INTERACTION PSEUDOPOTENTIALS IN QUANTUM HALL SYSTEMS: NOVEL CORRELATIONS AND INCOMPRESSIBLE STATES

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1. INTRODUCTION

Incompressible quantum fluid states have recently been observed at unexpected values of the electron filling factor (e.g. ν= 4/11, 3/8, 5/13, 3/10, and 4/13). Some of these states have been attributed to composite Fermions (CF's) of different "flavor" with the notation ^{2}CF, ^{4}CF, ... used to denote CF's with different numbers of attached Chern–Simons (CS) flux quanta [1]. The idea of forming new quasiparticles (QP's) by attaching additional CS flux to Laughlin QP's is not new [2], and it is known to be invalid for certain values of ν_{QP}, the QP filling factor [3]. In this paper we investigate QP correlations in light of our understanding of correlations in Laughlin states [4] and in the Moore–Read state [5]. Through numerical diagonalization of a system of N_{QP} QP's interacting through a QP pseudopotential $V_{\mathrm{QP}}(L_2)$, incompressible states are found at $\nu_{\mathrm{QE}} = 2/3$, 1/2, 1/3 and at $\nu_{\mathrm{QH}} = 1/4$, 1/5, corresponding to novel observed fractions at electron filling factors $\nu = 5/13$, 3/8, 4/11 and 3/10, 4/13, respectively [6]. However, the interpretation of the numerical results in terms of QP correlations is not completely clear. Laughlin correlations among the QP's can be ruled out quite easily for spin polarized states [3,7]. Pairing correlations of the Moore–Read type do not fit the numerical results for all values of N_{QP} and do not always have the correct relationship between N_{QP} and the degeneracy, $2l_{\mathrm{QP}}+1$, of the QP angular momentum shell [7]. Despite the lack of final resolution, some general features of the correlations occurring with different pseudopotentials seem worth reviewing.

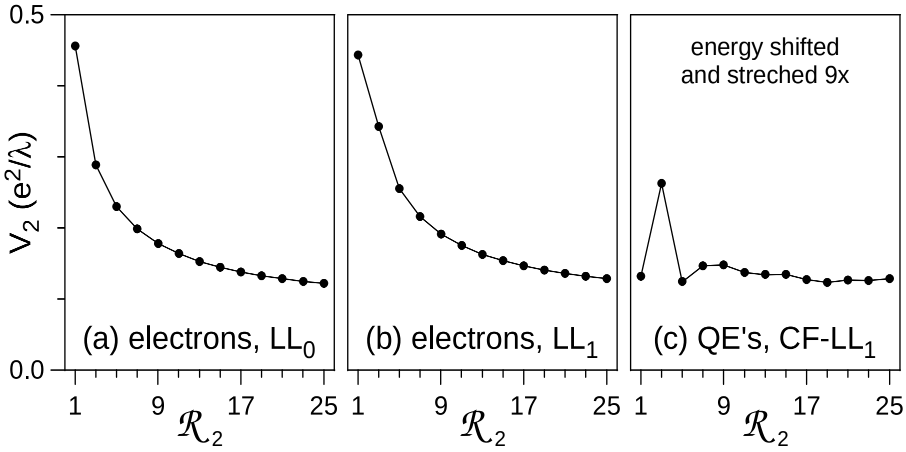

Figure 1. Pair interaction pseudopotentials (pair interaction energy V_2 vs. relative angular momentum $\mathcal{R}_2$) for electrons in the lowest (a) and first excited LL (b), and for QE's of the Laughlin $\nu = 1/3$ state (c).

2. PAIR PSEUDOPOTENTIALS

For an interacting two dimensional Fermion system in the presence of a perpendicular magnetic field $\mathbf{B}$, the energy of interaction of a pair (in the Haldane spherical geometry [8]) depends on the pair angular momentum L_2 (whose allowed values are twice the single particle angular momentum l minus an odd integer). This energy is referred to as the pair pseudopotential $V_2(L_2)$. For electrons interacting through Coulomb forces it depends on Landau level (LL) index n as illustrated in Fig. 1 (a) for the $n = 0$ (LL$_0$) and (b) $n = 1$ (LL$_1$) LL's. In this figure we display V_2 as a function of the *relative* angular momentum defined by $\mathcal{R}_2 = 2l - L_2$. The smallest allowed values of $\mathcal{R}_2$ correspond to the smallest pair separations and largest repulsion. Also shown in Fig. 1 (c) is the pseudopotential $V_{\rm QE}(\mathcal{R}_2)$ describing the interaction of two Laughlin quasielectrons (QE's) [9] of the $\nu = 1/3$ condensed state. For the smallest values of $\mathcal{R}_2$, $V_{\rm QP}$ can be obtained with reasonable accuracy from numerical diagonalization of small systems of electrons at appropriate values of the magnetic monopole strength $2Q$ [9,10]. The fact that the energies $E_{\rm 2QP}(\mathcal{R}_2)$ of the two QP states are not all degenerate results from their *residual* interactions with one another. In fact, up to an overall constant, which has no effect on correlations, $E_{\rm 2QP}(\mathcal{R}_2)$ is the QP pseudopotential $V_{\rm QP}(\mathcal{R}_2)$. In Fig. 2 we compare the low lying spectrum of 12 electrons at $2l = 29$ with that of four QE's of the Laughlin $\nu = 1/3$ state interacting through $V_{\rm QP}(\mathcal{R}_2)$. The agreement gives us confidence in treating larger QP systems interacting through the pseudopotential $V_{\rm QP}(\mathcal{R}_2)$ obtained as described above.

