Novel Excitonic States and Photoluminescence in Quantum Hall Systems

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Abstract

The formation and possible decay processes of neutral and charged excitonic complexes in electronic integral and fractional quantum Hall systems are discussed. The excitonic complexes are bound states of a small number of the relevant negatively and positively charged quasiparticles (e.g., conduction electrons and valence holes, reversed-spin electrons and spin holes, Laughlin quasielectrons and quasiholes, composite Fermions) that can occur in a 2D electron in the presence of a strong magnetic field. Examples of such bound states are interband neutral and charged excitons, fractionally charged "anyon excitons," spin waves, skyrmions, or "skyrmion excitons." Possible decay processes include radiative recombination, experimentally observed in photoluminescence or spin transitions, important in the context of nuclear spin relaxation.

1 Introduction

Photoluminescence (PL) in quantum Hall systems has been studied for more than a decade [1, 2, 3]. The possibility of extracting from the experimental data information about correlations within the underlying two-dimensional electron gas (2DEG) in a high magnetic field has stimulated their effort. For the ideal theoretical model, in which electrons and holes lie on the same defect free 2D layer at extremely high magnetic field, the PL spectrum is predicted to consist of a single sharp line, resulting from radiative decay of a neutral exciton (X), and to give no information about electron-electron correlations [4]. This results from a "hidden symmetry" associated with the equality of the magnitudes of the particle-particle interactions, $V_{ee} = V_{hh} = -V_{eh}$.

In realistic systems, the admixture of higher Landau levels (LL's) by the Coulomb interaction, the alteration of the interactions by form factors associated with finite quantum well width, and the finite separation of the electron and hole layers by a built-in or an applied electric field lead to a much richer spectrum. Both neutral (X) and charged (X^-) excitons contribute to PL in fractional quantum Hall systems at filling factors $\nu \leq \frac{1}{3}$ [3]. In the ideal model only the triplet $X^-_{\rm td}$ (with electronic spin, S, of unity) is

bound and, of course, it is non-radiative (subscript "d" denotes dark). In real experimental systems, both the singlet, $X_{\rm s}^-$, and triplet, $X_{\rm t}^-$, have one or more bound states. The PL intensity of the $X_{\rm td}^-$ ground state is non-zero, but it is weak compared to the $X_{\rm s}^-$ and the neutral exciton, X. In fact, it was observed only recently [3], when special care (e.g., very low temperature) was taken to detect its weak PL signal. In order to understand the PL process in real systems, we study small systems (confined to the surface of a sphere) by numerical diagonalization. For the simplest systems, the Hilbert space is not restricted to the lowest LL as in the ideal theoretical model, but linear combinations containing up to five LL's are employed. The Coulomb interaction is modified so as to mimic that of a symmetric quantum well of width $w = 11.5$ nm. For systems containing more than three electrons and one hole, we fall back to the single LL approximation, but we allow the electrons and holes to reside on distinct 2D layers separated by a distance d. This breaks the "hidden symmetry" in the simplest way, but it is a reasonable approximation only for the highest magnetic fields.

