Pairs and triplets of composite fermions in partially filled shells

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Abstract. Two- and three-body correlation functions (the number of pairs or triplets vs relative angular momentum) are calculated for several quantum Hall liquids. It is shown that quasielectrons (i.e., composite fermions in their first excited Landau level, CF-LL$_1$) form pairs at electron filling factor $\nu_e = 4/11$, and (most likely) three-body clusters at $\nu_e = 5/13$.

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INTRODUCTION

New fractions of quantum Hall effect observed by Pan [1] in a spin-polarized two-dimensional electron gas at electron Landau level (LL) filling factors $\nu_e = 4/11$, $3/8$, $5/13$, etc. do not belong to the well-known Jain [2] series at $\nu_e = n(2pn \pm 1)^{-1}$ (with integral n and p).

Jain states can be understood by assumming that each electron attaches $2p$ magnetic flux quanta $\phi_0 = hc/e$ to form a so-called composite fermion (CF). In mean field, the CFs feel effective magnetic field $B^* = B - 2p\phi_0\rho$ (ρ being electron concentration) and occupy effective LLs, called CF-LL$_n$. The incompressible ground states at fractional ν_e coincide with integral fillings of CF LLs.

Fractional quantum Hall effect at $\nu_e = 4/11$, $3/8$, or $5/11$ corresponds to the situation when CFs fill completely their lowest CF-LL$_0$, but only a fraction $\nu = 1/3$, $1/2$, and $2/3$ of the second level, CF-LL$_1$, respectively. Incompressibility of these state might be predicted in the hierarchical model [4], where once again the CF procedure (attaching fluxes) is applied to CFs from partially occupied CF-LL$_1$, thus creating "second generation" of CFs [3]. Unfortunately, because of completely different short-range interaction between the CFs and between electrons, this procedure is unjustified. Instead, spontaneous cluster formation is more plausible.

In this work we study numerically two- and three-body correlations of these new states, and compare them with analogous data for electrons. Although the origin of incompressibility is not clear, we found direct evidence for the CF pairing in the state $\nu_e = 4/11$.

CALCULATIONS

Calculation have been done in Haldane spherical geometry [4], where N particles are confined to a spherical surface of radius R. Dirac monopole of strength $2Q$ placed in the centre of the sphere is the source of magnetic field B, and $2Q\phi_0 = 4\pi R^2 B$. Each particle on its n-th LL has angular momentum $l = Q + n$ and degeneracy of each shell $g = 2l + 1$.

Studying the $\nu_{\rm CF} \equiv 1 + \nu = 1 + 1/3$ state ($\nu_e = 4/11$) we neglected CFs on their lowest, completely filled, LL and take into consideration only particles on the second CF-LL$_1$, equivalent to Laughlin quasielectrons (QEs). Interactions between the QEs is given by pseudopotential $V_{\rm QE}(\mathscr{R})$, where $\mathscr{R} = 2l - L$ is relative pair angular momentum, taking odd integer values for identical particles. Larger $\mathscr{R}$ means larger average squared distance between particles. The psudopotential $V_{\rm QE}(\mathscr{R})$ is known from independent calculation, and its most important feature is the strong maximum at $\mathscr{R} = 3$ and a small value at $\mathscr{R} = 1$ [5, 6]. Large systems with filling factor $\nu = N/g$ are represented on a sphere by finite size combinations $2l = N/\nu - \gamma$, where $\gamma(\nu)$ is integral "shift" function set in the way that the ground states has $L = 0$ and is significantly separated from excited states by energy gap. For Laughlin states $\gamma = 3$, $\nu_{\rm QE} = 1/3$ state is represented by configuration $2l = 3N - 7$ [7].

To study clusterization properties we examine systems containing $N = 12$ particles by calculating number of nearest pairs $\mathscr{N}_2(\mathscr{R} = 1)$ in the ground state, understood in that way that energy of the state can be written as:

$$E = \sum_{\mathscr{R}} \mathscr{N}_2(\mathscr{R}) V_2(\mathscr{R}). \qquad (1)$$

Fig. 1(a) shows the results of calculation – the number of pairs as a function of twice the shell angular momentum $2l$. For electrons on LL$_1$ and for QEs on CF-LL$_1$ we can identify the $\nu = 1/2$ series at $2l = 2N - 3$, the $\nu = 1/3$ series at $2l = 3N - 7$, and their particle-hole conjugates at $2l = 2N + 1$ and $3N/2 + 2$. In the CF-LL$_1$ there are more pairs than in electron LL$_0$ and LL$_1$, but there

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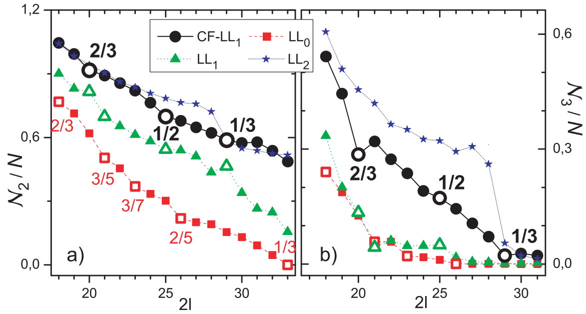

FIGURE 1. The number of pairs $\mathcal{N}_2(\mathscr{R}=1)$ (a) and triplets $\mathcal{N}_3(\mathscr{T}=3)$ (b) as a function of double shell angular momentum $2l$ for interaction in the first excited CF Landau level (CF-LL$_1$) and in different electron Landau levels (LL$_n$).