3. PAIR ANGULAR MOMENTA AND SUM RULES

A simple theorem involving the total angular momentum L of an angular momentum eigenstate $|l^N; L\eta>$ of N Fermions, each of angular momentum l, can be

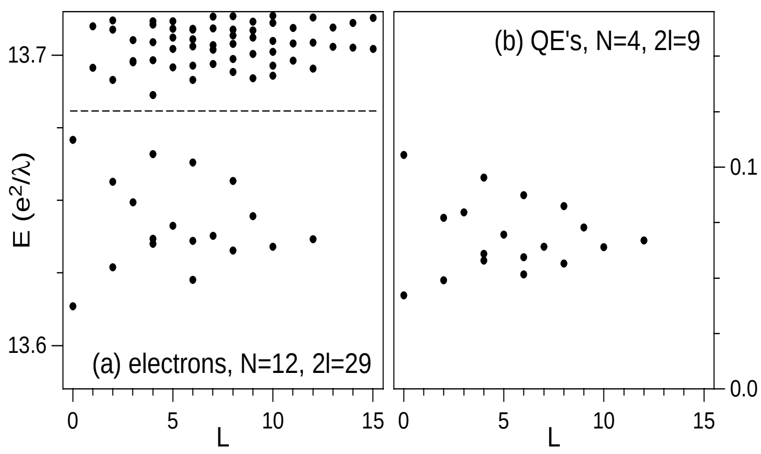

Figure 2. Energy spectra for $N = 12$ electrons in the lowest LL with $2l = 29$ and for $N = 4$ QE's in the first excited CF LL with $2l = 9$. The energy scales are the same, but the QE spectrum obtained using $V_{\rm QE}(\mathcal{R}_2)$ is determined only up to an arbitrary constant.

used to obtain insight into the nature of the correlations [11]. It can be written

$$\sum_{i<j} \hat{L}_{ij}^2 = \hat{L}^2 + N(N-2)\hat{l}^2. \tag{1}$$

Here $\hat{L}_{ij} = \hat{l}_i + \hat{l}_j$ is the angular momentum of pair $< i,j >$. The usefulness of this theorem results from the fact that energy of a state $|l^N; L\eta\rangle$ can be written [12]

$$E_\eta(L) = \frac{N!}{2!(N-2)!} \sum_{L_2} \mathcal{G}_{L\eta}(L_2) V_2(L_2). \tag{2}$$

The pair amplitude $\mathcal{G}_{L\eta}(L_2)$ is defined as the probability of having pairs with pair angular momentum L_2 in the multiplet $|l^N; L\eta\rangle$. $\mathcal{G}_{L\eta}(L_2)$ can be expressed in terms of the coefficients of fractional grandparentage [13], or in terms of the expectation value of an operator $\mathcal{P}_{ij}(L_2)$ which projects the many body eigenfunction $|l^N; L\eta\rangle$ onto a subspace in which the pair $< i,j >$ is in pair eigenstate $|l^2; L_2\rangle$. For an antisymmetric eigenfunction $|l^N; L\eta\rangle$, $\mathcal{G}_{L\eta}(L_2)$ can be written

$$\mathcal{G}_{L\eta}(L_2) = \langle l^N; L\eta | \mathcal{P}_{12}(L_2) | l^N; L\eta\rangle. \tag{3}$$

Because $\mathcal{G}_{L\eta}(L_2)$ is the probability that $|l^N; L\eta >$ contains pairs with pair angular momentum L_2, and because of Eq. (1), the relation between $\hat{L}^2$ and $\sum_{i<j} \hat{L}_{ij}^2$, we can obtain two useful sum rules:

$$\sum_{L_2} \mathcal{G}_{L\eta}(L_2) = 1, \tag{4}$$

and

$$\frac{1}{2}N(N-1)\sum_{L_2} L_2(L_2+1)\mathcal{G}_{L\eta}(L_2) = L(L+1) + N(N-2)l(l+1). \tag{5}$$

An immediate result of these sum rules and Eq. (2) is the absence of correlations for a pair pseudopotential given by $V_H(L_2) = A + BL_2(L_2+1)$ which we call a *harmonic* pseudopotential. By the absence of correlations we mean that $E_\eta(L)$ is independent of the index η. All multiplets having the same total angular momentum L have the same energy.

