

Interaction of spin excitations in quantum Hall systems

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Abstract. In a two dimensional electron system, a spin wave (SW) of a filled Landau level consists of an electron–hole pair in the minority and majority spin levels respectively. The energy of a single SW is a well known function of wave number k (or of angular momentum $\ell = kR$ on a spherical surface of radius R). Numerical results for the energy of a pair of SW's as a function of their total angular momentum are used to study SW–SW interactions.

INTRODUCTION

The spectrum of spin excitations of a two-dimensional N -electron system at filling factor $\nu = N(2l + 1)^{-1} = 1$ can be determined as a function of total angular momentum L and spin S by numerical diagonalization of the Coulomb interaction [1] in the Haldane spherical geometry [2]. At high magnetic field these excitations involve only the lowest Landau level (LL₀), and they consist of $K = \frac{1}{2}N - S$ spin flips, i.e., K e - h pairs, with each e and h having angular momentum $l = \frac{1}{2}(N - 1)$. The energy of a single spin wave (SW) can be evaluated analytically [3] (as well as numerically); it consists of the Zeeman energy plus the Coulomb energy. The Coulomb energy itself is the sum of E_X , the exchange energy lost due to the spin flip, and the interaction energy V_{eh} of the e - h pair. V_{eh} varies with the SW angular momentum ℓ , but E_X does not. At $\ell = 0$, the Coulomb energy vanishes resulting in the uniform ($k = 0$) spin resonance occurring at the bare Zeeman energy.

Since it consists of a pair of fermions, a SW is often thought of as a boson. For $\ell > 0$, the SW has a finite electric dipole moment, and one might be tempted to think of a pair of SW's as bosons interacting via electric dipole coupling. This picture gives completely erroneous results because it ignores the composite nature of the SW's, thereby neglecting the Pauli exclusion principle for identical fermions belonging to different SW's.

HILBERT SUBSPACE OF SPIN WAVES

The simplest illustration of the importance of the composite nature of the SW's can be obtained for a system of $N = 4$ electrons at $2l = 3$. The allowed values of the total angular momentum L for different values of

the total spin S can be determined by noting that each pair of identical fermions has allowed angular momenta $l_{2e} = l_{2h} = 2l - j$, where j is an odd integer (giving $l_{2e} = 0 \oplus 2$). Adding l_{2e} and l_{2h} , treated as distinguishable objects, gives $L = 0^2 \oplus 1 \oplus 2^3 \oplus 3 \oplus 4$. These values belong to $|L, S\rangle$ multiplets with values of $S = 0, 1$, and 2 . It is not difficult to see that the multiplets $|L, S\rangle^{n_{LS}}$ are $|0, 2\rangle; |1, 1\rangle, |2, 1\rangle, |3, 1\rangle; |0, 0\rangle, |2, 0\rangle^2, |4, 0\rangle$, where n_{LS} is the number of independent multiplets of given L and S . The total number of states contained in these eight multiplets is $\sum_{LS}(2L + 1)(2S + 1)n_{LS} = 70$, the number of distinct ways of putting 4 fermions in $(2l + 1)(2s + 1) = 8$ states.

Treating the individual SW's with $\ell = 0 \oplus 1 \oplus 2 \oplus 3$ as bosons and ignoring their composite nature gives rise to a much larger Hilbert space of SW states; e.g. states with $\ell = \ell' = 3$ would give states with $L = 6$. Antisymmetrization with respect to identical fermions constituents eliminates these extraneous states.

SIMPLE NUMERICAL RESULTS

We illustrate our numerical results and their interpretation with a slightly larger “toy” system having $N = 6$ and $2l = 5$. The allowed multiplets $|L, S\rangle^{n_{LS}}$ with a given value of $S_z = \frac{1}{2}(N_\uparrow - N_\downarrow)$ are determined in the standard way, and they are: $|0, 3\rangle; |1, 2\rangle, |2, 2\rangle, |3, 2\rangle, |4, 2\rangle, |5, 2\rangle; |0, 1\rangle^2, |1, 1\rangle, |2, 1\rangle^5, |3, 1\rangle^3, |4, 1\rangle^5, |5, 1\rangle^2, |6, 1\rangle^3, |7, 1\rangle, |8, 1\rangle; |1, 0\rangle^3, |2, 0\rangle, |3, 0\rangle^5, |4, 0\rangle^2, |5, 0\rangle^3, |6, 0\rangle^2, |7, 0\rangle^2$, and $|9, 0\rangle$. There are $[(2l + 1)(2s + 1)]! \{N! [(2l + 1)(2s + 1) - N]!\}^{-1} = 924$ states in these 48 multiplets.

