Moore-Read states on a sphere: Three-body correlations and finite-size effects

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Abstract. Energy spectra of a model short-range three-body repulsion are calculated for a half-filled Landau level. The Moore–Read ground state and its quasielectron (QE), quasihole (QH), magnetoroton (QE+QH), and pair-breaking excitations are all identified. Two- and three-body correlations of these states are analyzed. The QE/QH excitations are described by a composite fermion model for Laughlin-correlated electron pairs. Comparison with the results obtained for the Coulomb interaction suggests that finite-size effects are important in numerical diagonalization for the $\nu = 5/2$ quantum Hall state.

INTRODUCTION

The Moore–Read (MR) wavefunction [1] was proposed as a trial state for the half-filled first excited Landau level (LL_1). Although it has commonly been accepted to explain the $\nu = \frac{5}{2}$ fractional quantum Hall (FQH) effect [2], earlier diagonalization studies on a sphere [3] indicated that realistic Coulomb pseudopotentials in LL_1, V_C^1, are too weak at short range to support the MR state. We find that the discrepancy is a finite-size effect.

Laughlin-correlated states of electron pairs were proposed by Halperin [4]. But because pair–pair interaction does not conserve relative pair angular momentum $\mathscr{R}_2$, its pseudopotential is not well-defined, and Laughlin correlations cannot be rigorously established. In fact, they were incorrectly anticipated [5] for *e–e* pseudopotentials $V_2(\mathscr{R}_2)$ that were attractive (rather than "harmonically repulsive," as in LL_1) at short range, and the idea was largely ignored in the context of $\nu = \frac{5}{2}$ FQH effect.

The MR state is an exact zero-energy ground state of a short-range three-body repulsion $W_0(\mathscr{R}_3) = \delta_{\mathscr{R}_3,3}$ [5], where $\mathscr{R}_3$ is the relative triplet angular momentum. In spherical geometry [6], we calculate the energy spectra of W_0 and pair and triplet amplitudes (correlation functions) [7] in the low-energy states. We find that Halperin's picture [4, 8] correctly describes the MR state as well as its quasielectron (QE), quasihole (QH), magnetoroton (QE+QH), and pair-breaking excitations.

Let us also mention that an idea of composite fermion (CF) pairing and condensation at $\nu = \frac{5}{2}$ is unjustified. The CF model relies on Laughlin correlations that only occur if $V_2(\mathscr{R}_2)$ is superharmonic at short range (and in LL_1 it is nearly harmonic). It was shown directly [7, 8] that CF's carrying two flux quanta do not form in LL_1.

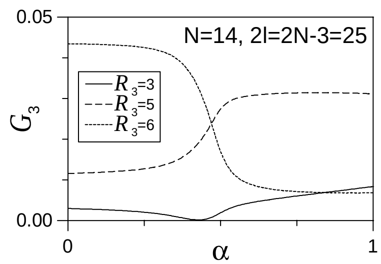

FIGURE 1. Triplet amplitudes $\mathscr{G}_3$ as a function of α for the lowest $L = 0$ states of 14 particles at $\nu = \frac{1}{2}$ interacting via U_α.

MODEL

We consider N electrons on a sphere of radius R, in a $LL_{n=1}$ shell of angular momentum $l = Q + n$. Magnetic monopole strength $2Q$ and the magnetic length λ are related via $R^2 = Q\lambda^2$. The relation between total (L_2) and relative ($\mathscr{R}_2$) pair angular momenta is $\mathscr{R}_2 = 2l - L_2$.

3-BODY CORRELATIONS

Let us define pair interaction $U_\alpha(\mathscr{R}_2) = (1-\alpha)\,\delta_{\mathscr{R}_2,1} + \frac{1}{2}\alpha\,\delta_{\mathscr{R}_2,3}$ with parameter α controlling anharmonicity at short range. $U_{1/2}$ is harmonic for $\mathscr{R}_2 = 1$ through 5 and models well V_C^1. In Fig. 1 we plot the leading triplet amplitudes $\mathscr{G}_3(\mathscr{R}_3)$ as a function of α, calculated in the lowest $L = 0$ states at half-filling ($2l = 2N - 3$). Clearly, $\mathscr{G}_3(3)$ vanishes at $\alpha \approx \frac{1}{2}$. Just as Laughlin correlations at $\nu \approx \frac{1}{3}$ could be defined as the minimization of pair amplitude $\mathscr{G}_2(1)$, the correlations at $\nu \approx \frac{5}{2}$ have a sim-

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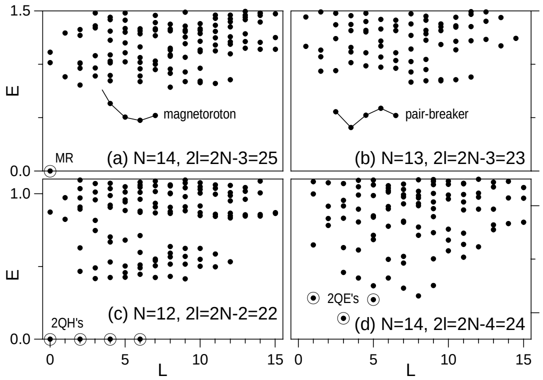

FIGURE 2. Energy spectra $E(L)$ of three-body repulsion W_0.

