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Physica E 40 (2008) 1111–1114
PHYSICA E
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On the microscopic origin of fractional quantum Hall states with partially filled quasiparticle shells

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Available online 29 August 2007

Abstract

Residual interaction among quasiparticles in partially filled composite Fermion (CF) level and the types of correlations they support are investigated. The pseudopotential describing the interaction between quasielectrons (QEs) as a function of relative angular momentum $\mathscr{R}$ is "subharmonic" for $\mathscr{R}=1$. As a consequence the QEs at $\nu_{\rm QE}=\frac{1}{3}$ cannot be Laughlin correlated, but they tend to form pairs or larger clusters. Interpretation of numerical results for small systems can be problematic, because a given value of particle number N and level degeneracy $2\ell+1$ can correspond to several different filling factors.

PACS: 73.43.Cd; 71.10.Pm

Keywords: Fractional quantum Hall; Quasiparticle correlations; Composite Fermions; Residual interactions

1. Introduction

The experimental observation of new fractional filling factors [1] in fractional quantum Hall (FQH) systems [2] opens an interesting debate about the nature of correlations among quasiparticles of quantum Hall liquids. There have been several attempts [3–5] to explain the spin polarized FQH states at filling factor $\nu=\frac{4}{11}$ as a "second generation" of composite Fermions quasielectrons (CFQEs) whose residual interaction produces an incompressible daughter state at $\nu_{\rm QE}=\frac{1}{3}$. Sitko et al. [6] introduced this CF hierarchy picture but found, by numerical diagonalization, that there was no FQH daughter state at $\nu_{\rm QE}=\frac{1}{3}$. Wójs and Quinn [7] demonstrated that the CF picture is applicable only if the Fermions support Laughlin correlations (LCs). This required the pseudopotential $V_{\rm QE}(L)$ describing the interactions to be "superharmonic" (i.e. to rise faster than the eigenvalue of the square of L, the pair angular momentum). $V_{\rm QE}(L)$ has been determined by numerical diagonalization [6,7] and it is not "superharmonic" at relative angular momentum $\mathscr{R}=2\ell-L$ equal to unity for QEs of the Laughlin $\nu=\frac{1}{3}$ state. Instead of being Laughlin correlated, the QEs tend to form pairs or larger clusters. $V_{\rm QE}(L)$ has been used to diagonalize systems up to $N_{\rm QE}=12$ QEs for different values of $2\ell_{\rm QE}$. A family of FQH daughter states is found at $2\ell_{\rm QE}=3N_{\rm QE}-7$ (and also at $2N_{\rm QE}+1$ and $4N_{\rm QE}-9$ or $(\frac{7}{2})N_{\rm QE}-6$ for some values of $N_{\rm QE}$). Chang and Jain [3] have reported numerical studies of N electron systems ($N=8,12,16,20,24$) and 2ℓ values selected so that first generation of CFQE occurs at $2\ell_{\rm QE}=3N_{\rm QE}-3$. Only the $(N_e,2\ell_e)=(12,29)$ and $(20,51)$ systems were found to have incompressible ground states. We suggest that these two states belong to our FQH families at $\ell=2N_{\rm QE}+1$ and $2\ell_{\rm QE}=4N_{\rm QE}-9$ or $2\ell_{\rm QE}=7N_{\rm QE}/2-6$, corresponding to $\nu_{\rm QE}=\frac{1}{2}$ and $\frac{1}{4}$ or $\frac{7}{2}$, respectively [8,9].

2. Residual interactions

The residual interactions between quasiparticles are determined by studying the spectrum of a system containing N_e electrons in angular momentum shell $2\ell=3N_e-5$

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1386-9477/$ - see front matter
doi:10.1016/j.physe.2007.08.018

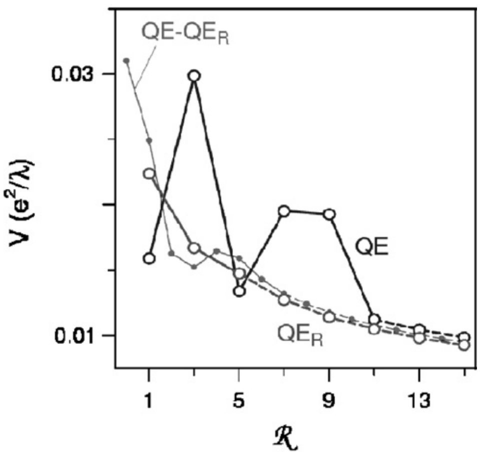

Fig. 1. Pseudopotentials describing QP interaction.