The numerical studies [5] at $\nu \leq \frac{1}{3}$ were done for electrons systems which are maximally spin polarized (except for the spin reversed electron of the $X_{\rm s}^-$). It is known that for filling factor $\nu = 1$ the ground state is spin polarized, and the lowest energy excitations are spin waves. However, when ν is slightly smaller than unity, a "spin hole," h, in the $\nu = 1$ level is present, and it can spontaneously create and bind a spin wave [6]. Depending on the Zeeman energy, the ground state will contain either free spin holes or "antiskyrmions," S^+, bound states of one or more spin waves and the spin hole, h. For ν slightly larger than unity, the reversed spin electrons, $e_{\rm R}$ (above the $\nu = 1$ filled level) can form skyrmions, S^-, states containing one or more spin waves bound to $e_{\rm R}$. We review some results on spin excitations within the ideal model [7], with the thought of simply replacing on of the spin holes of the $\nu = 1$ states by a valence band hole, v. In this ideal model the spin hole, h, and the valence band hole, v, are distinguishable (they can be assigned a pseudospin to distinguish them), but their interactions with other charged particles are equal in magnitude. This implies that an X^+ made up of a v bound to a spin wave, that is $X^+ = (vhe_{\rm R})^+$, must exist. We consider PL resulting from this antiskyrmion-like excitonic complex. For ν slightly smaller than unity, some antiskyrmions, S^+, are present before introduction of the valence hole. These will avoid the X^+ due to Coulomb repulsion, so the PL at $1-\nu \ll 1$ should be due to isolated X^+ excitonic complexes (and, of course, neutral excitons [8]). For ν slightly larger that unity, some S^- complexes will be present before introduction of the valence band hole. It is possible that the valence hole captures reversed spin electrons from these S^- complexes, that is $v^+ + S^- \rightarrow X + \mathrm{SW}$ or $v^+ + 2S^- \rightarrow X^- + 2$ SW (where SW denotes the spin wave) or that the v creates and binds a spin wave to form a $(vhe_{\rm R})^+ = X^+$. The X^+ has an attractive interaction with the skyrmions that could result in $X^+ + S^- \rightarrow X^- + h^+ + \mathrm{SW}$, or it could decay radiatively before reaching the (X^-, h^+) state, particularly when the initial S^- density is very small. From analogy with the dark triplet $X_{\rm td}^-$ at $\nu \leq \frac{1}{3}$, we can guess that isolated X^- will be "dark," and that the $v-e$ recombination will be primarily with majority spin electrons as suggested by Cooper and Chklovskii [8]. Definitive predictions will require numerical work based on more realistic models than used in much of their work. This makes the present work qualitative and suggestive, and not intended for detailed comparison with experiment.

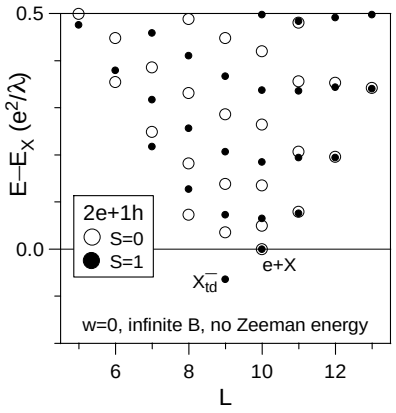

Figure 1: Energy spectrum (energy E vs. angular momentum L) of the $2e+v$ system in the lowest LL, calculated on a Haldane sphere for monopole strength $2Q = 20$. E_X is the exciton energy, and λ is the magnetic length.

2 Energy spectrum and PL for $\nu \leq \frac{1}{3}$

It has become rather standard to diagonalize numerically the Coulomb interaction for a finite system of N electrons confined to a spherical surface which contains at its center a magnetic monopole of strength $2Q$ flux quanta [9]. In the ideal theoretical model only states of the lowest LL are included. For realistic experimental systems (having a finite quantum well width w in a finite magnetic field B) both higher LL's and the modification for the Coulomb matrix elements associated with the envelope functions of the quantum well must be included.

In Fig. 1 we present the energy spectrum for a simple system consisting of two electrons and one valence hole at $2Q = 20$ evaluated in the ideal theoretical model and excluding the Zeeman energy [5]. The solid dots are triplet states (total spin of the electron pair is $S = 1$), and the open circles are singlets ($S = 0$). The state labeled $e+X$ at angular momentum $L = 10$ is a "multiplicative state" consisting of an unbound electron and a neutral exciton (X). Notice that only one bound state ($X_{\rm td}^-$) occurs. It is at $L = 9$ and is called the "dark triplet" because it is forbidden to decay radiatively.

In Fig. 2 similar results are presented for a realistic system consisting of a symmetric GaAs quantum well of width $w = 11.5$ nm at the finite values of the magnetic field $B = 13$, 30, and 68 T. The appropriate electron Zeeman splitting has been included. To achieve even qualitative agreement with experimental data, it has also been necessary to include a number of higher LL's, particularly at the lower magnetic fields. Five LL's were needed to obtain convergence in our calculations.