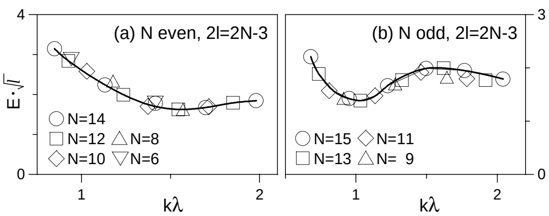

FIGURE 3. Energy dispersion $E(k)$ for the magnetoroton (a) and pair-breaking (b) bands in the spectra of W_0.

ple three-body form, consisting of the minimization of $\mathscr{G}_3(3)$, i.e., the tendency to avoid the $\mathscr{R}_3 = 3$ triplet state. And just as Laughlin $\nu = \frac{1}{3}$ state is an $E = 0$ eigenstate of U_0, the MR $\nu = \frac{5}{2}$ state is an $E = 0$ eigenstate of W_0.

Large values of $\mathscr{G}_2(1)$ and, at the same time, the vanishing of $\mathscr{G}_3(3)$ support Halperin's idea of $\mathscr{R}_2 = 1$ pairing and Laughlin pair–pair correlations that can be modeled by a flux attachment in a standard way.

SPECTRA OF 3-BODY REPULSION

Since W_0 induces the same correlations as V_C^1, we can identify elementary excitations of the MR state in the spectra of W_0. In Fig. 2(a) we show the spectrum for $N = 14$ at $2l = 2N - 3$. The MR ground state occurs at $E = L = 0$. The excited band is a magnetoroton [9] (at $L \le \frac{1}{2}N$, as expected for Laughlin state of pairs). Its continuous dispersion and a minimum at $k\lambda \approx 1.5$ are visible Fig. 3(a), where we plot data for $N = 6$ to 14 as a function of wavevector $k = L/R$. In bottom frames of Fig. 2 we show spectra for $2l = (2N - 3) \pm 1$, whose low-energy states contain a pair of QH's (c) or QE's (d) in the Laughlin state of pairs. The neutral-fermion pair-breaking excitation [5] is identified in Fig. 2(b) for odd N and $2l = 2N - 3$, and its continuous dispersion and a minimum at $k\lambda \approx 1$ are shown in Fig. 3(b).

FINITE-SIZE EFFECTS

Earlier studies using V_C^1 [3, 5, 8] showed $L = 0$ ground states with a gap at $2l = 2N - 3$, but no clear sign of the QE, QH, or pair-breaking excitations. We have calculated overlaps between the eigenstates of U_α, V_C^1, and W_0. For $N = 14$ and $2l = 2N - 3$, the MR state and the Coulomb ground state both turn out excellent ground states of U_α, but for different values of α ($\alpha_{MR} \approx 0.425$ and $\alpha_C \approx 0.5$). This discrepancy (and small direct squared overlaps $\zeta^2 \sim 0.5$ between eigenstates of V_C^1 and W_0) raises the question of whether the MR state and its excitations actually occur in the FQH $\nu = \frac{5}{2}$ state. Fortunately, it is largely artificial. The size-dependence of α_{MR} can be traced to that of the pair amplitudes $\mathscr{G}_2(\mathscr{R}_2)$ of the triplet eigenstates, caused by the surface curvature, which makes α_{MR} smaller than α_C. For large N, we expect that $\alpha_{MR} \to \alpha_C = \frac{1}{2}$ and that Coulomb and W_0 spectra become similar. Hence, "$\mathscr{R}_3 > 3$" correlations, electron pairing, MR state, and QE, QH and pair-breaking excitations are all relevant for the $\nu = \frac{5}{2}$ FQH effect.

CONCLUSION

At half-filling of LL_1, correlations consist of the maximum avoidance of the triplet state with $\mathscr{R}_3 = 3$. They result in incompressible MR state, described by Halperin's picture of a Laughlin state of electron pairs. Small overlaps of numerical ground states on a sphere with the MR state is a finite-size (surface curvature) effect.

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