4. CORRELATIONS

The pair pseudopotential $V(\mathcal{R})$ (we omit the subscript 2 on V_2 and $\mathcal{R}_2$) can be written as the sum of a harmonic and an anharmonic contribution; $V(\mathcal{R}) = V_H(\mathcal{R}) + \Delta V(\mathcal{R})$. Correlations are completely determined by ΔV. Consider, for example, the model pseudopotential in which

$$\Delta V(\mathcal{R}) \propto U_\alpha(\mathcal{R}) = (1-\alpha)\delta_{\mathcal{R},1} + \frac{1}{2}\alpha\delta_{\mathcal{R},3}, \tag{6}$$

where $0 \leq \alpha \leq 1$. For $\alpha = 0$, the lowest energy state for each value of the total angular momentum L is the state with the smallest value of $\mathcal{G}_{L\eta}(\mathcal{R}=1)$, which we will call $\mathcal{G}_{L0}(\mathcal{R}=1)$. Such states can be selected from the set of multiplets $|l^N; L\eta\rangle$ by defining $l^* = l-(N-1)$ and forming the set $|l^{*N}; L\eta\rangle$. In fact, if $\Delta V(\mathcal{R}=1)$ is infinite, only states which completely avoid $\mathcal{R}=1$ pairs have finite energy. This complete avoidance of $\mathcal{R}=1$ pairs corresponds exactly to the Laughlin–Jastrow factor $\prod_{<i,j>}(z_i-z_j)^2$ in the Laughlin wavefunction for the $\nu = 1/3$ state [4]. The lowest value of the single particle angular momentum l at which such states can occur satisfies $2l = 3(N-1)$, so that $2l^* = N-1$ corresponds to a filled *effective* Landau level with an $L=0$ Laughlin incompressible ground state. Selection of the subset $|l^{*N}; L\eta\rangle$ from the original set of multiplets [14] always yields states with low total angular momentum and low energy (since they avoid the largest pair repulsion). This is exactly what is meant by Laughlin correlations.

This approach can be used to justify Jain's composite Fermion picture [15] without the necessity of introducing a *mean* Chern–Simons field and a new energy scale associated with CS gauge interactions among fluctuations. The only energy scale is the Coulomb scale e^2/λ, where λ is the magnetic length, and its pseudopotential coefficients $V(\mathcal{R})$. For example, the $\nu = 2/5$ state can be obtained by starting with electrons at filling factor $\nu = 2$ in a dc magnetic field B_i. By adiabatically increasing the dc magnetic field to $B_f = 5B_i$, and simultaneously adding adiabatically to each electron a CS flux tube carrying two flux quanta oriented opposite to the dc field, one automatically obtains a Laughlin correlated state at $\nu = 2/5$. The change in the dc magnetic field decreases the magnetic length, and the CS flux causes Laughlin correlations between the electron pairs. To eliminate any remnant of the kinetic energy associated with electrons which were initially in LL_1, the resulting wavefunction must be projected onto LL_0. Jain has shown in detail how the mean field picture [15] with CF filling factor ν^* satisfying ${\nu^*}^{-1} = \nu^{-1} - 2p$, where p is an integer, gives rise to the Jain sequence of incompressible states at $\nu = n(1 \pm 2pn)^{-1}$ when ν^* is equal to an integer n. If ν^* is not equal to an integer, the excess CF's go into the next CF LL as QE's of the Jain state (holes in nearly filled CF levels act as quasiholes (QH's) of the Jain state). If the QP pseudopotential $V_{\mathrm{QP}}(L_2)$ satisfies the necessary conditions (i.e. behaves *superharmonically* at the appropriate value of the QP filling factor ν_{QP}) the QP's can form incompressible daughter states. For spin polarized systems this does not happen at $\nu_{\mathrm{QE}} = 1/3$ and at $\nu_{\mathrm{QH}} = 1/5$.

For the first excited LL $\alpha = 1/2$ makes Eq. (6) a model pseudopotential that can be used to investigate the nature of the correlations. It is not difficult to demonstrate that in this case Laughlin correlations (i.e. minimum value of $\mathcal{G}_{L0}(\mathcal{R}=1)$) will not produce the lowest energy state [7]. One can transfer some pair probability away from $\mathcal{G}_{L0}(\mathcal{R}=3)$ to $\mathcal{G}_{L0}(\mathcal{R}=1)$ and $\mathcal{G}_{L0}(\mathcal{R}>3)$ in such a way that the two sum rules, Eqs. (4) and (5), are still satisfied. The energy can be lowered by such a transfer if $\alpha \geq 1/2$. The increase in $\mathcal{G}_{L0}(\mathcal{R}=1)$ from its Laughlin correlated value together with the decrease in $\mathcal{G}_{L0}(\mathcal{R}=3)$ can be interpreted as a indication of formation of pairs. These $\mathcal{R}=1$ pairs have angular momentum $L_2 = 2l-1$. The pairs can be treated as Bosons or as Fermions (a Chern–Simons transformation can change statistics in two dimensional systems), but the pairs cannot get too close to one another without violating the Pauli principle with respect to constituent electrons belonging to different pairs. This can be accomplished by restricting the allowed angular momentum of a pair (treated as a Fermion) to the value

$$l^*_{\mathrm{FP}} = 2l - 1 - \frac{3}{2}(N_{\mathrm{p}} - 1). \tag{7}$$

Here N_{p} is the number of Fermion pairs, and subtracting $\frac{3}{2}(N_{\mathrm{p}}-1)$ from the value $L_2 = 2l-1$ changes the Boson pairs to Fermions and keeps the pairs from getting too close to one another.