In Fig. 2c, at the high magnetic field of 68 T, the $X_{\rm td}^-$ at $L = 9$ is still the lowest energy state, but singlet and another triplet bound states occur at $L = 8$, 9, and 10. The singlet at $L = 8$ (no label) and $L = 10$ ($X_{\rm s}^-$), and the triplet at $L = 10$ ($X_{\rm tb}^-$) have roughly

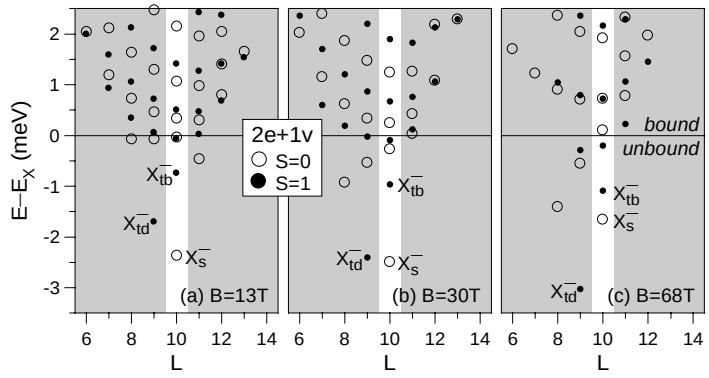

Figure 2: Same as Fig. 1, but for a realistic GaAs quantum well of width $w = 11.5$ nm at the finite values of magnetic field B as shown. The Zeeman energy has been included, and five LL's for both electrons and hole have been used in the diagonalization.

half the binding energy of the $X_{\rm td}^-$ ground state. As the magnetic field B is lowered, the $X_{\rm s}^-$ at $L = 10$ moves down in energy relative to the triplet states, becoming the ground state for B smaller than 30 T (as shown in Figs. 2b and a). This is in agreement with the results obtained by Whittaker and Shields [10]. The spectra are quite sensitive to the experimental parameters. The well width w enters the Coulomb interaction [5] through $V(r) = e^2/\sqrt{r^2+d^2}$, where d is proportional to w. The cyclotron frequencies $\omega_{ce}(B)$ and $\omega_{cv}(B)$ for the electrons and valence hole, and the Zeeman energy $E_{\rm Z}(B)$, are taken from experiment, after Refs. [11] and [12].

Because exact diagonalization gives the eigenfunctions as well as the eigenvalues, it is straightforward to evaluate matrix elements of the luminescence operator $\hat{L} = \int {\rm d}^2 r \hat{\Psi}_e(r)\hat{\Psi}_v(r)$ between an initial state Φ_i of N electrons and one valence hole, and final states Φ_f containing $N-1$ electrons. $\hat{\Psi}_e$ and $\hat{\Psi}_v$ are the annihilation operators for an electron and valence hole respectively. The oscillator strength for the transition [13] from $|\Phi_i\rangle$ to $|\Phi_f\rangle$ is proportional to $|\langle\Phi_f|\hat{L}|\Phi_i\rangle|^2$. For an isolated charged exciton X^- (where $N = 2$) angular momentum conservation forbids the lowest triplet ($X_{\rm td}^-$) from decaying radiatively. The singlet $X_{\rm s}^-$ and the excited triplet $X_{\rm tb}^-$ ("b" stands for "bright") both have finite oscillator strengths which are of the same order of magnitude. These radiative states appear in the light color area at $L = 10$ in Fig. 2.

When additional electrons are present ($N > 2$) radiative decay of the $X_{\rm td}^-$ is not strictly forbidden, since in the recombination process an unbound electron can scatter, changing the momentum of the final state. However, it was found that for $\nu \leq \frac{1}{3}$ such decays are weak because Laughlin correlations of the X^- with unbound electrons inhibit close collisions. The amplitude for radiative decay of the $X_{\rm td}^-$ is estimated [5] to be smaller by one or more orders of magnitude than those of the $X_{\rm s}^-$ and $X_{\rm tb}^-$. It was suggested in [5] that the $X_{\rm td}^-$ would be difficult to see in PL, and that the non-crossing peaks observed by Hayne *et al.* [14] were the $X_{\rm s}^-$ and $X_{\rm tb}^-$. The presence of impurities

relaxes the $\Delta L = 0$ selection rule, and the $X_{\rm td}^-$ peak is clearly observed at very low temperature where the excited $X_{\rm tb}^-$ and $X_{\rm s}^-$ states are sparsely populated [3]. The agreement of experiment [3] and the numerical predictions [5] reinforce the hope of using PL to understand correlations in fractional quantum Hall systems.

3 Fractionally charged excitons

For systems containing more than two or three electrons and a valence hole, it is very difficult to include the admixture of higher LL's caused by Coulomb interactions. However, the "hidden symmetry" can be broken by separating the 2D planes on which the hole and the N electron system reside by a finite length d. Because only the lowest LL is included, such a simple model will be useful only at the highest magnetic fields.