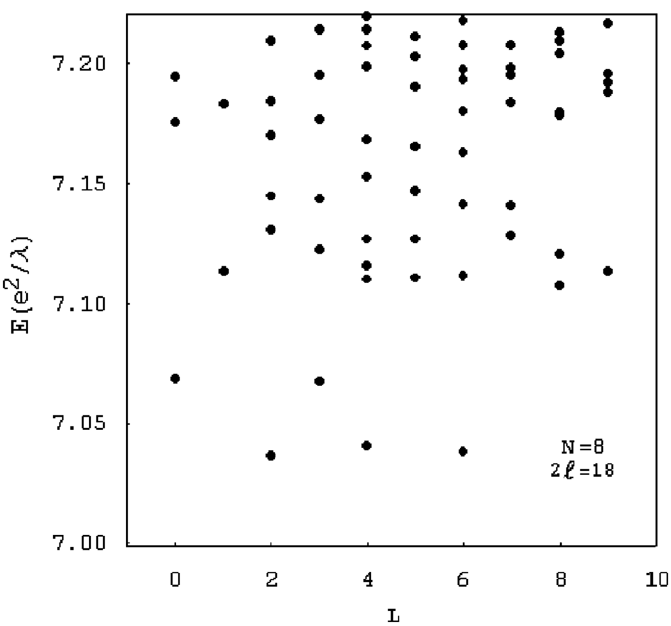

Fig. 2. The energy spectrum of eight electrons in the angular momentum shell of $2\ell = 18$.

in Haldane's spherical geometry [10]. It contains two QEs or two QEs with reversed spin (QERs). The pseudopotential describing the residual interactions of QEs $V_{\mathrm{QE-QE}}$ is determined, up to an irrelevant constant, from the lowest band of states with the total spin $\Sigma = \frac{1}{2}N_e$, while its counterpart for QERs $V_{\mathrm{QER-QER}}$ is determined from the system with spin $\Sigma = \frac{1}{2}N_e - 2$ (two flipped spins). This type of calculation produces accurate results for relatively small relative angular momentum $\mathscr{R}$. To account for the finite size errors, the pseudopotentials obtained for different values of N_e are plotted vs. N_e^{-1} and the results are extrapolated for infinitely large systems. The results are shown in Fig. 1.

Using similar pseudopotentials, Wójs et al. [11] suggests that a spin phase transition would occur when parallel magnetic field and/or width of quantum well are changed for a $\frac{4}{11}$ filling factor.

3. Quasiparticle correlations

The pseudopotential $V_{\mathrm{QE-QE}}$ has an important feature: it is "subharmonic" [12] for $\mathscr{R} = 1$, making the residual interaction different from Coulomb interaction. As a consequence of the "subharmonic" behavior of $V_{\mathrm{QE-QE}}$ at $\mathscr{R} = 1$, the incompressible quantum liquid (IQL) state at $\nu_{\mathrm{QE}} = \frac{1}{3}$ cannot be a Laughlin correlated one. The QEs tend to form pairs or larger clusters. The angular momentum