One can easily see from Eq. (7) that the Fermion pair filling factor ν_{FP} and electron filling factor ν_1 (in the first excited LL, LL_1) are related by

$$\frac{1}{\nu_{\mathrm{FP}}} = \frac{4}{\nu_1} - 3. \tag{8}$$

Pairing would be expected for values of ν_1 satisfying $2/3 \geq \nu_1 \geq 1/3$, where Jain states that avoid $\mathcal{R}=1$ occur for a superharmonic potential, as in the lowest LL, LL_0. At $\nu_{\mathrm{FP}} = 1/3$, $1/5$, $1/7$, and $1/9$ one might expect Laughlin correlations among the Fermion pairs. From Eq. (8) this would suggest condensed states at electron filling factors (in LL_1) of $\nu_1 = 2/3$, $1/2$, $2/5$, and $1/3$. Adding to these values 2, for the occupied spin up and spin down LL_0's, gives $\nu = 2 + \nu_1 = 8/3$, $5/2$, $12/5$, and $7/3$. Certainly the $\nu = 5/2$ state is the most studied state of the LL_1. The $\nu = 8/3$ and $7/3$ are also observed, but have not been studied in detail.

In Fig. 3 we display the spectra obtained for 8, 10, and 12 electrons at angular momentum l satisfying $2l = 2N+1$ using the Coulomb pseudopotential for the LL_1 (a)–(a$''$) and the model pseudopotential, Eq. (6), with $\alpha = 1/2$, (b)–(b$''$). In each case the $L=0$ Moore–Read ground state is clearly separated from the excitations by a gap. The excited state spectra of the Coulomb and model pseudopotentials are qualitatively similar, but no harmonic contribution to the energies (which increases as $L(L+1)$) has been included in the model potential.

It is informative to evaluate the pair amplitude $\mathcal{G}_2(\mathcal{R})$ for the model pseudopotential given by Eq. (6). This is easily accomplished by using the eigenstates obtained

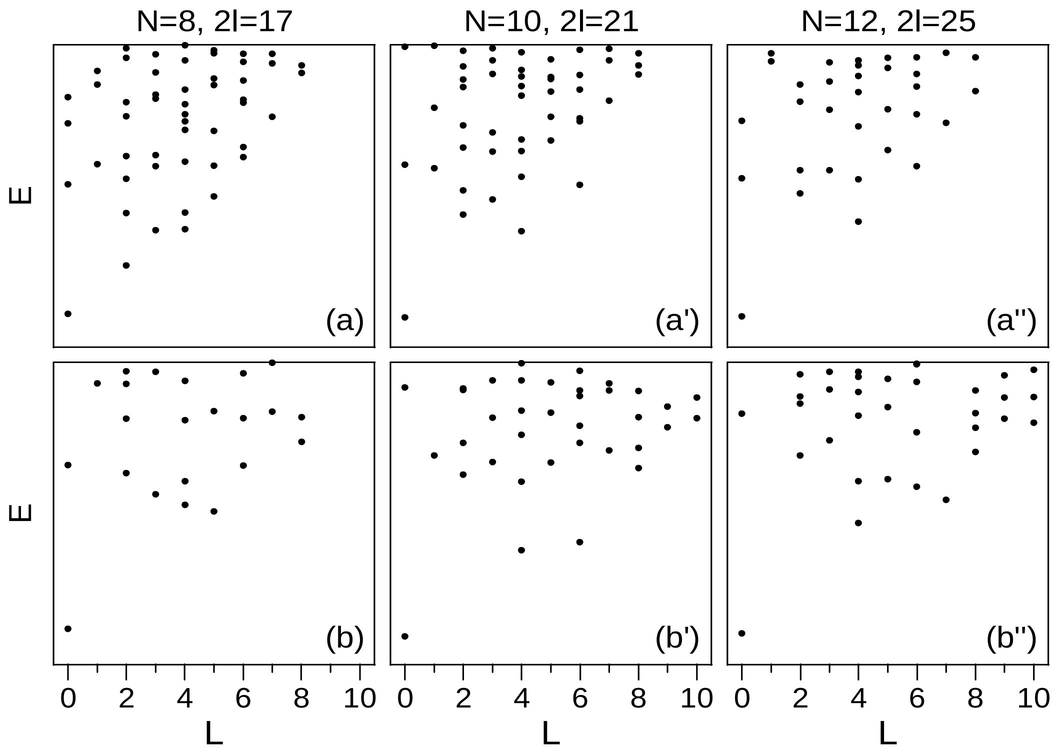

Figure 3. Energy spectra for N electrons on a Haldane sphere: $N = 8$ at $2l = 17$, $N = 10$ at $2l = 21$, and $N = 12$ at $2l = 25$, calculated for the Coulomb pseudopotential of LL_1 (a)–(a$''$) and for the model pseudopotential of Eq. (6) with $\alpha = 1/2$ (b)–(b$''$).