When d is measured in units of magnetic length $\lambda = \sqrt{\hbar c/eB}$, we can identify three regimes: strong coupling (of the valence hole to the electron system) when $d \ll 1$; weak coupling when $d \gg 1$, and intermediate coupling when $d \sim 1$. In the strong coupling region, neutral (X) and charged triplet excitons (X^-) are formed due to the strong $e-v$ interactions. Neutral excitons in their $L = 0$ ground state are almost decoupled from the remaining $N-1$ electron system. At $d = 0$, the hidden symmetry holds, and the excitons are completely decoupled, these states are called multiplicative states. The X^- is a charged Fermion that interacts with the remaining $N-2$ electrons. The generalized composite Fermion (CF) picture [15] describes the low lying excitations and the Laughlin correlations very well.

For $d \gg 1$, the $e-v$ interaction is a weak perturbation on the energies of the N electron system. The low energy spectrum contains bands obtained by adding the angular momentum of the hole ($l_v = Q$) to that of the low lying quasiparticle states of the electrons, obtained from the CF picture or from the numerical diagonalization [13].

For intermediate coupling, the $e-v$ interaction is not strong enough to bind a full electron, but bound states of one or more Laughlin quasielectrons (QE) do occur. These are referred to as fractionally charged excitons (FCX) or anyon excitons.

In Fig. 3 we display the energy spectra for a system of nine electrons and one valence hole at $2Q = 21$, 22, and 23, for $d = 0$, 1.5, and 4. This data is taken from Ref. [16]. It is worth noting that for $d = 0$, the X and the X^- appear at each value of $2Q$, while for $d = 4$ only bound states of the valence hole and Laughlin quasielectrons (QE) occur. In frame (a) the low lying state at $L = 0$ is a multiplicative state consisting of one decoupled neutral X and a Laughlin condensed state of the remaining eight electrons [$2Q = 3(N-1) = 21$ for this Laughlin $\nu = \frac{1}{3}$ state]. A band of states extending from $L = 1$ to $L = 6$ consists of an X^- with $l_{X^-} = \frac{5}{2}$ and a single Laughlin quasihole (QH) with $l_{\rm QH} = \frac{7}{2}$. The solid line drawn through these states represents the pseudopotential $V_{X^-,{\rm QH}}(L)$ of the interacting X^-–QH pair. Initially, this band of states was incorrectly interpreted as a neutral exciton interacting with a magneto-roton [17]. The generalized CF picture [15] gives a simple and natural interpretation of all of the low lying states for all values of the parameter d. In frame (b") the nine electron system contains two QE's. Since $l_{\rm QH} = Q-(N-1) = 3$ and $l_{\rm QE} = l_{\rm QH}+1$, therefore the lowest CF shell is filled by $2l_{\rm QH}+1 = 7$ of the CF's leaving two as QE's in the first excited CF shell.