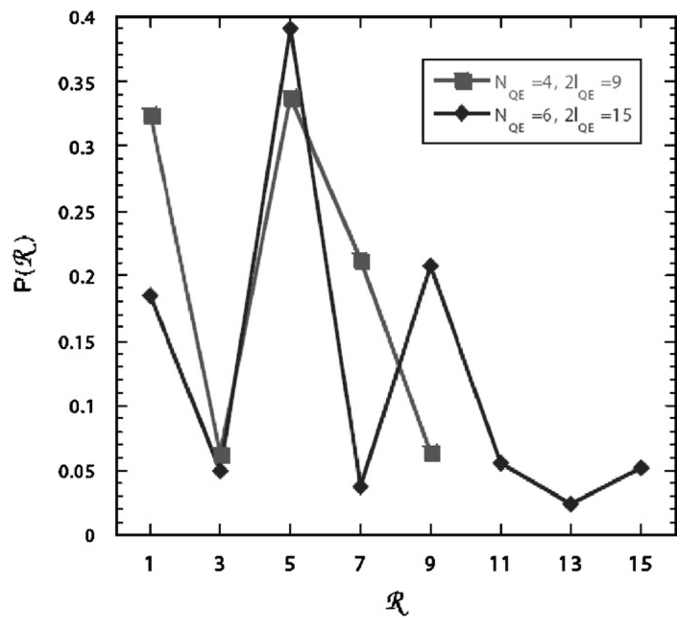

Fig. 3. Plot of $P(\mathscr{R})$ at $(N_{\mathrm{QE}}; 2\ell_{\mathrm{QE}}) = (4;9)$ and $(6;15)$ in the $L = 0$ ground state.

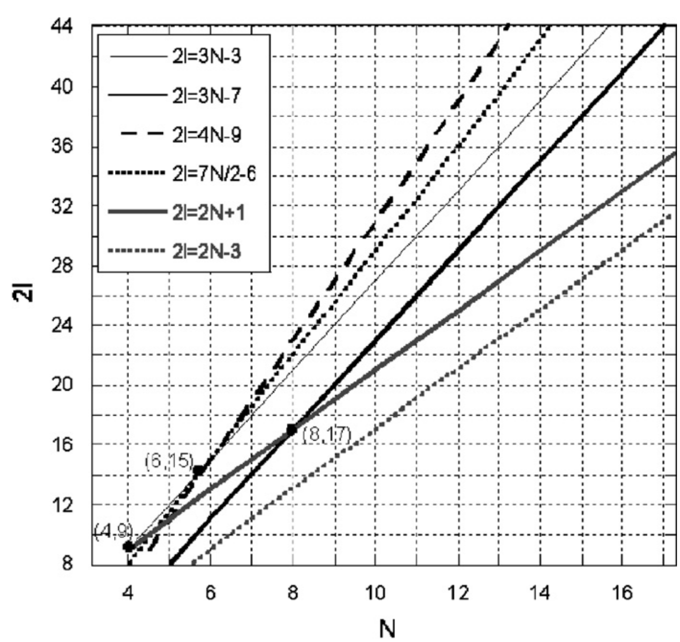

Fig. 4. 2ℓ vs. N for Laughlin correlated states ($2\ell = 3N - 3$) and for the families $2\ell = 3N - 7$ ($\frac{1}{3}$ filling factor for QEs), $2N + 1$, $2N - 3$ ($\frac{1}{2}$ filling factor for QEs) and $4N - 9$ and $7N/2 - 6$.

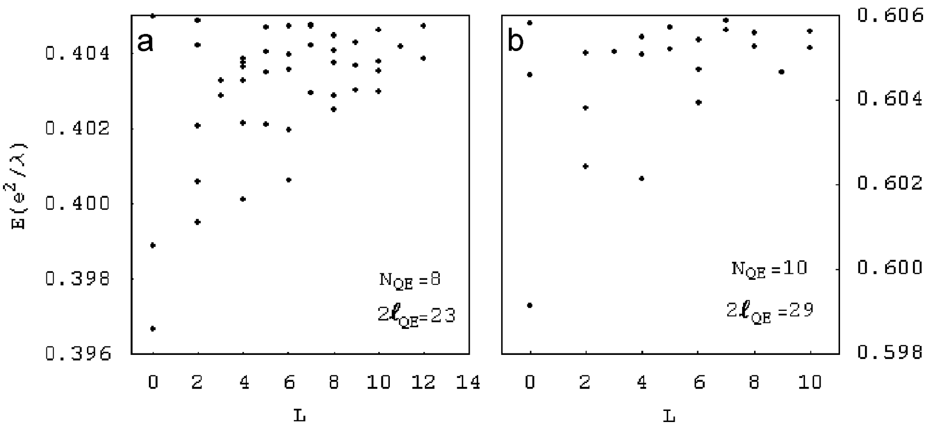

Fig. 5. The energy spectrum of (a) 8 QEs with $2\ell = 23$ (corresponding to a filling factor of $\frac{1}{4}$) and (b) 10 QEs with $2\ell = 29$ (corresponding to a filling factor of $\frac{2}{7}$).

shell at which IQL states occur is different from those of electrons. For example, the IQL state with $\nu_{QE} = \frac{1}{3}$ occurs for $2\ell = 3N - 7$ (at all values of N) rather than the well-known $2\ell = 3N - 3$ for a Laughlin correlated $\nu = \frac{1}{3}$ state.