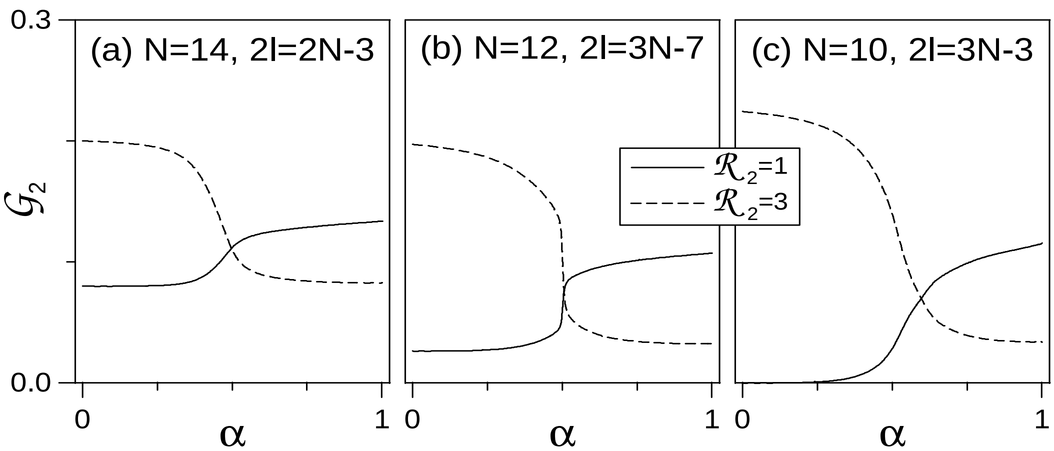

Figure 4. Dependence of pair amplitudes $\mathcal{G}_2(\mathcal{R})$ on parameter α of pair interaction U_α defined by Eq. (6), calculated on a Haldane sphere for the lowest $L = 0$ states of N-particle systems.

from exact diagonalization. The resulting $\mathcal{G}_2(\mathcal{R})$ are displayed as a function of the parameter α for $\mathcal{R} = 1$ and 3 in Fig. 4.

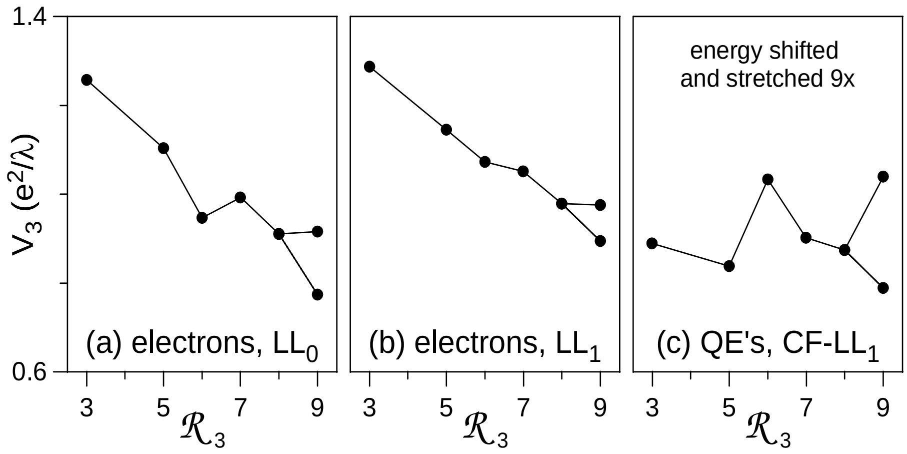

Figure 5. Triplet interaction pseudopotentials (triplet interaction energy V_3 vs. relative triplet angular momentum $\mathcal{R}_3$) for pair pseudopotentials shown in Fig. 1.

5. LARGER CLUSTERS

The theorem relating the total angular momentum $\hat{L}$ of a multiplet $|l^N; L\eta\rangle$ can be extended to larger clusters. It is not difficult to show for n-particle clusters [16] that

$$\sum_{i_1<i_2<\cdots<i_n} \hat{L}^2_{i_1\cdots i_n} = \frac{(N-2)!}{(n-2)!(N-n)!}\left[\hat{L}^2 + N(N-n)(N-1)^{-1}\hat{l}^2\right]. \qquad (9)$$

Here $\hat{L}_{i_1\cdots i_n} = \hat{l}_{i_1} + \cdots + \hat{l}_{i_n}$, and the sum is over all distinct n-particle clusters. For $n=3$ the energy of a multiplet $|l^N; L\eta\rangle$ can be written

$$E_\eta(L) = \frac{N!}{3!(N-3)!}\sum_{\mathcal{R}_3,\beta_3} \mathcal{G}^{(3)}_{L\eta}(\mathcal{R}_3,\beta_3)V_3(\mathcal{R}_3,\beta_3). \qquad (10)$$

Here $\mathcal{G}^{(3)}_{L\eta}(\mathcal{R}_3,\beta_3)$ is the probability amplitude of having a triplet with relative angular momentum $\mathcal{R}_3 = 3l - L_3$ in multiplet β_3. The allowed values of $\mathcal{R}_3$ are 3, 5, 6, 7, …, but for values of $\mathcal{R}_3$ smaller than 9 there is only a single multiplet for each $\mathcal{R}_3$. For these values, $V_3(\mathcal{R}_3)$ can be considered a single three particle pseudopotential.