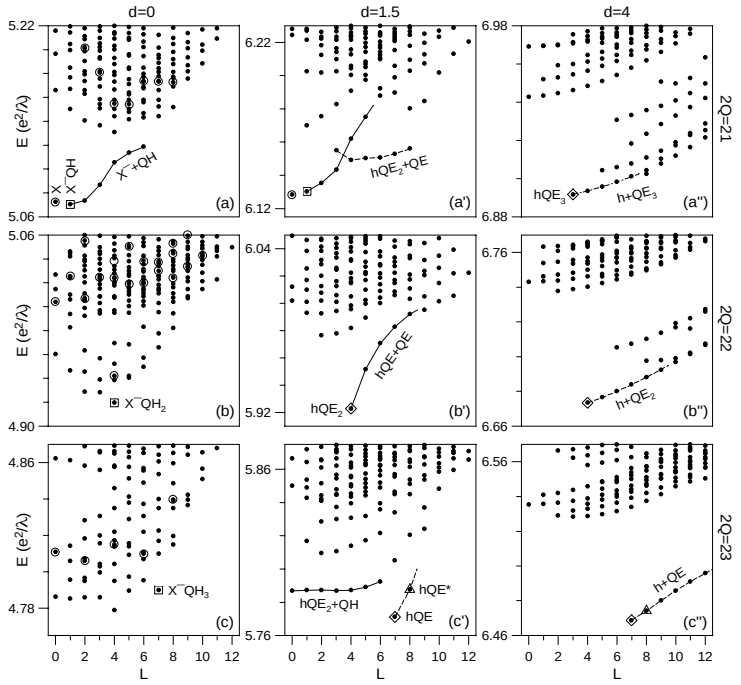

Figure 3: Energy spectra (energy E vs. angular momentum L) of the $9e+\nu$ system in the lowest LL, calculated on a Haldane sphere for different monopole strengths $2Q = 21$, 22, and 23, and e–ν layer separations $d = 0$, 1.5, and 4. Lines and open symbols mark the low-energy states containing different bound excitonic complexes.

The allowed values of the total angular momentum of their pair are $L_{2\mathrm{QE}} = 1 \oplus 3 \oplus 5 \oplus 7$. In the mean field CF picture, these pair states would form a degenerate band. Exact diagonalization (which includes QE-QE interactions beyond mean field) gives the ordering of the energies $E(L_{2\mathrm{QE}})$ as $E(7) < E(3) < E(5) < E(1)$. The interaction of the valence hole (with angular momentum $l_h = Q = 11$) with the QE pair leads to four bands, each increasing with $\vec{L} = \vec{L}_{2\mathrm{QE}} + \vec{l}_h$. Detailed discussion of all these spectra is given in [16]. In this review we restrict the discussion to a few illustrative examples.

4 Spin excitations near $\nu = 1$

In order to understand the excitonic complexes that can be formed near filling factor $\nu = 1$, it is first necessary to study the kinds of elementary excitations that can occur in the absence of valence band holes. For filling factor ν equal to unity, the lowest energy excitations are spin flip excitations which create a reversed spin electron, e_{R}, in the

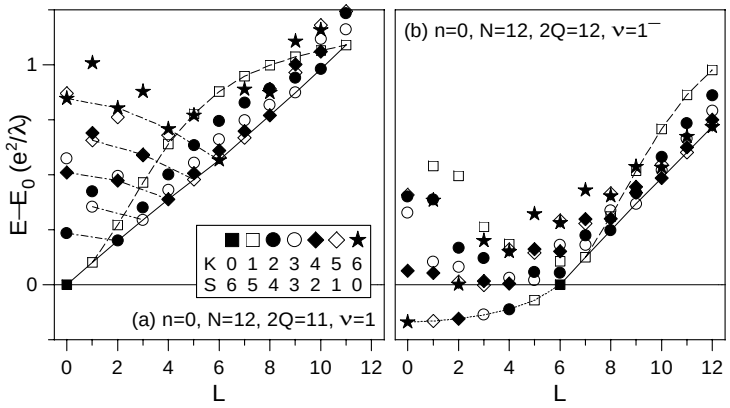

Figure 4: Energy spectra (energy E vs. angular momentum L) of the spin unpolarized 12e system in the lowest LL, calculated on a Haldane sphere for monopole strength $2Q = 11$ (a) and 12 (b).

same $n = 0$ LL leaving behind a spin hole, h, in the otherwise filled $\nu = 1$ state. Even when the Zeeman energy $E_{\rm Z}$ is zero, the Coulomb exchange energy will spontaneously break the spin $(\uparrow,\downarrow)$ symmetry giving a spin polarized ground state.

In Fig. 4a we show the low lying spin excitations of the $\nu = 1$ state (with $E_{\rm Z}$ taken to be zero) for a system of $N = 12$ electrons [7]. The solid square at $L = 0$ is the spin polarized $\nu = 1$ ground state with spin $S = 6$. The symbol $K = \frac{1}{2}N - S$ is the number of spin flips away from the fully spin polarized state. The band of open squares connected by a dashed line gives the spin wave dispersion $\varepsilon_{\rm SW}(L)$. The angular momentum L is related to wave number k by $L = kR$, where R is the radius of the spherical surface to which the N electrons are confined. The SW consists of a single $e_{\rm R}h$ pair; its dispersion can be evaluated analytically [18]. The solid circles, open circles, etc. represent states containing 2, 3, ... spin flips (i.e., 2, 3, ... $e_{\rm R}h$ pairs). Dot-dashed lines connect low lying states with equal numbers of spin flips. It is interesting to note the almost straight line connecting the lowest energy states at $0 \le L \le 6$. This can be interpreted as band of K SW's each with $l_{\rm SW} = 1$ with $L = K$. The near linearity suggests that these K SW's are very nearly non-interacting in the state with $L = K$.

