Interaction and particle–hole symmetry of Laughlin quasiparticles

Arkadiusz Wójs
Department of Physics, University of Tennessee, Knoxville, Tennessee 37996
and Institute of Physics, Wroclaw University of Technology, 50-370 Wroclaw, Poland
(Received 7 January 2001; published 31 May 2001)

The pseudopotentials describing interaction of Laughlin quasielectrons (QE) and quasiholes (QH) in an infinite fractional quantum Hall system are studied. The QE and QH pseudopotentials are similar, which suggests the (approximate) particle–hole symmetry recovered in the thermodynamical limit. The problem of the hypothetical symmetry-breaking QE hard-core repulsion is resolved by the estimate that the ''forbidden'' QE pair state has too high an energy and is unstable. Strong oscillations of the QE and QH pseudopotentials persist in an infinite system, and the analogous QE and QH pair states with small relative angular momentum and nearly vanishing interaction energy are predicted.

DOI: 10.1103/PhysRevB.63.235322 PACS number(s): 73.43.Lp, 71.10.Pm

An important element in our understanding of the incompressibile-fluid ground states[1–3] formed in a two-dimensional electron gas (2DEG) in high magnetic fields has been the identification of Laughlin correlations[1] in a partially filled lowest Landau level (LL). These correlations can be defined[4,5] as a tendency to avoid pair eigenstates with the largest repulsion (smallest relative pair angular momentum $\mathcal{R}$) in the low-energy many-body states. The incompressibility results at a series of filling factors (number of particles divided by the number of states) $\nu=(2p+1)^{-1}$ at which the p leading pair states at $\mathcal{R}=1,3,\ldots,2p-1$ are completely avoided in the nondegenerate (Laughlin) ground state, but not in any of the excited states.

Each Laughlin-correlated state can be understood in terms of two types of quasiparticles (QP's): quasielectrons (QE's) and quasiholes (QH's), moving in an underlying Laughlin ground state (''reference'' or ''vacuum'' state). The QP's are the elementary excitations of the Laughlin fluid and correspond to an excessive (QH) or missing (QE) single-particle state, compared to an exact $\nu=(2p+1)^{-1}$ filling. They have finite size and (fractional) electric charge of $\pm(2p+1)^{-1}e$, and thus (in analogy to LL's of electrons) the single-QP spectrum in a magnetic field is degenerate at a finite energy denoted as $\varepsilon_{\mathrm{QP}}$. For the QP's at a complex coordinate $z=0$, their wave functions are obtained by applying the prefactors $\Pi_k\partial/\partial z_k$ (QE) and $\Pi_k z_k$ (QH) to the Laughlin wave function $\Phi_{2p+1}=\Pi_{i<j}(z_i-z_j)^{2p+1}$.

The partially filled lowest LL is not the only many-body system with Laughlin correlations, which generally occur when the single-particle Hilbert space is degenerate and the two-body interaction is repulsive and has short range.[4,5] Among other Laughlin-correlated systems are a two-component system of electrons and charged excitons (X^-, two electrons bound to a valence hole) formed in an electron–hole plasma in a magnetic field,[6,7] or a system of (bosonic) electron pairs formed near the half-filling of the first excited LL[5] (Moore–Read[8] state at $\nu=\frac{5}{2}$).

Due to their LL-like macroscopic degeneracy and the Coulomb nature of their interaction, Laughlin correlations can also be expected in a system of Laughlin QP's. The concept of Laughlin ground states formed by Laughlin QP's gave rise to Haldane's hierarchy[9] of incompressible ''daughter'' states at $\nu_{\mathrm{QP}}=(2p_{\mathrm{QP}}+1)^{-1}$, in addition to the ''parent'' states at $\nu=(2p+1)^{-1}$. For example, the incompressible $\nu=\frac{2}{7}$ state can be viewed as the $\nu_{\mathrm{QH}}=\frac{1}{3}$ state of QH's in the parent $\nu=\frac{1}{3}$ state of electrons.

The criterion for the ''short range'' of the two-body repulsion that causes Laughlin correlations is expressed[4,5] in terms of the interaction pseudopotential $V(\mathcal{R})$, defined[10] as the pair interaction energy V as a function of $\mathcal{R}$. Therefore, the knowledge of $V(\mathcal{R})$ is necessary to predict the type of correlations (and possible incompressibility) in a given many-body system.

In this note we continue our earlier study[11] of interactions between Laughlin QP's. The QE and QH interaction pseudopotentials are calculated for the Laughlin $\nu=\frac{1}{3}$ and $\frac{1}{5}$ states of up to 8 and 12 electrons on a Haldane sphere,[9] respectively, and extrapolated to an infinite planar system. Our results lead to the following two main conclusions.