In Fig. 1 we show the energy as a function of L of the multiplets with $0 \leq S \leq 3$. The ground state occurs at $L = 0$ and $S = 3$. This multiplet contains seven states with $-3 \leq S_z \leq 3$ and $L = 0$. The lowest excited state

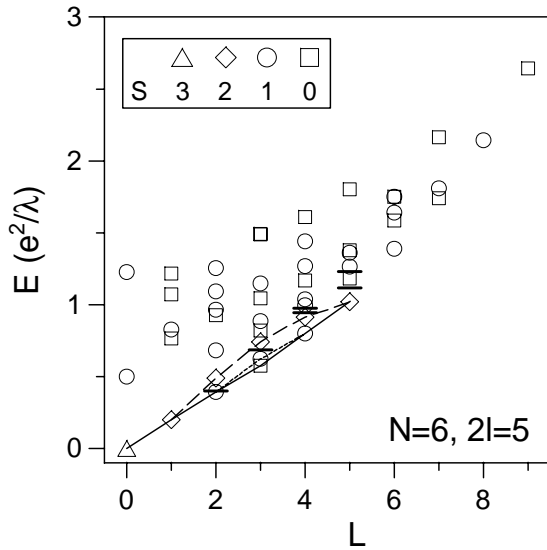


FIGURE 1. Spin excitation spectrum of $N = 6$ electrons at $2l = 5$. Different symbols correspond to total spin $S = 0, 1, 2,$ and 3 . The low-energy states marked by lines and the energies marked by the horizontal dashes are described in the text.

has $S = 2$ and $L = 1$, and is part of a band of one SW states marked by a dashed line connecting diamonds at $L = 1$ up to 5 . One interesting result is the almost straight line connecting the origin with the lowest eigenstates at $L = 1$ to 5 . We interpret these states as states containing $1 \leq n \leq 5$ SW's, each with angular momentum $\ell = 1$. Because this band is almost linear, these SW's are almost non-interacting. The open circles at $2 \leq L \leq 4$ form an arc going from this band at $L = 2$ to 4 , and the value $L = 3$ lies between the arc of a single SW states and the straight line representing n non-interacting $\ell = 1$ SW's. The $S = 0$ state at $L = 3$ is, of course, part of the band of n non-interacting $\ell = 1$ SW's with $n = 3$. This behavior (arcs of eigenvalues consisting of different partitions (ℓ, ℓ') with $1 \leq \ell, \ell' \leq N - 1$) persists for large systems.

Also, marked on Fig. 1 by a heavy dash at different values of L are energies $\varepsilon_\ell + \varepsilon_{\ell'}$ corresponding to two SW's with $\ell + \ell' = L$, where ε_ℓ is the energy of a non-interacting SW with angular momentum ℓ . The difference between these energies and the lowest open circles (energy of fully interacting states containing two SW's) can be interpreted as the binding energy of two SW's. Because ℓ is not a conserved quantity in a system containing more than one SW, the eigenstates are actually linear combinations of different partitions (ℓ, ℓ') with $1 \leq \ell, \ell' \leq N - 1$.

In Fig. 2 we show the results for the interaction energy of two SW's with wave number $k = \ell/R$ (and $\ell \leq \frac{1}{2}N$) traveling parallel to one another [4]. The interaction energy V is measured in the units of $e^2/2\pi R$, where R is the

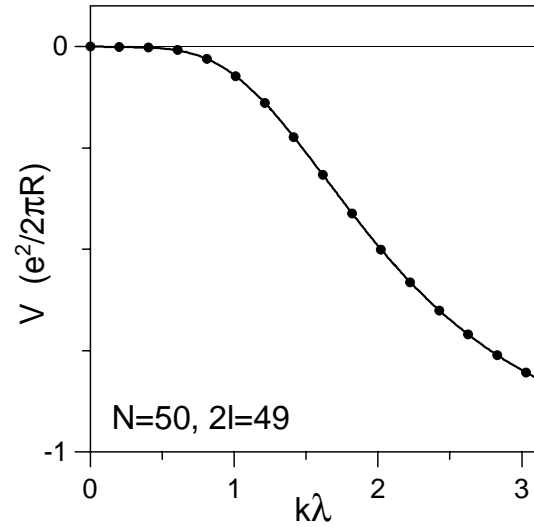


FIGURE 2. The interaction energy V in units of $e^2/2\pi R$ as a function of $k\lambda$ for two spin waves of wave number $k = \ell/R$ and $L = 2\ell$. The data for $N = 30$ and 50 are essentially indistinguishable.

radius of the spherical surface, and the abscissa is $k\lambda$, where λ is the magnetic length. The calculations were carried out for $N = 30$ and 50 , and the two curves are essentially indistinguishable. Other features of the numerical data are susceptible to simple physical interpretation and will be reported elsewhere [4].

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