The first attempt to explain new states like that with $\frac{4}{11}$ filling factor was made by Sitko et al. [6]. Their idea was to reapply the Chern–Simons transformation to the partially filled CF shells. It would eventually lead [13] to a hierarchy of filling factors that generalizes the Jain's sequence [14,15] and generates all Haldane's fractions. In this way the $\frac{4}{11}$ state could be thought as an LC state of QEs with $\frac{1}{3}$ filling factor.

A simple example proves that such generalization does not work in the case of QEs. The CF transformation applied to a system of eight electrons in the angular momentum shell of $2\ell = 18$ would generate three QEs with $2\ell_{QE} = 6$. This corresponds to a filling factor of $\nu_{QE} = \frac{1}{3}$. But, as one can notice from Fig. 2, this does not produce an $L = 0$ ground state [12,16,6].

The ground state pair amplitude, obtained from our numerical diagonalization, plotted in Fig. 3 as a function of relative angular momentum $\mathscr{R}$ indicates a clear tendency of QEs in the ground state to avoid pairs with $\mathscr{R} = 3$, in favor of $\mathscr{R} = 1, 5$. This is different from the LC electron liquids where pair states with $\mathscr{R} = 1$ are avoided. The states plotted here correspond to filling factors of $\nu_{QE} = \frac{1}{2}$ and $\frac{1}{4}$. The fact that pairing probability $P(\mathscr{R})$ is a minimum for $\mathscr{R} = 3$, not for $\mathscr{R} = 1$ as in an LC state, is incontrovertible evidence that QEs do not form a "second generation" of CFs.

The interpretation of the results involving small systems poses a difficult problem. In Fig. 4 we draw straight lines (for QEs) corresponding to Laughlin correlated $2\ell = 3N - 3$, and the "clustered QE" ($2\ell = 2N + 1$, $3N - 7$, $4N - 9$ and $7N/2 - 6$) states. It can be noticed that lines representing different series of IQL states cross in the region of small N. The FQH states at $\nu_{QE} = \frac{1}{2}$ and $\frac{1}{4}$ or $\frac{7}{2}$ happen to fall at the same values of (N_{QE}, $2\ell_{QE}$) as a LC $\nu_{QE} = \frac{1}{3}$ state would, if one existed. In such conditions it is easy to misinterpret various points as being LC states. One such example is the system containing six QEs in the angular momentum shell of $2\ell = 15$. Fig. 3 shows clearly that this cannot be a Laughlin correlated state with $2\ell = 3N_{QE} - 3$.

IQL ground states ($L = 0$) have been observed for angular momentum shells of $2\ell_{QE} = (7/2)N_{QE} - 6$ and $2\ell_{QE} = 4N_{QE} - 9$ as noticed in Fig. 5. The existence of these states is strongly dependent on the features of the QEs pseudopotential for intermediate values of $\mathscr{R}$. As a consequence, the use of small electron systems reduces the accuracy of such calculations.

4. Conclusions

The properties of the pseudopotentials describing the quasiparticle interaction are studied. The "subharmonic" pseudopotential, describing the QE interaction of a spin polarized system is different from a Coulomb potential. It does not produce Laughlin correlations, but favors the formation of pairs or larger clusters. This prevents the second generation of CF in partially filled QE shells. Finite size effects make it difficult to identify with certainty the filling factor of such small systems. The case of ($N_{QE} = 6$, $2\ell_{QE} = 15$) does not exhibit Laughlin correlations. It does not correspond to a filling factor of $\nu_{QE} = \frac{1}{3}$, but to a clustered state. More accurate calculations of pseudopotential values are required for better understanding of the nature of these type of ground states.

Acknowledgments

KSY's work was supported in part by Korean Research Foundation Grant KRF-2006-005-J02804. GS and JQ acknowledge the partial support form Basic Science Program of DOE.

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