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Pair-Distribution Functions of Laughlin Quasielectrons in Partially Filled Composite Fermion Levels

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Pair-distribution functions $g(r)$ of the Laughlin quasielectrons are calculated in the fractional quantum Hall states at electron filling factors $\nu = 4/11$ and 3/8. They all have a shoulder at a medium range, supporting the idea of quasielectron cluster formation. The intra- and inter-cluster contributions to $g(r)$ are identified. The average cluster sizes are estimated; pairs and triplets of quasielectrons are suggested at $\nu = 4/11$ and 3/8, respectively.

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1. Introduction

Pan et al. [1] have recently observed fractional quantum Hall effect (FQHE) in a spin-polarized two-dimensional electron gas (2DEG) at electron filling factors $\nu = \frac{4}{11}$, $\frac{3}{8}$, and $\frac{5}{13}$. These values correspond to $\nu_{\mathrm{QE}} = \frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{3}$ of the Laughlin quasielectrons (QE's), respectively. In the composite fermion (CF) model [2], each QE corresponds to a CF in the second Landau level (LL). Pan's discovery implies that the QE's or CF's can also form incompressible states when partially filling a shell. This could not be predicted by a simple analogy with known fractional electron liquids (Laughlin [3], Jain [2], or Moore–Read [4] states), because of a different form of the QE–QE interaction [5].

From exact numerical diagonalization on a Haldane sphere [6], we have obtained the energy spectra and wave functions of up to 14 interacting QE's. We have identified the series of finite-size liquid ground states with a gap, which extrapolate to the experimentally observed incompressible FQH states. In these states, we have calculated QE–QE pair-distribution functions $g(r)$, and showed that they increase as $\sim r^2$ at short range and have a pronounced shoulder at a medium range. This behavior supports the idea of QE cluster formation, suggested earlier [7] from the analysis of QE–QE interaction pseudopotential. The $g(r)$ is decomposed into short- and long-range contributions, interpreted as correlations between the QE's from the same or different clusters. The inter-cluster QE–QE correlations appear to be the same in all three $\nu_{\rm QE} = \frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{3}$ states. The cluster size on the other hand does depend on ν, and we argue that the QE's form pairs at $\nu_{\rm QE} = \frac{1}{3}$ and triplets at $\nu_{\rm QE} = \frac{1}{2}$.

2. Numerical calculations

To investigate the system we use Haldane's idea [6] of putting N particles of charge q on a spherical surface of radius R. Dirac monopole of strength $2Q$ placed in the center of the sphere is the source of magnetic field B, and $2Q\phi_0 = 4\pi R^2 B$. Here $\phi_0 = hc/q$ is the elementary flux. Using the definition of the magnetic length, $\lambda = \sqrt{\hbar c/qB}$, this can be written as $Q\lambda^2 = R^2$. In the following, λ denotes the QE magnetic length corresponding to the fractional charge $q = -e/3$.

Each particle on the lowest LL has angular momentum $l = Q$ and its z-component m can take on values from $-l$ to l so degeneracy of the shell is equal to $\Gamma = 2l + 1$. The single-particle configurations $|m_1, m_2, \ldots, m_N\rangle$ can be chosen as a basis of the Hilbert space. Diagonalizing the matrix of interaction Hamiltonian we obtain energy as a function of total angular momentum L. Since number of electrons required to represent FQHE at $\nu = \frac{3}{8}$, $\frac{4}{11}$, and $\frac{5}{13}$ is too large to be diagonalized exactly, we neglect quasiparticles on the lowest CF-LL and take into account only CF's from the second, partially filled CF-LL. Interaction between these particles is given by QE–QE pseudopotential $V(\mathcal{R})$, where the relative pair angular momentum $\mathcal{R} = 2l - L$ increases with average pair separation $\sqrt{\langle r^2\rangle}$.

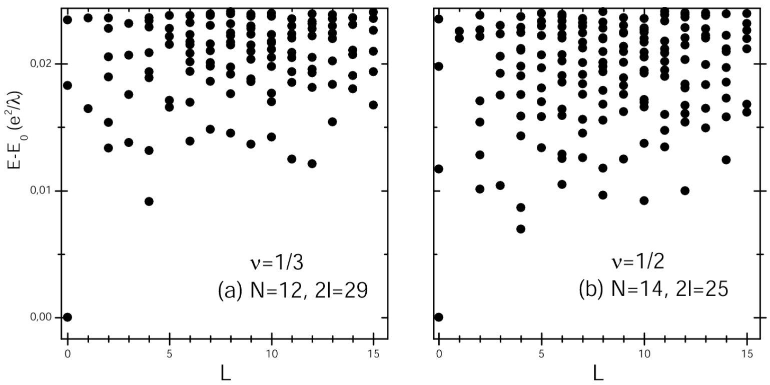

Fig. 1. Excitation energy spectra (E_0 is the ground state energy) of N interacting QE's on a sphere, at the values of CF-LL degeneracy $\Gamma = 2l + 1$ corresponding to the incompressible ground states at the QE filling factors $\frac{\nu=1}{3}$ (a) and $\frac{1}{2}$ (b).