(i) Opposite to what seemed to follow from finite-size calculations,[11,12] the sign and magnitude of the pseudopotential coefficients calculated for an infinite plane agree with the expectation that, being charge excitations, the QP's of the same type must repel and not attract one another. However, the oscillations in the QP charge density cause oscillations in $V(\mathcal{R})$, and the QP pair states with small $\mathcal{R}$ (small radius) and nearly vanishing interaction energy are predicted. The vanishing of repulsion in these states rules out incompressibility of such hypothetical[11] ground states in Haldane's hierarchy as $\nu=\frac{6}{17}$ or $\frac{6}{19}$, and limits the family of valid hierarchy states to the (experimentally observed) Jain sequence[13] at $\nu=n(2pn\pm1)^{-1}$. This vanishing is also essential for the stability of fractionally charged excitons[14] $h\mathrm{QE}_n$ (n QE's bound to a valence hole) observed[15] in photoluminescence of the 2DEG.

(ii) From the similarity of QE and QH pseudopotentials we conclude that it is only due to its large repulsion energy that the QE pair state with $\mathcal{R}=1$ (the counterpart of the highest-energy QH pair state) is not a stable eigenstate of an underlying electron system. We find that the highest QE pseudopotential parameter, $V_{\mathrm{QE}}(1)$, exceeds the Laughlin gap $\Delta=\varepsilon_{\mathrm{QE}}+\varepsilon_{\mathrm{QH}}$ to create an additional QE–QH pair. This makes the QE pair state at $\mathcal{R}=1$ unstable by pushing it into

the 3QE+QH continuum. The most important implication of this result is that the asymmetry between the two-QE and two-QH spectra that is observed in numerics (and that can be accounted for by introduction of a phenomenological hard-core QE–QE repulsion[16] at $\mathcal{R}=1$) does not originate from a hypothetical asymmetry between QE and QH Hilbert spaces. This agrees with Haldane's intuitive picture of both QE's and QH's being bosonic QP's placed ''between'' the electrons,[9] and thus, for example, having equal angular momenta on a sphere. We hope to clarify that the unobserved highest-energy QE pair state is not mysteriously ''forbidden'' or absent in the two-QE Hilbert space, but that it is simply energetically unstable. This instability explains why Jain's composite fermion (CF) picture[13] correctly predicts the lowest-energy bands of states, despite the unjustified asymmetry of QE and QH LL's introduced by an (unphysical) effective magnetic field. A minor conclusion is that the similarity of the QE and QH pair states and energies precludes qualitatively different response of a Laughlin-correlated 2DEG to a positively and negatively charged perturbation.[14]

The knowledge of pseudopotentials defining interactions of Laughlin QP's is essential in Haldane's hierarchy[9] of the fractional quantum Hall effect,[1–3] in which they determine those of Laughlin fillings at which the QP's form (daughter) Laughlin incompressible states of their own. Although they are to a large extent equivalent, Haldane's hierarchy differs from Jain's CF picture in the ''symmetric'' description of the two types of QP's. Haldane's elegant argument[9] that both QE and QH excitations are bosons placed ''between'' the N (effectively one-dimensional) electrons yields equal numbers of possible QE and QH states, $\tilde{g}_{\rm QE}=\tilde{g}_{\rm QH}=N+1$ (tildes mean bosons), which on a sphere correspond to equal single-particle angular momenta, $\tilde{l}_{\rm QE}=\tilde{l}_{\rm QH}=\frac{1}{2}N$ (because $\tilde{g}=2\tilde{l}+1$; the lowest LL on a Haldane sphere is an angular momentum shell of $l=S$, half the strength of Dirac's magnetic monopole in the center[4,9,17]).

In a system of n QP's, a mean-field (MF) Chern–Simons (CS) transformation[18–20] can further be used to convert such bosonic QP's to more convenient fermions with $g=\tilde{g}+(n-1)$, yielding $l=\tilde{l}+\frac{1}{2}(n-1)$. However, for QE's this value of l seemed to predict an incorrect number of low-energy states in the numerical energy spectra unless the pair state at the maximum angular momentum $L_{\rm max}=2l-1$ was forbidden.[16] On a sphere, the relation between $L=|\mathbf{l}_1+\mathbf{l}_2|$ and $\mathcal{R}$ is $L=2l-\mathcal{R}$, and thus the exlusion of the pair state at $L_{\rm max}$ is equivalent to a hypothetical hard-core repulsion, $V_{\rm QE}(1)=\infty$.

