Pairing and Condensation
of Laughlin Quasiparticles
in Fractional Quantum Hall Systems

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From the analysis of their interaction pseudopotentials, it is argued that (at certain filling factors) Laughlin quasiparticles can form pairs. It is further proposed that such pairs could have Laughlin correlations with one another and form condensed states of a new type. The sequence of fractions corresponding to these states includes all new fractions observed recently in experiment (e.g., $\nu = 5/13$, $3/8$, or $4/11$).

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1. Introduction

In a recent experiment Pan et al. [1] observed the fractional quantum Hall (FQH) effect [2, 3] at novel filling fractions ν of the lowest Landau level (LL). Among other features, the FQH effect is a macroscopic manifestation of an incompressible character of the many-body ground state formed at a specific filling ν. The new spin-polarized FQH states occur at filling factors outside the Jain sequence [4] of composite fermion (CF) states. Some of them, as $\nu = 4/11$ or $4/13$, appear in the Haldane hierarchy [5] of quasiparticle (QP) condensates, and therefore at first sight it might only be surprising that they have not been observed earlier. However, the "hierarchical" interpretation of these states was questioned [6] because of the specific form of the QP–QP interaction, which leaves their interpretation quite uncertain. Others, such as the $\nu = 3/8$ or $3/10$ states, do not belong to the Haldane hierarchy, and thus the origin of their incompressibility is puzzling in an even more obvious way. In the CF picture, these even-denominator

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fractions correspond to the half-filled first excited ($n = 1$) CF LL. While for
the electrons half-filling their $n = 1$ LL (including the double degeneracy of the
lowest LL, this amounts to the total electron filling of $\nu = 2 + 1/2 = 5/2$) the
FQH effect has been known for quite some time [7] (the incompressible $\nu = 5/2$
state is known as a Moore–Read state [8]), the similar behavior of electrons and
CF's at this filling is rather unexpected. Pan et al. take their observations as ev-
idence for residual CF–CF interactions, sufficiently strong to cause emergence of
new FQH states (that, unlike virtually all observed fractions, cannot be predicted
from the noninteracting CF model or standard Haldane hierarchy). However, they
ignore the theoretical investigations in which these interactions were studied in
detail [6, 9, 10].

   In this paper we propose an explanation for these new states involving the
formation of QP pairs which display Laughlin correlations with one another. First,
we explain the connection between the CF model and the QP hierarchy [5, 6, 9].
Second, we recall two simple types of two-body correlations, Laughlin correlations
and pairing, that may occur in an interacting system depending on the filling
factor ν and on whether $V(\mathcal{R})$ is super- or subharmonic at the relevant range [11].
Then, knowing the QP–QP pseudopotential $V_{\mathrm{QP}}(\mathcal{R})$, we apply the concept of
Laughlin condensed states of (bosonic) pairs (used earlier for the electrons in the
$n = 1$ LL to describe such FQH states as $\nu = 5/2$ or $7/3$ [12]) to the particles
or holes in a partially filled CF LL, i.e., to Laughlin quasielectrons (QE's) or
quasiholes (QH's). Finally, we propose the novel hierarchy of FQH states in which
the incompressibility results from the condensation of QP pairs (QE$_2$'s or QH$_2$'s)
into Laughlin correlated pair states. The series of FQH states derived from the
parent $\nu = 1/3$ state include all novel fractions: $\nu = 5/13, 3/8, 4/11$, and $6/17$ for
the QE's and $\nu = 5/17, 3/10, 4/13$, and $6/19$ for the QH's.

2. Laughlin QP's and CF's

   Let us begin with recalling the connection between Laughlin QP's [3] and the
CF model (equivalent to the mean-field Chern–Simons transformation) [4]. Laugh-
lin QP's are the actual elementary excitations of a two-dimensional electron liquid
filling a fraction of the lowest LL. They have well-defined and known wave func-
tions (and thus also density profiles, size, etc.) and single-particle energy. They
carry (fractional) electric charge, counted with respect to the uniformly spread
charge of the underlying Laughlin state. The negatively and positively charged
QP's are called quasielectrons (QE's) and quasiholes (QH's), respectively. Moving
in the external magnetic field, both kinds of QP's follow cyclotron orbits, conve-
niently labeled by angular momentum.