In Fig. 4b we present the low energy spectrum for $\nu = 1^-$ (i.e., a single spin hole in the $\nu = 1$ state). In both Fig. 4a and b only the lowest energy states at each L and S are shown. Of particular interest in Fig. 4b is the band of states with $L = S = Q - K$ and negative energy. These are antiskyrmion states, $S_K^+ = Ke_{\rm R} + (K+1)h$, bound states of one spin hole and K spin waves [6, 7]. They are analogous to interband charged excitons [5], but they can be equilibrium states not subject to radiative decay at the appropriate value of the Zeeman energy. Skyrmion states are $S_K^- = Kh + (K+1)e_{\rm R}$. Electron-hole symmetry requires their existence for $\nu > 1$.

It has been demonstrated [7] that in the fractional quantum Hall regime analogous excitations occur with $\mathrm{QE_R}$ and QH replacing $e_{\rm R}$ and h of the integral quantum Hall

case. Spin waves, skyrmions, and antiskyrmions made from Laughlin quasiparticles occur for $\nu \approx \frac{1}{3}$. The most stable skyrmion or antiskyrmion size depends weakly on the quantum well width for the $\nu \approx 1$ state, but for $\nu \approx 3$, 5, ... the well width w must be of the order of a few times the magnetic length in order to obtain stable bound states of SW's and spin holes or reversed spin electrons [7, 19]. As reported by Melik-Alaverdian *et al.* [20], the inclusion of the admixture of higher LL's caused by the Coulomb interaction weakly affects the skyrmion energy spectrum, particularly when the finite well width w is also taken into account.

The skyrmion and antiskyrmion states $S_K^{\pm}$ are quite analogous to the excitonic $X_K^{\pm}$ states of valence band holes interacting with conduction band electrons. In the ideal theoretical model, a valence hole has exactly the same interactions as a spin hole in the $\nu = 1$ state of the conduction band. In fact these two types of holes can probably be distinguished by an pseudospin as is done for electrons on different layers of a bilayer system [21]. The spectrum and possible condensed states of a multicomponent Fermion liquid containing electrons, X_1^-, X_2^-, etc., has been considered in Ref. [15]. Exactly the same ideas are applicable to a liquid of electrons and skyrmions or antiskyrmions of different sizes. The only difference is that the skyrmion $S^- = h(e_{\rm R})_2$ is stable while the $X^- = ve_2$ has a finite lifetime for radiative recombination of an e–v pair.

When there are N_h spin holes in the $\nu = 1$ level (or N_e reversed spin electrons in addition to the filled $\nu = 1$ level) and when N_h (or N_e) is much smaller than $N \approx 2Q+1$, the degeneracy of the filled lowest LL, then the most stable configuration will consist of N_h antiskyrmions (or N_e skyrmions) of the most stable size. These antiskyrmions (or skyrmions) repel one another. They are positively (or negatively) charged Fermions with standard LL structure, so it is not surprising that they would form either a Wigner lattice or a Laughlin condensed state with ν for the antiskyrmion (or skyrmion) equal to an odd denominator fraction as discussed in Refs. [7, 22, 23].

5 Photoluminescence near $\nu = 1$

In the ideal theoretical model, a valence hole acts exactly like a spin hole in the $\nu = 1$ level of the conduction band. Therefore we would expect an excitonic complex consisting of K SW's bound to the valence hole to be the lowest energy state, in the same way that the antiskyrmion consisting of K SW's bound to a spin hole in the $\nu = 1$ level gives the lowest energy state when $E_{\rm Z}$ is sufficiently small. For a small number of valence holes, the $X_K^+ = v(e_{\rm R}h)_K$ excitonic complexes formed by each valence hole will repel one another. If a small number of antiskyrmions are already present (for $\nu < 1$), the positively charged antiskyrmion–charged-exciton repulsion will lead to Laughlin correlations or Wigner crystallization of the multicomponent Fermion liquid. Just as for the X^- excitons in the dilute regime, the PL at low temperature will be dominated by the $X_K^+ \rightarrow S_{K'}^+ + \gamma$ process, with $K' = K$ or $K-1$ depending on spin of the annihilated valence hole (i.e., on the circular polarization of the emitted photon γ). This corresponds to the most stable X_K^+ undergoing radiative ev or $e_{\rm R}v$ recombination and leaving behind an antiskyrmion consisting of K or $K-1$ SW's bound to a spin hole of the $\nu = 1$ state. Because the valence hole and the spin hole in the $\nu = 1$ conduction level are distinguishable (or have different pseudospin) even in the ideal theoretical