Such a hard-core interaction can be formally removed by an appropriate redefinition of the single-particle Hilbert space. This is accomplished by a fermion-to-fermion MF CS transformation,[4,11,21,22] which replaces g by $g^*=g-2(n-1)$, and $l=\frac{1}{2}N+\frac{1}{2}(n-1)$ by $l^*=\frac{1}{2}N-\frac{1}{2}(n-1)$. By ''elimination'' we mean that the angular momenta $L_{n\rm QE}$ of states containing n QE's can be obtained by simple and unrestricted addition of n individual angular momentum vectors $\mathbf{l}^*_{\rm QE}$ followed by antisymmetrization (QE's are treated as indistinguishable fermions), just as $L_{n\rm QH}$ could be obtained by an antisymmetric combination of n vectors $\mathbf{l}_{\rm QH}$. Although they seem to agree with the ''numerical experiments,''[4,11,16] no explanation exists for a hard core in the QE–QE repulsion (and its absence in $V_{\rm QH}$) or the resulting asymmetry between $l_{\rm QH}$ and $l^*_{\rm QE}$.

This asymmetry is inherent in Jain's CF picture,[13] in which QE's and QH's are converted into particles and vacances in different CF LL's whose (different) angular momenta are equal to $l^*_{\rm QE}$ and $l_{\rm QH}$, respectively. However, the effective magnetic field leading to the correct values of $g^*_{\rm QE}$ and $g_{\rm QH}$ in the CF picture does not physically exist. While for the QH states the effective field is one of possible physical realizations of the MF CS transformation describing Laughlin correlations (the avoidance of the most strongly repulsive pair states) in the underlying electron system,[4] no explanation for $g^*_{\rm QE}$ being smaller than $g_{\rm QH}$ is possible within the CF model itself.

To resolve this puzzle we have examined the QE and QH pseudopotentials calculated for the systems of $N\leqslant 12$ electrons at $\nu=\frac{1}{3}$ and $\frac{1}{5}$. In Fig. 1(a) we compare $V_{\rm QE}$ and in Fig. 1(b) $V_{\rm QH}$ obtained at $\nu=\frac{1}{3}$ for different values of N. In both frames, $\mathcal{R}=2l-L$, with $l_{\rm QE}=l_{\rm QH}=\frac{1}{2}(N+1)$. To obtain the values of V, the energies of the Laughlin ground state and of the two QP's are subtracted from the energies of the appropriate QP pair states[11,12] (such as the QE pair states for $N=11$ and 12 shown in the insets). The energy is measured in the units of e^2/λ, and λ is the magnetic length.

In the limit of $N\rightarrow\infty$, the sphere radius $R\sim\sqrt{N}$ diverges and the numerical values of $V(\mathcal{R})$ converge to those describing an infinite 2DEG on a plane. In this (planar) geometry, $\mathcal{R}$ is the usual relative pair angular momentum. Remarkably, when $\mathcal{R}_{\rm QE}$ is defined as $2l_{\rm QE}-L$ rather than $2l^*_{\rm QE}-L$, the QE and QH pseudopotentials become quite similar. The main difference is the obvious lack of the $\mathcal{R}_{\rm QE}=1$ state and stronger oscillations in the $V_{\rm QE}(\mathcal{R})$, but the maximum at $\mathcal{R}=5$ and the minima at $\mathcal{R}=3$ and 7 are common for both $V_{\rm QE}$ and $V_{\rm QH}$. The same structure occurs also for the QP's in the $\nu=\frac{1}{5}$ state.[11] Most unexpected in Fig. 1 are the negative signs of $V_{\rm QP}$. The only positive pseudopotential coefficient is $V_{\rm QH}(1)$, which might indicate that, despite QP's being charge excitations, both QE–QE and QH–QH interactions are generally attractive.

In Fig. 2 we plot a few leading pseudopotential coefficients (those at the smallest values of $\mathcal{R}$) $V_{\rm QH}$ and $V_{\rm QE}$ at $\nu=\frac{1}{3}$ and $\frac{1}{5}$ as a function of N^{-1}. Clearly, the corresponding coefficients of all four pseudopotentials behave similarly, which confirms the correct use of $l_{\rm QE}$ rather than $l^*_{\rm QE}$ in the definition of $\mathcal{R}_{\rm QE}$. It is also clear that all coefficients V increase with increasing N (although at a different rate for QE's and QH's), and it seems that none of them will remain negative in the $N\rightarrow\infty$ limit. In attempt to estimate the magnitude of V in this limit we have drawn straight lines that approximately extrapolate our data for some of the coefficients. The most noteworthy values are $V_{{\rm QH},\nu=1/3}(1)\approx 0.03\ e^2/\lambda$ being about three times larger than