In Fig. 5 we display the triplet pseudopotential $V_3(\mathcal{R}_3)$ associated with each of the pair pseudopotentials given in Fig. 1. As in the case of the pair pseudopotential, normalization of the triplet probability amplitude $\mathcal{G}^{(3)}_{L\eta}(\mathcal{R}_3,\beta_3)$ together with Eq. (9) imply that if a triplet pseudopotential is linear in $\hat{L}_3^2$ (the square of the triplet angular momentum), every multiplet $|l^N; L\eta\rangle$ with the same value of L has the same energy. In analogy with the occurrence of Laughlin correlations in the presence of short range anharmonicity of the pair pseudopotential ($\Delta V_2(\mathcal{R}_2) = \delta_{\mathcal{R}_2,1}$), short range triplet anharmonicity $\Delta V_3(\mathcal{R}_3) = \delta_{\mathcal{R}_3,3}$ is known to produce Moore–Read correlations [5]. By this we mean the avoidance of triplets with $\mathcal{R}_3 = 3$. The CF picture associated with attaching Chern–Simons flux to the Moore–Read electron pairs gives incompressible

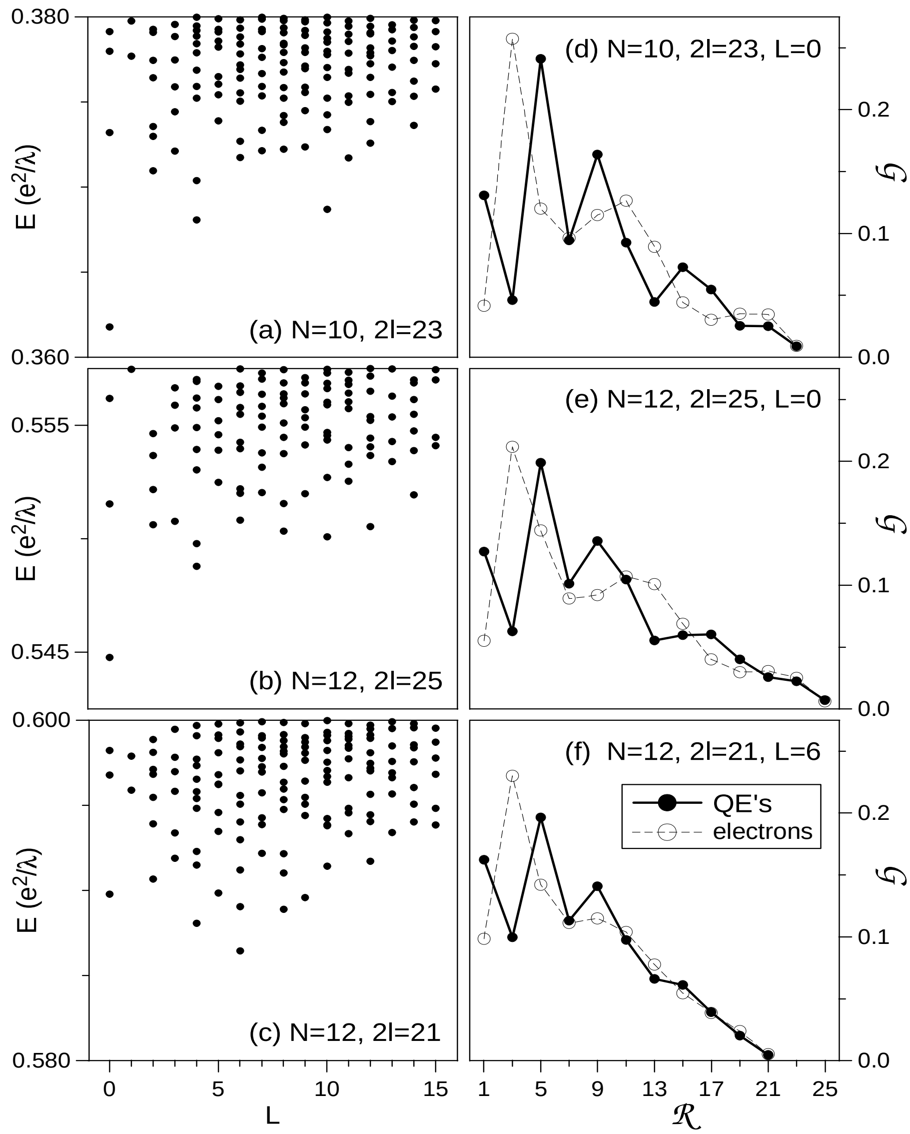

Figure 6. Low energy spectra and pair amplitude functions: Frames (a), (b), and (c) show the energy spectra for $N = 10$ QE's at $2l = 23$, for $N = 12$ QE's at $2l = 25$, and for $N = 12$ QE's at $2l = 21$ as a function of total angular momentum L. Frames (d), (e), and (f) display pair amplitude functions $\mathcal{G}(\mathcal{R})$ for the ground states of the case presented in (a), (b), and (c), as a function of relative pair angular momentum $\mathcal{R}$. The solid circles are the ground state values of $\mathcal{G}(\mathcal{R})$ for the QE pseudopotentials. The open circles are the values for the superharmonic electron pseudopotential.

$L = 0$ ground states at $2l = 2N - 3$ and $2l = 2N + 1$, corresponding to the electron–hole conjugate states for a half filled LL_1. In addition, the low energy excitations obtained numerically are in good qualitative agreement with the CF picture [16].