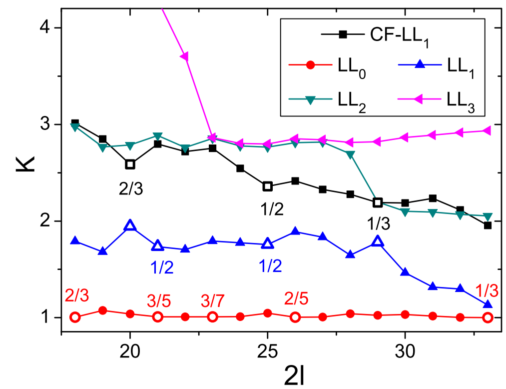

FIGURE 2. Average cluster size estimated for $N=12$ interacting quasielectrons and electrons on their Landau levels.

is visible similarity between $\nu=1/2$ state on the electron LL$_1$ (Moore–Read state, known to be paired) and $\nu_{\mathrm{QE}}=1/3$ – in both cases $\mathcal{N}_2 \approx N/2$.

In a similar way, we found the number $\mathcal{N}_3$ of compact triplets as the expectation value of operator $\hat{\delta}_{\mathscr{T},3}$, where $\mathscr{T}=2l-L$ is the three-body relative angular momentum ($\mathscr{T}=3$ or $\mathscr{T}\geq 5$). Fig. 1(b) shows that $\mathcal{N}_3$ quickly decreases for both LL$_0$ and LL$_1$, and at $2l=21$ it actually vanishes. It means that for sufficiently low ν, there are no close triplets in these LLs (clusters larger than pairs do not form). The number of QE triplets in CF-LL$_1$ is also a nearly linear function of $2l$, but it vanishes at $2l=3N-7=29$, i.e., $\nu=1/3$. The vanishing of $\mathcal{N}_3$ together with having $\mathcal{N}_2=N/2$ is the evidence of QE pairing at $\nu_e=4/11$. A paired state at $\nu=1/2$ (if it were stable) would be a "second generation" Moore–Read state of CFs. Moore–Read state has two types of excitations: quasiholes (QHs) and pair-breakers. Since the $\nu=1/3$ ground state contains the same number ($N/2$) of pairs as the hypothetical Moore–Read paired state, it only contains QHs. Hence, we interpret the $\nu=1/3$ ($\nu_e=4/11$) state as an incompresible state of the "second generation" Moore–Read QHs. (Note that neither is the "second generation" Moore–Read state itself actually stable at $\nu_e=3/8$, nor does our interpretation explain the observed incompressibility at $\nu_e=4/11$).

The number of triplets $\mathcal{N}_3=N/3$ at $\nu=2/3$ in CF-LL$_1$ (i.e. $\nu_e=5/13$ at $2l=20$) suggests that N QEs can be divided into $N/3$ clusters, each containing 3 particles. For $\nu=1/2$, i.e. $\nu_e=3/8$ ($2l=25$), $\mathcal{N}_3=N/6$ implies a more complicated cluster configuration.

To estimate average cluster size we calculated $\mathcal{N}_2$ for a single K-size cluster and compared the values for N/K such clusters with those for real interacting N electrons or CFs. In the Fig. 2 we see that pairs forms in LL$_1$ at $\nu>1/3$ and for CF-LL$_1$ $\nu=1/3$. Triplets are likely in in CF-LL$_1$ at $\nu=2/3$ and in some fractions for electrons at LL$_2$ and LL$_3$.

CONCLUSION

Numerical calculations of $N=12$ particle systems show that the number of "maximum density" CF triplets (with the minimum allowed $\mathcal{N}_3=3$) decreases as a function of $2l$ from $\mathcal{N}_3=N/3$ at $\nu=2/3$ to zero at $\nu=1/3$. At the same time, the number of "maximum density" CF pairs (with $\mathscr{R}=1$) decreases from $\mathcal{N}_2=N$ to $N/2$. This is the evidence for CF pairing in the $\nu=4/11$ quantum Hall state in analogy to similar signatures of electron pairing in the half-filled Moore–Read state. It is shown that $\nu_e=4/11$ can be interpreted as a condensate of the quasiholes of the Moore–Read state of the CFs.

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REFERENCES

1. W. Pan, H. L. Störmer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **90**, 016801 (2003);
2. J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989); Science **266**, 1199 (1994); Phys. Today **53**, No.4, 39 (2000).
3. J. H. Smet, Nature **422**, 391 (2003).
4. F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983).
5. A. Wójs and J. J. Quinn, Phys. Rev. B **61**, 2846 (2000).
6. S.-Y. Lee, V. W. Scarola, and J. K. Jain, Phys. Rev. Lett. **87**, 256803 (2001); Phys. Rev. B **66**, 085336 (2002).
7. A. Wójs, K.-S. Yi, and J. J. Quinn, Phys. Rev. B **69**, 205322 (2004).