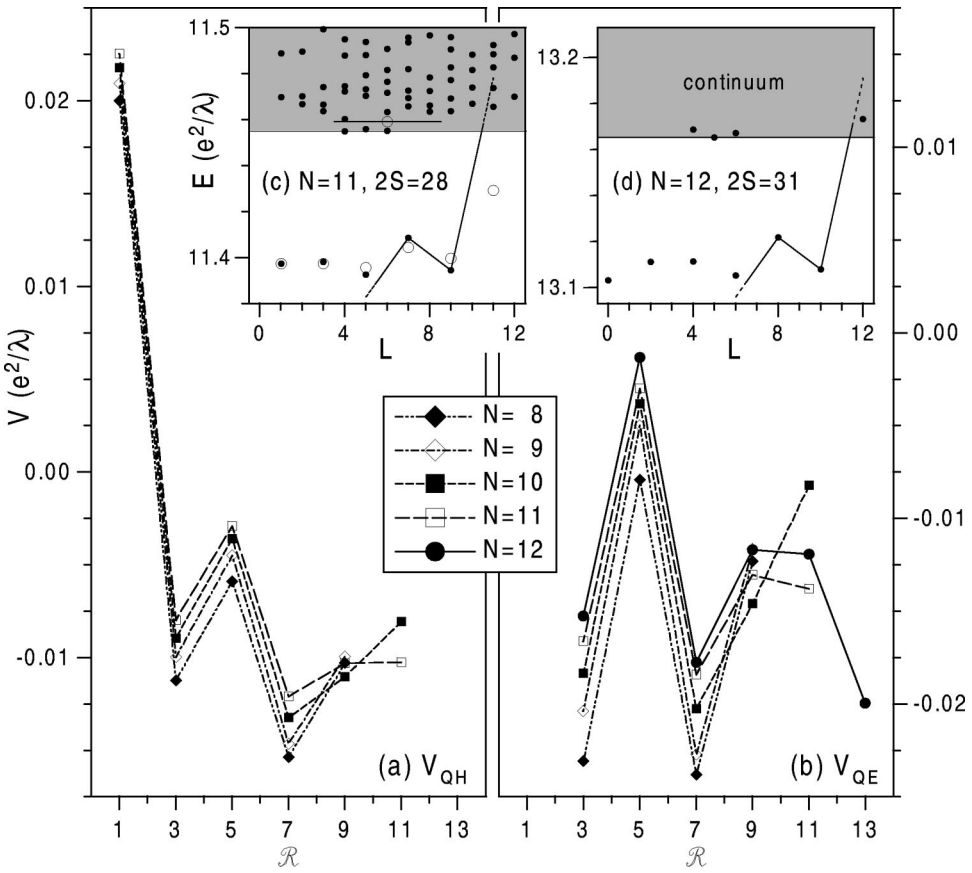

FIG. 1. The comparison of (a) quasihole and (b) quasielectron pseudopotentials $V(\mathcal{R})$ calculated at $\nu=\frac{1}{3}$ in N-electron systems on a Haldane sphere. Insets: The comparison of (c) 11-electron and (d) 12-electron energy spectra in which the lowest-energy band contains two quasielectrons. In (c), open circles show the (shifted in energy) 11-electron spectrum of two quasiholes.