Showing small value at $\mathcal{R} = 1$ and strong maximum at $\mathcal{R} = 3$ [5, 8, 9], this pseudopotential differs from a superlinearly decreasing one, describing the Coulomb interaction among electrons. This difference precludes the Laughlin correlations among QE's.

Incompressible states are represented by these combinations of particles number N and shell degeneracy Γ for which ground state is significantly separated from excited states. For the Laughlin liquid it occurs for $2l = 3N - 3$. In contrast, for QE such situation takes place for sequence $2l = 3N - 7$ (Fig. 1) representing $\nu_{\mathrm{QE}} = \frac{1}{3} \approx N/\Gamma$ and for its particle–hole conjugates series (i.e. sequences with N replaced by $\Gamma - N$) $2l = \frac{3}{2}N + 2$ ($\nu = \frac{2}{3}$) [7].

Another sequence was anticipated at $2l = 2N + 1$ and $2l = 2N - 3$ to represent the infinite $\nu = \frac{3}{8}$ FQH state. Unfortunately, we are unable to compute the spectra for $N \geq 18$ and it seems that in this case incompressible states occur only for odd values of $\frac{1}{2}N$. Hence we have only two configurations for each sequence. Nevertheless, we expect that the ground state for $N = 14$ and $2l = 25$ (and its particle–hole counterpart at $N = 12$ and the same $2l = 25$) may possibly represent the $\nu = \frac{3}{8}$ FQH state (i.e., have similar correlations causing incompressibility).

3. Pair-distribution function

Pair-distribution functions $g(r)$ were calculated as expectation value of operator $\hat{g}(r) = (2/N)^2\delta(R\theta - r)$ for previously found wave function of non-degenerate ground state of QE system. Here, θ is the relative angle on a sphere and r measures interparticle distance along the sphere surface. Denoting infinitesimal area by $\mathrm{d}S = 2\pi R^2 \mathrm{d}(\cos\theta)$ or (in magnetic units) by $\mathrm{d}s = \mathrm{d}S/2\pi\lambda^2$, we get a normalization condition in large systems

$$\int [1 - g(r)]\mathrm{d}s = \frac{2l}{N} \to \nu^{-1}. \tag{1}$$

Since $\mathrm{d}s = l\mathrm{d}(\cos\theta)$, a "local filling factor" can also be defined as $\nu(r) = \mathrm{d}N/\mathrm{d}s = (N/2l)\, g(r)$, and it satisfies $\nu(\infty) = \nu$ and $\int \nu(r)\mathrm{d}s = N - 1$.

For a full lowest LL ($\nu_{\mathrm{e}} = 1$) the pair-distribution function is [10]:

$$g_0 = 1 - \exp(-r^2/2\lambda^2). \tag{2}$$

For the sequence $2l = 3N - 7$, representing $\nu_{\mathrm{QE}} = \frac{1}{3}$ state, the behavior of $g(r)$ looks like in Fig. 1a. For small r it is similar to g_0, i.e., $g(r) \propto r^2$, but the curve bends around $r = 2.5\lambda$. The shoulder appears for all calculated N, and evidently it is not an effect of a finite number of particles. Similar shoulders occur also in $g(r)$ for $\nu_{\mathrm{QE}} = \frac{2}{3}$, i.e. $\nu = \frac{5}{13}$ state. In Fig. 2b we have compared this curve with pair-correlation functions obtained for other incompressible FQH states (full LL, Laughlin $\nu = \frac{1}{3}$ state, or Moore–Read half-filled state). Qualitative difference between these functions indicates that correlations responsible for the FQHE at new $\nu = \frac{4}{11}$ and $\frac{3}{8}$ fillings states cannot be the same.

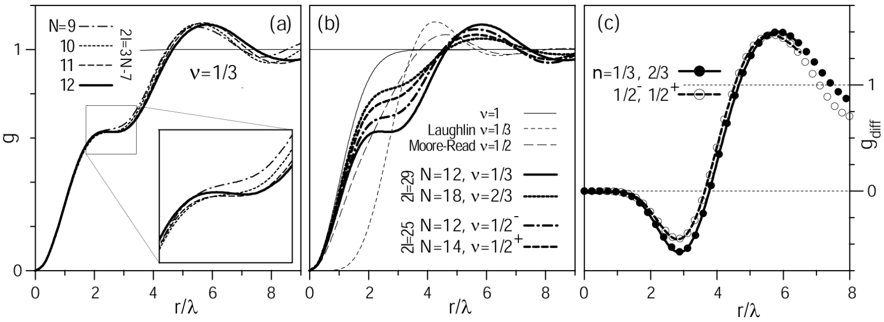

Fig. 2. QE–QE pair-distribution functions $g(r)$ of the incompressible ground states at different QE filling factors ν. (a) Curves for $\nu = \frac{1}{3}$ and different QE numbers N; inset — blowup of the plateau region; (b) curves for QE's at different ν (thick lines) compared to some known incompressible states of electrons; (c) the "remainder" $g_{\text{diff}}(r)$ defined by Eq. (3).