6. NUMERICAL RESULTS FOR N_{QP} QUASIPARTICLES

In Fig. 6 we present the low energy spectra for (a) $N = 10$ QE's at $2l = 23$, which corresponds to $\nu_{\text{QE}} = 1/3$ and $\nu = 4/11$; (b) $N = 12$ QE's at $2l = 25$, which corresponds to $\nu_{\text{QE}} = 1/2$ and $\nu = 3/8$; and (c) $N = 12$ QE's at $2l = 21$, which in a simple pairing model like that used in discussing the Moore–Read $\nu = 5/2$ state would be expected to give a $\nu_{\text{QE}} = 1/2$ and $\nu = 3/8$ state. The pseudopotential $V_{\text{QE}}(\mathcal{R}_2)$ used in the numerical evaluations is taken from the work of Lee, Scarola, and Jain [10]. The $\nu_{\text{QE}} = 1/3$ state is one of a sequence of states occurring at $2l = 3N - 7$ whose spectra we have evaluated numerically for $4 \leq N \leq 12$. The other two states belong to the sequence $2l = 2N + 1$, which together with their conjugate states at $2l = 2N - 3$ correspond to $\nu_{\text{QE}} = 1/2$ and $\nu = 3/8$. Frames (a) and (b) show $L = 0$ ground states separated by a substantial gap from excited states. Frame (c) does not have an $L = 0$ ground state, though a simple pairing model [6,17] would predict one for this case. In frames (d), (e), and (f) the values of the pair amplitude functions $\mathcal{G}(\mathcal{R})$ for the ground states of (a), (b), and (c) are shown as solid dots. For the sake of contrast, $\mathcal{G}(\mathcal{R})$'s for a superharmonic electron pseudopotential are shown as open circles. The pairing at $\mathcal{R} = 1$ and avoidance of $\mathcal{R} = 3$ QP states are quite clear.

A very simple pairing model based on Halperin's idea [18] was used [6,17] earlier which assumed that all the QE's formed $\mathcal{R} = 1$ pairs. The pairs can be treated as Fermions [6] or as Bosons [17], and if Laughlin correlations between the pairs are assumed, incompressible ground states are formed at $\nu_{\text{QE}} = 1/3$, $1/2$, and $2/3$ and $\nu_{\text{QH}} = 1/5$ and $1/4$, giving novel condensed states at the values observed experimentally [1]. However, the simple complete pairing model is probably too simple. Two major difficulties are not yet understood. First, the states obtained in our numerical calculations occur at $2l = 3N - 7$ (for $\nu_{\text{QE}} = 1/3$) for $N = 8$, 9, 10, 11, and 12, and at $2l = \frac{3}{2}N + 2$ (for $\nu_{\text{QE}} = 2/3$) for $N = 10$, 12, 14, 16, and 18. Complete pairing can *only* occur for N even, and the sequence at $2l = 3N - 7$ occurs for both odd and even values of N. In addition, the simple complete pairing model would predict the $\nu_{\text{QE}} = 1/3$ state at $2l = 3N - 5$ and the $\nu_{\text{QE}} = 2/3$ state at $2l = \frac{3}{2}N + 1$, instead of at the values of $2l$ observed in the numerical study. Although this discrepancy is a finite size effect which becomes negligible for large N, we consider it important and are trying to understand its cause.

It is worth noting that the formation of clusters of k Fermions of angular momentum l (when the clusters themselves are treated as Fermions) results in condensed liquid states of Laughlin correlated clusters when $2l = mN - [(m-1)k + 1]$. This would give correlated pair states at $2l = 2N - 3$ and correlated triplet states at $2l = 3N - 7$, as observed in our numerical results. Of course, the occurrence of complete triplet formation requires N to be divisible by 3, so it would only explain selected states in the $2l = 3N - 7$ sequence. We are still investigating what happens when incomplete clustering (simultaneously having single Fermions, Fermion pairs, Fermion triplets, etc.) occurs. The second problem is that the paired states at $2l = 2N - 3$ (and its electron–hole conjugate states at $2l = 2N + 1$) do not occur at every expected even value of N in the numerical experiments.

Our numerical results are summarized in Fig. 7, a plot of N versus $2l$ which contains four straight lines $2l = 3N - 7$, $2l = \frac{3}{2}N + 2$, $2l = 2N - 3$, and $2l = 2N + 1$. The last two are conjugate pair states for $\nu_{\text{QE}} = 1/2$. The values at which $\nu_{\text{QE}} = 1/3$ and $\nu_{\text{QE}} = 2/3$ states found in our numerical experiments are shown as solid squares and solid dots, respectively. The values at which we find $\nu_{\text{QE}} = 1/2$ states are shown