   Since the uniform-density Laughlin states only occur at a discrete series of
filling factors ($\nu = 1/3, 1/5, \ldots$), and Laughlin QP's are simply the least-energy
excitations able to carry charge in excess of the value corresponding to the nearest

Laughlin state, their type and number depend on ν. At ν precisely equal to any of the Laughlin values $\nu_p = (2p+1)^{-1}$ no QP's exist in the ground state and they may only appear in a form of excitonic, charge-neutral QE–QH pairs. At ν slightly smaller (or larger) than ν_p, a number N_{QP} of the "p–type" QE's (or QH's) appear in the Laughlin liquid. The QP number is equal to the difference between the total magnetic flux through the sample and its value corresponding to the nearest "parent" Laughlin state, measured in units of elementary flux, $N_{QP} = |\Phi - \Phi_p|/\phi_0$.

For electrons, the filling factor is expressed through the flux Φ and the electron number N as $\nu = \Phi/(N\phi_0)$, For the QP's of the parent ν_p Laughlin state, each carrying charge $\pm e/(2p+1)$, the LL degeneracy is reduced by a factor of $(2p+1)$ compared to the electron value. Consequently, the QP filling factor is equal to $\nu_{QP} = \Phi/[(2p+1)N_{QP}\phi_0]$. Note that ν_{QP} is a linear function of ν near each value of ν_p, The situation with ν far away from the nearest ν_p, corresponds to a large filling factor for appropriate QP's whose mutual interactions and filling of higher QP LL's both become important.

In the mean-field CF picture, the reduction of the QP LL degeneracy is attributed to a reduced magnetic field $B^* = B/(2p+1)$ rather than to a reduced QP charge. In the simplest formulation of the model, the effective field B^* results from the capture of an even number of magnetic flux quanta $2p\phi_0$ by each electron (such bound state is called a CF), and the QE's and QH's are pictured as particles and vacancies in the otherwise empty (full) CF LL's.

The CF picture turns out very useful for the description of many properties of the QP's (e.g., size of cyclotron orbits or LL degeneracy) or of the FQH systems in general (e.g., the values of ν at which the incompressibility results from a complete filling of the QP/CF LL's). However, the QP–QP interactions important for the present problem in the CF model arise as a combination of rather complicated two- and three-body gauge interactions between charges and fluxes, and because of this difficulty, are usually neglected. It is therefore important to realize that the QP–QP interaction really is a Coulomb interaction between a pair of charged particles. For example, for two identical QP's it is repulsive, and at long range it is similar to the electron–electron repulsion, only reduced in magnitude by $(2p+1)^2$ because of a smaller QP charge. On the other hand, the exact form of the QP–QP interaction at short range (where it is different from the electron–electron repulsion because of the particular QP charge-density profile) has been quite accurately calculated numerically [6].

3. Haldane hierarchy and Jain sequence

Knowing that the QE–QE and QH–QH interactions are generally repulsive, Haldane proposed [5] condensation of QE's and QH's into the hierarchy of "daughter" states at the series of Laughlin values of ν_{QE} or ν_{QH}. In these states, the appropriate QP's correlate with one another in the same way as the electrons do in the Laughlin states, and their elementary excitations are simply a new gen-

eration of QP's. Assuming such QP condensation at each level of the hierarchy one would predict incompressibility of the whole electron system at all filling factors given by any odd-denominator fraction. This is in striking disagreement with the experiments, and the reason for this discrepancy is that although Coulombic, the QP–QP interaction at short range is not quite identical to the electron–electron repulsion responsible for the Laughlin correlations. As a result, the QP's form Laughlin liquids only at very few of the Laughlin fractions, which eliminates all but a few valid "hierarchy" fractions [6], in good agreement with the experiment. The same series of fractions arise naturally in the CF picture. These are the states at $\nu = (2p + 1/n)^{-1}$, corresponding to a number n of completely filled CF LL's each carrying flux $2p\phi_0$.

The new FQH states [1] occur at the values of ν from outside the Jain sequence, and thus corresponding to only partial filling of a CF LL. Hence, in the CF picture, their incompressibility implies role of CF–CF interactions. In the QP hierarchy picture, the new states either coincide with the "invalid" fractions (e.g., $\nu = 4/11$) or are new fractions altogether (e.g., $\nu = 3/8$). In both cases it is clear that the origin of observed incompressibility lies in the special form of QP–QP correlations, and that these correlations are of a new (non-Laughlin) type.

