying FCX of the weak-coupling regime, and the transition between the two states (which occurs at $d \approx \lambda$) is continuous. For example, hQE_2 is mixed with X^-QH_2 in Fig. 5b, and hQE is mixed with X^-QH_3 in Fig. 5a.

In Fig. 6 ($d = 2\lambda$, weak-coupling regime), well developed FCX bands occur. The isolated hQE, hQE_2, and hQE_3 states are the ground states in the spectra corresponding to $N_{QE} = 1$, 2, and 3, respectively. Their angular momenta l_{FCX} are obtained by adding $l_h = S$ and $l_{QE} = S^* + 1$, where $2S^* = 2S - 2(N-1)$ is the effective monopole strength in the composite fermion picture [16, 36] and $2S = 3(N-1) - N_{QE}$. Similarly, the angular momenta of states containing an FCX and excess QPs result from adding l_{FCX} and l_{QP}.

5.2 Selection rules and photoluminescence

Similarly as it was for X^- states, the translational symmetry of an isolated FCX leads to the conservation of L and L_z in the emission process. This leads to the strict optical selection rules that can only be broken by collisions or disorder. The recombination of an FCX state hQE_n formed in a Laughlin $\nu = (2p+1)^{-1}$ electron liquid occurs through annihilation of a well defined number of QEs and creation of an appropriate number of QHs. It turns out that the processes involving more than the minimum number of QPs all have negligible intensity, which for $p = 1$ ($\nu = 1/3$) leaves only the following four possible recombination events: $h + nQE \rightarrow (3-n)QH + \gamma$, where $n = 0$, 1, 2, or 3, and γ denotes the photon. Let us apply the angular momentum conservation law, $\Delta L = \Delta L_z = 0$, to the above recombination events. In the fermionic picture [37], the angular momenta of QE and h in the initial Ne–1h state at a given monopole strength $2S$ are $l_{QE} = S - N + 2$ and $l_h = S$. By adding l_{QE} and l_h, the following values are obtained for the angular momenta of FCX complexes [38]: $l_{hQE} = N - 2$, $l_{hQE_2} = (N-1)/2$, and $l_{hQE_3} = 3$. The angular momentum of QH in the final $(N-1)\,e$ state at the same $2S$ is $l_{QH} = S - N + 2$. By comparing the values of $l_h QE_n$ with the angular momenta allowed for $(3-n)$ identical QHs in the final state, we obtain that (i) the "decoupled hole" state h is radiative and can recombine to create a QH_3 molecule with the maximum $L = 3l_{QH} - 3$ allowed for three QHs, (ii) l_{hQE} is different from any L allowed for two QHs and thus hQE is non-radiative; however, the first excited state, hQE* at $l_{hQE^*} = l_{hQE} + 1$ is radiative and recombines to create a QH_2 molecule with $L = 2l_{QH} - 1$; (iii) l_{hQE_2} is radiative and its recombination leaves behind a single QH; (iv) neither l_{hQE_3} nor its excitations are radiative.

The above analysis leaves h, hQE*, and hQE_2 as the only radiative FCX states, while hQE and hQE_3 are found dark. It is expected that a valence hole introduced into the 2DEG at $\nu < 1/3$ (in the absence of free QEs) and at $d > \lambda$ will decouple and recombine from the initial state h (local filling factor $\nu = 1/3$). On the other hand, at $\nu > 1/3$ the valence hole will bind one or more free QEs and recombine from either hQE* or hQE_2 initial state, depending on d, temperature, and the QE density. Since the initial state from which the hole recombines changes at $\nu = 1/3$, and since different FCX

states have different emission energy and intensity [38], the PL spectrum of a 2DEG created in a strongly asymmetric structure is expected to change discontinuously at the corresponding magnetic field.

6. Conclusions

The PL spectrum of a 2DEG was studied as a function of the separation d between e and h layers. Two types of response of the 2DEG to the optically injected hole were identified. In the strong-coupling regime ($d \ll \lambda$) the most strongly bound states are X and X^-, and the PL spectrum measures their optical properties rather than the original correlations of the 2DEG. In particular, the dark triplet X^-_{td} remains virtually non-radiative for $\nu < 1/3$. In the weak-coupling regime ($d > \lambda$), the X^- states unbind and, depending on ν, the hole either decouples from the 2DEG or binds one or more QEs to form a FCX state hQE_n. The most stable radiative bound state for $\nu \leq 1/3$ is found to be the "decoupled hole," while for $\nu > 1/3$ it is either hQE_2 or hQE* (excited state of hQE). Since different FCX states have different optical properties, discontinuities are expected in the PL spectrum for $\nu = 1/3$.

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