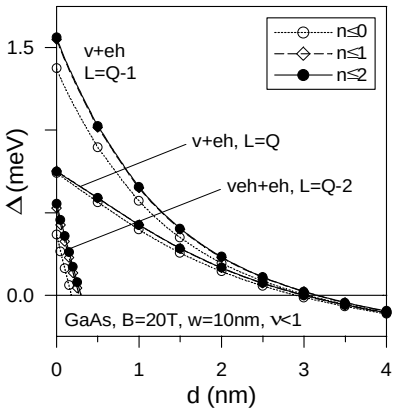

Figure 5: Binding energies Δ of charged skyrmion exciton $X_1^+ = \nu(e_{\rm R}h)$ at angular momentum $L = Q-1$ and Q, and of charged skyrmion biexciton $X_2^+ = \nu(e_{\rm R}h)_2$ at $L = Q-2$ calculated on a Haldane sphere for monopole strength $2Q = 30$ and plotted as a function of the e–ν layer separation d. The calculations are for a GaAs quantum well of width 10 nm at a magnetic field $B = 20$ T. Different curves include one, two, and three LL's for the valence hole.

model this PL is not forbidden. It will be very interesting to see how realistic sample effects (finite well width, LL admixture, finite separation between the electron and valence hole layers) alter the conclusions of the ideal theoretical model.

For $\nu \geq 1$, negatively charged skyrmions are present before the introduction of the valence holes. The skyrmions are attracted by the X_K^+ charge exciton, but how this interaction affects the PL can only be guessed. It is possible that the interaction of the valence hole with the skyrmions will lead to the formation of an X or an $X_{\rm td}^-$ and spin waves. The $X_{\rm td}^-$ will be very weakly radiative (just as in the case of $\nu \leq \frac{1}{3}$). However, the recombination can occur with a majority spin electron. This case was considered in Ref. [8] for the case of a single $X_{\rm td}^-$. The largest oscillator strength occurred for the process $X_{\rm td}^- \to S_1^- + \gamma$; in other words the spin hole left in the $\nu = 1$ level by the *ve* recombination formed a bound state with the two reversed spin electrons of the $X_{\rm td}^-$. Lower energy photon resulted when the two reversed spin electrons and the spin hole did not form the S_1^- bound state.

We are currently investigating more realistic models in systems containing a small number of skyrmions (or antiskyrmions) and valence band excitonic complexes. As a preliminary example of our results we show in Fig. 5 the binding energy, Δ, of the $X_1^+ = \nu(e_{\rm R}h)$ and $X_2^+ = \nu(e_{\rm R}h)_2$ complexes for different values of the total angular momentum L as a function of the separation between the electron and valence hole layers. Δ is defined as the binding energy of the spin wave to the appropriate smaller charged complex, that is $\Delta[\nu(e_{\rm R}h)] = E[\nu] + E[e_{\rm R}h] - E[\nu(e_{\rm R}h)]$ and $\Delta[\nu(e_{\rm R}h)_2] = E[\nu(e_{\rm R}h)] + E[e_{\rm R}h] - E[\nu(e_{\rm R}h)_2]$. The calculation was done for parameters correspond-

ing to a GaAs quantum well of width $w = 10$ nm, at a magnetic field of 20 T, but the Zeeman energy, E_Z, has been omitted. The different symbols (open circles, open diamonds, and solid circles) are for calculations in which one, two, or three LL's for the valence hole have been included (inter-LL excitations of conduction electrons are less important due to their smaller effective mass). It is clear that the binding energies decrease with increasing *e–v* layer separation as expected.

We believe that numerical diagonalization for realistic models including LL admixture and finite well width should explain the behavior of PL for electron filling factors close to unity. The qualitative behavior expected has been discussed in this note. Realistic "numerical experiments" are being carried out to check whether the expected behavior is correct. These results will be reported elsewhere.

Acknowledgment

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