4. QP–QP pseudopotential

The nature of QP correlations depends critically on the form of pseudopotential $V_{\mathrm{QP}}(\mathcal{R})$ describing their pair interaction energy V_{QP} as a function of relative pair angular momentum $\mathcal{R}$. We have shown earlier [11, 12] that the correlations are of the Laughlin type (i.e., the particles tend to avoid pair states with one or more of the smallest values of $\mathcal{R} = 1, 3, \ldots$) only if $V(\mathcal{R})$ is "superharmonic" at the relevant values of $\mathcal{R}$ for a given filling factor ν (specifically, at $\mathcal{R} = 2p - 1$ for $\nu \sim (2p + 1)^{-1}$, where $p = 1, 2, \ldots$). Laughlin correlations defined in this way justify reapplication of the CF picture to the QP's to select the lowest states of the whole many-body spectrum, and lead to the incompressible QP "daughter" states of the standard CF hierarchy [9]. The superharmonic repulsion is defined as one for which V decreases more quickly than linearly as a function of the average particle–particle separation $\langle r^2 \rangle$ for the consecutive pair eigenstates labeled by $\mathcal{R}$. In spherical geometry [5], most convenient for finite-size calculations, this means that V increases more quickly than linearly as a function of $L(L + 1)$, i.e., of the squared total pair angular momentum $L = 2l - \mathcal{R}$, where l is the single-particle angular momentum.

The qualitative behavior of the QP–QP interaction pseudopotential $V_{\mathrm{QP}}(\mathcal{R})$ at short range is well known from numerical studies of small systems [6, 9, 10]. On the other hand, the repulsive character of the QP–QP interaction and the long-range behavior of $V_{\mathrm{QP}}(\mathcal{R}) \sim \mathcal{R}^{-1/2}$ follow from the fact that QP's are charged particles (the form of QP charge density affects V_{QP} only at short range, comparable to the QP size). Combining the above arguments, it is clear that the dominant

features of V_{QE} are the small value at $\mathcal{R} = 1$ and a strong maximum at $\mathcal{R} = 3$. Analogous analysis for the QH's yields maxima at $\mathcal{R} = 1$ and 5, and nearly vanishing $V_{QH}(3)$.

5. QP pairing

It is evident that V_{QE} does not support Laughlin QE–QE correlations. Instead, we expect that at least some of the QE's will form pairs (QE$_2$) at $\mathcal{R} = 1$. A paired state would be characterized by a greatly reduced fractional parentage $\mathcal{G}$ [11] from the strongly repulsive $\mathcal{R} = 3$ state compared to the Laughlin correlated state, and have the lower total interaction energy $E = \frac{1}{2}N(N-1)\sum_{\mathcal{R}} \mathcal{G}(\mathcal{R})V(\mathcal{R})$. Let us stress that such pairing is not a result of some attractive QE–QE interaction, but due to a tendency to avoid the most strongly repulsive $\mathcal{R} = 3$ pair state. At sufficiently high QE density this can only be achieved by having significant $\mathcal{G}(1)$, which can be interpreted as pairing into the QE$_2$ molecules. By analogy, the QH pairing is expected at $\mathcal{R} = 3$. The range of ν_{QP} at which pairing can be considered is limited by the condition that the separation between the pairs must exceed the pair size. While for the QE pairs this is satisfied at any $\nu_{QE} < 1$, the QH pairing can only occur at $\nu_{QH} < 1/3$.

6. Laughlin correlations between pairs

Having established that the QP fluid consists of (bosonic) QP$_2$ molecules, the QP$_2$–QP$_2$ interactions need be studied to understand correlations. The QP$_2$–QP$_2$ interaction is described by an effective pseudopotential $V_{QP_2}(\mathcal{R})$ that includes correlation effects caused by the fact that the two-pair wave function must be symmetric under exchange of the whole QP$_2$ bosons and at the same time antisymmetric under exchange of any pair of the QP fermions. This problem is analogous to that of interaction between the electron pairs in the $n = 1$ LL [12].

Although we do not know $V_{QP_2}(\mathcal{R})$ accurately, we expect that since it is due to the repulsion between the QP's that belong to different QP$_2$ pairs, it might be superharmonic at the range corresponding to the QP$_2$–QP$_2$ separation. Our preliminary numerical results for four QE's seem to support this idea. However, in contrast to the $n = 1$ electron LL [12], the lack of accurate data for V_{QP} at the intermediate range makes such calculations uncertain.