The shoulder in $g(r)$ may result from different correlation between QE's close to each other and between distant ones, consistently with an idea of clusters formation [7, 11, 12]. To find inter- and intra-cluster correlation we decompose $g(r)$ into g_0 describing a full lowest LL and a (properly normalized) "remainder" g_{diff}:

$$g(r) = \alpha g_0(r) + (1-\alpha) g_{\text{diff}}(r). \tag{3}$$

Parameter α is calculated as the limit of g/g_0 at $r \to 0$. The curves g_{diff} obtained for $\nu_{\text{QE}} = \frac{1}{3}$ and $\frac{1}{2}$ are very similar and for the pairs of particle–hole conjugate states they are identical (Fig. 2c). The vanishing of $g_{\text{diff}}(r)$ at short-range reflects isolation of QE's belonging to different clusters. Negative value around $r = 3$ indicates that intra-cluster correlations are accurately described by g_0 only within a certain radius.

Similar form of $g(r)$ was found [13] for broken-symmetry Laughlin states, in which the shoulder results from angular averaging of an anisotropic function $g(r, \phi) \sim r^2$ or r^6, depending on ϕ. However, the present case of QE's is different, because $g(r)$ is isotropic (nondegenerate ground state wave functions have $L = 0$) and the shoulders result from *radial* averaging of inter- and intra-cluster correlations (beginning as $\sim r^2$ and a higher power of r at short range, respectively).

4. Average cluster size

Let us assume that our system consists of independent clusters, each with K particle. By independence of the clusters we mean that inter-cluster correlations do not affect the local filling factor $\nu(r)$ at short range. Such system for small r should have $\nu(r)$ similar to $\nu_K(r)$ of a single K-cluster. For that cluster, which on a sphere is the K-particle state with the maximum total angular momentum

$L = Kl - \frac{1}{2}K(K-1)$, we have calculated $g_K(r)$ and then by taking $\nu_K(r) = \nu g_K(r) \approx \beta_K g_0(r)$ we found prefactor β_K for different values of K and $2l$. Some results are listed in Table. Now we can compare these values β_K with $\beta = (N/2l)\alpha$ obtained for our investigated incompressible N-QE systems.

TABLE

Parameters β_K of the short-range approximation $\nu(r) \sim \beta g_0(r)$ obtained for independent clusters of size K.

$2l$	β_2	β_3	β_4	β_5	β_6
25	0.2768	0.4196	0.5110	0.5765	0.6269
29	0.2730	0.4134	0.5029	0.5669	0.6159
60	0.2609	0.3938	0.4778	0.5372	0.5821
∞	0.2500	0.3763	0.4555	0.5110	0.5527

Obviously, assumption that our clusters are independent is only approximation. Apart from that, we know that $\nu_{\mathrm{QE}} = \frac{1}{3}$ states occur for all N (not only divisible by two or three) so different clusters could have different sizes K. Also, parameters α and β are size-dependent and their extrapolation to large systems is not very reliable based on limited number of N-QE systems we are able to diagonalize. Nevertheless, we can test the method using the Moore–Read state known to be paired [4, 14]. Our calculation for $N = 14$ and $2l = 25$ gives $\beta_{\mathrm{MR}} = 0.336$, somewhat larger than β_2. Hence, we shall assume that β_K in general underestimates the actual value of β in a many-body K-clustered state.

For the QE system at $\nu_{\mathrm{QE}} = \frac{1}{3}$ ($N = 12$ and $2l = 29$), we got $\beta = 0.319 \approx \beta_{\mathrm{MR}}$, and $\beta = 0.479$ for $\nu_{\mathrm{QE}} = \frac{1}{2}^{+}$ ($N = 14$ and $2l = 25$). In the light of these facts it seems probable that QE's (on the average) form pairs at $\nu_{\mathrm{QE}} = \frac{1}{3}$ and triplets at $\nu_{\mathrm{QE}} = \frac{1}{2}$.

5. Conclusion

Pair-distribution functions of new FQH states differ significantly from these known for electrons at $\nu = 1$, $\frac{1}{3}$ (Laughlin), or $\frac{1}{2}$ (Moore–Read). For small r they behave like $g(r)$ for electrons occupying completely the lowest LL, and then for $r \approx 2.5\lambda$ they have a shoulder. Our results support hypothesis that QE's form clusters. Short- and long-range contribution to $g(r)$, describing correlations between the QE's from the same and different clusters, have been found. Both correlations depend rather weakly on ν. We also estimated average size of the clusters, which seem to form pairs at $\nu_{\mathrm{QE}} = \frac{1}{3}$ and triplets at $\nu_{\mathrm{QE}} = \frac{1}{2}$.

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