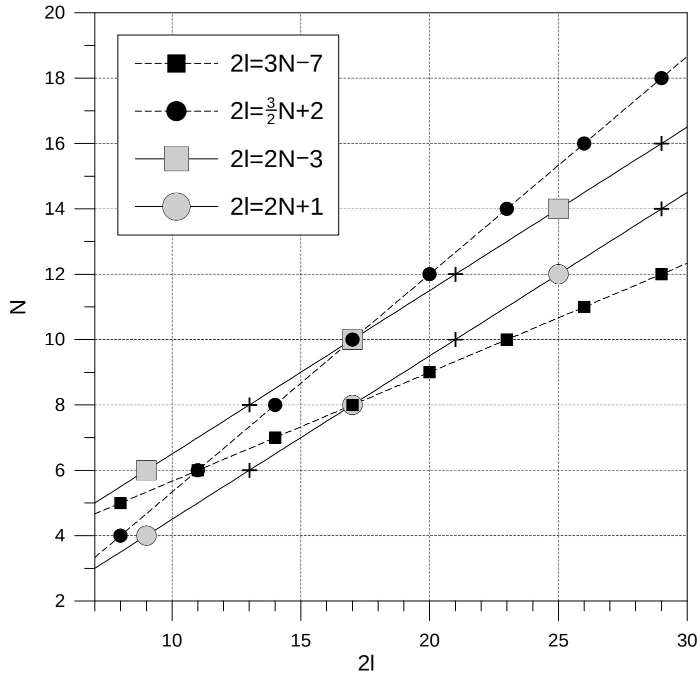

Figure 7. The sequences of $\nu_{\rm QE}=1/3$ (at $2l=3N-7$), $\nu_{\rm QE}=2/3$ (at $2l=\frac{3}{2}N+2$), and $\nu_{\rm QE}=1/2$ (at $2l=2N-3$ and $2N+1$) states shown as straight lines. The values of N and $2l$ at which $L=0$ ground states separated from excited states by a substantial gap are shown as solid dots and solid squares (for $\nu_{\rm QE}=1/3$ and $2/3$, respectively) and by open circles and open squares (for $\nu_{\rm QE}=1/2$). The locations where $L=0$ ground states of N QP's each with angular momentum l would be expected in the simple pairing model but are not found numerically are indicated by the symbol '+'.

as open circles and squares (the circles and squares surround the solid dots and solid squares at $2l=17$, where $\nu_{\rm QE}=1/2$ and $\nu_{\rm QE}=1/3$ or $\nu_{\rm QE}=2/3$ fit the observed states). The expected but unobserved states at $2l=13$ (for $N=6$ and 8), $2l=21$ (for $N=10$ and 12), and $2l=29$ (for $N=14$ and 16) are indicated by the symbol '+'.

We know [6] that for a model pseudopotential with $U_\alpha(\mathcal{R}=1)=1-\alpha$ and $U_\alpha(\mathcal{R}=3)=\alpha/2$, having approximately $\alpha\le 0.25$ and $\alpha\ge 0.75$ leads to Laughlin correlations with $\mathcal{G}(\mathcal{R}=3)\gg\mathcal{G}(\mathcal{R}=1)$ and *anti*-Laughlin correlations with $\mathcal{G}(\mathcal{R}=3)\ll\mathcal{G}(\mathcal{R}=1)$, respectively. For $\alpha\approx 0.5$ (as in the first excited electron LL), $\mathcal{G}(\mathcal{R}=3)\approx\mathcal{G}(\mathcal{R}=1)$. For this case, the Moore–Read state is considered a good description, and it is directly applicable to the $\nu=5/2$ state which corresponds in the U_α to $\alpha\approx 1/2$.

A model three-body pseudopotential [5] $V_3(\mathcal{R}_3)=\delta_{\mathcal{R}_3,3}$ (where $\mathcal{R}_3=3l-L'$ and L' is the three-particle angular momentum) can be used to describe the Moore–Read correlations. In Fig. 8 we display $\mathcal{G}_3(\mathcal{R}_3=3)$, the amplitude for triplets with $\mathcal{R}_3=3$ (the smallest allowed value) as a function of α, the parameter in the two-body pseudopotential $U_\alpha(\mathcal{R})$. It is clear that for $0.4\le\alpha\le 0.5$, triplets with $\mathcal{R}_3=3$ are maximally avoided. However, for $\alpha\approx 1$, $\mathcal{G}_3(\mathcal{R}_3=3)$ is restored to a value even larger

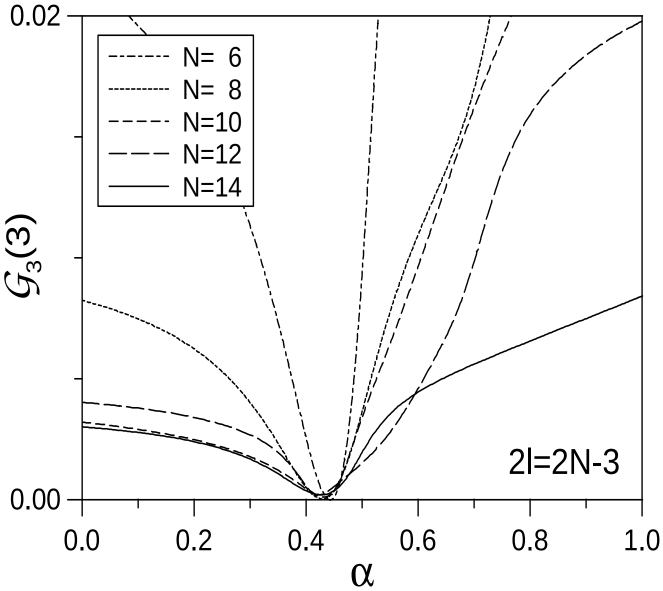

Figure 8. Triplet amplitude $\mathcal{G}_3(\mathcal{R}_3 = 3)$ plotted as a function of α in the lowest $L = 0$ state of different numbers of Fermions N interacting through U_α in a shell with $2l = 2N - 3$.

than that for $\alpha = 0$. This is certainly suggestive of clusters larger than pairs, and is currently being studied.

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