7. Condensed pair states

The assumption of Laughlin correlations between the QP$_2$ bosons implies the sequence of Laughlin condensed QP$_2$ states that can be conveniently described using the "composite boson" (CB) model [12]. Let us use spherical geometry and consider the system of N_1 fermions (QP's) each with (integral or half-integral)

angular momentum l_1 (i.e., in a LL of degeneracy $g_1 = 2l_1 + 1$). Neglecting the
finite-size corrections, this corresponds to the filling factor $\nu_1 = N_1/g_1$. Let the
fermions form $N_2 = \frac{1}{2}N_1$ bosonic pairs each with angular momentum $l_2 = 2l_1 - \mathcal{R}_1$,
where $\mathcal{R}_1$ is an odd integer. The filling factor for the system of pairs, defined as
$\nu_2 = N_2/g_2$ where $g_2 = 2l_2 + 1$, equals to $\nu_2 = \frac{1}{4}\nu_1$. The allowed states of two
bosonic pairs are labeled by total angular momentum $L = 2l_2 - \mathcal{R}_2$, where $\mathcal{R}_2$ is an
even integer. Of all even values of $\mathcal{R}_2$, the lowest few are not allowed because of the
Pauli exclusion principle applied to the individual fermions. The condition that the
two-fermion states with relative angular momentum smaller than $\mathcal{R}_1$ are forbidden,
is equivalent to the elimination of the states with $\mathcal{R}_2 \leq 4\mathcal{R}_1$ from the two-boson
Hilbert space. Such a "hard core" can be accounted for by a CB transformation
with $4\mathcal{R}_1$ flux quanta attached to each boson [13]. This gives effective CB angular
momentum $l_2^* = l_2 - 2\mathcal{R}_1(N_2 - 1)$, LL degeneracy $g_2^* = g_2 - 4\mathcal{R}_1(N_2 - 1)$, and
filling factor $\nu_2^* = (\nu_2^{-1} - 4\mathcal{R}_1)^{-1}$.

The CB's defined in this way condense into their only allowed $l_2^* = 0$
state ($\nu_2^* = \infty$) when the corresponding fermion system has the maximum den-
sity at which pairing is still possible, $\nu_1 = \mathcal{R}_1^{-1}$. At lower filling factors, the
CB LL is degenerate and the spectrum of all allowed states of the N_2 CB's
represents the spectrum of the corresponding paired fermion system. In partic-
ular, using the assumption of the superharmonic form of boson–boson repul-
sion, condensed CB states are expected at a series of Laughlin filling factors
$\nu_2^* = (2q)^{-1}$. Here, $2q$ is an even integer corresponding to the number of additional
magnetic flux quanta attached to each CB in a subsequent CB transformation,
$l_2^* \rightarrow l_2^{**} = l_2^* - q(N_2 - 1)$, to describe Laughlin correlations between the original
CB's of angular momentum l_2^*. From the relation between the fermion and CB
filling factors, $\nu_1^{-1} = (4\nu_2^*)^{-1} + \mathcal{R}_1$, we find the following sequence of fractions cor-
responding to the Laughlin condensed pair states, $\nu_1^{-1} = q/2 + \mathcal{R}_1$. Finally, we set
$\mathcal{R}_1 = 1$ for the QE's and $\mathcal{R}_1 = 3$ for the QH's, and use the hierarchy equation [6],
$\nu^{-1} = 2p + (1 \pm \nu_{QP})^{-1}$, to calculate the following sequences of electron filling
factors, ν, derived from the parent $\nu = (2p + 1)^{-1}$ state

$$\nu^{-1} = 2p + 1 \mp (2 + q/2)^{-1}, \tag{1}$$

where "+" corresponds to the QE's and "–" to the QH's. Remarkably, all the
fractions reported by Pan et al. are among those predicted for the $\nu = 1/3$ parent.
Note also that the same values of $q = 1$, 2, 4, and 8 describe both observed QE
and QH states. This indicates similarity of the QE–QE and QH–QH pseudopo-
tentials and suggests that both V_{QE_2} and V_{QH_2} may be superharmonic only at the
corresponding four values of $\mathcal{R}$ (in such case, the remaining fractions could not be
observed even in most ideal samples).

8. Conclusion

We have studied the QP–QP interactions leading to novel spin-polarized
FQH states in the lowest LL. Using the knowledge of QP–QP pseudopotentials

and a general dependence of the form of correlations on the super- or subharmonic behavior of the pseudopotential, we have shown that QP's form pairs over a certain range of filling factor ν_{QP}. Then, we argued that the correlations between the QP pairs should be of Laughlin type and proposed a hierarchy of condensed paired QP states. The proposed hierarchy of fractions agrees remarkably well with the recent experiment of Pan et al. [1].

Acknowledgments

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