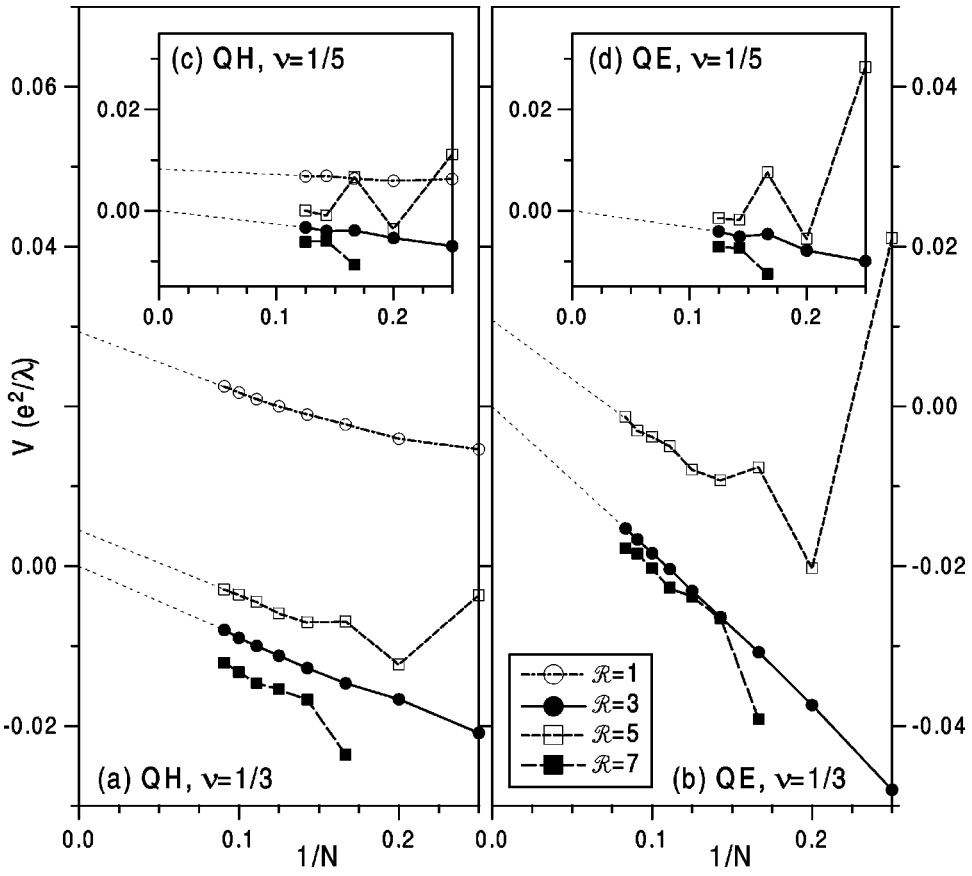

FIG. 2. The leading (a,c) quasihole and (b,d) quasielectron pseudopotential coefficients $V(\mathcal{R})$ calculated at (a,b) $\nu=\frac{1}{3}$ and (c,d) $\nu=\frac{1}{5}$ in N-electron systems on a Haldane sphere, plotted as a function of N^{-1}. Thin dotted lines show extrapolation to $N\rightarrow\infty$.

$V_{\mathrm{QH},\nu=1/5}(1)$ as expected from the comparison of interacting charges ($\frac{1}{3}e$ and $\frac{1}{5}e$, respectively), the $V(3)$ coefficients (seemingly) vanishing in all four plots, and $V_{\mathrm{QH},\nu=1/3}(5) \approx 0.005\ e^2/\lambda$ being about twice smaller than $V_{\mathrm{QE},\nu=1/3}(5)$.

The predicted small value of $V(3)$ and of some other leading coefficients is by itself quite interesting, although it can be understood from the fact that QP's are more complicated objects than electrons, and the oscillations in $V_{\mathrm{QP}}(\mathcal{R})$ reflect the oscillations in their more complicated charge density profile (similar oscillations occur in the electron pseudopotentials in higher LL's). The consequences of this fact are even more important.

First, from a general criterion[4,5] for Laughlin correlations at $\nu \approx (2p+1)^{-1}$ (defined as the avoiding of pair states with $\mathcal{R} < 2p+1$ in the low-energy many-body states) in a system interacting through a pseudopotential $V(\mathcal{R})$ we find that the QP's of the parent Laughlin state of electrons form Laughlin states of their own only at $\nu_{\mathrm{QP}} = \frac{1}{3}$. These states and their $\nu_{\mathrm{QE}} = \frac{1}{3}$ daughters exhaust Jain's $\nu = n(2pn \pm 1)^{-1}$ sequence. No other incompressible daughter states occur in the hierarchy, including the (ruled out earlier[11]) $\nu = \frac{4}{11}$ or $\frac{4}{13}$ states or the hypothetical[11] $\nu = \frac{6}{17}$ or $\frac{6}{19}$ states. Despite all the differences between Haldane's hierarchy and Jain's CF model, our conclusion makes their predictions of the incompressibility at a given ν completely equivalent.

Second, the (near) vanishing of $V_{\mathrm{QE}}(3)$ explains the stability of the $h\mathrm{QE}_2$ complex[14] in the 2DEG interacting with an (optically injected) valence hole. Being the most strongly bound and the only radiative state of all ''fractionally charged excitons'' $h\mathrm{QE}_n$, the $h\mathrm{QE}_2$ is most likely the complex observed[15] in the PL spectra of the 2DEG at $\nu > \frac{1}{3}$.

Third, since $V_{\mathrm{QE}}(5)$ is about twice larger than $V_{\mathrm{QH}}(5)$, it is also plausible that $V_{\mathrm{QE}}(1)$ could be much larger than $V_{\mathrm{QH}}(1)$, so that the $\mathcal{R}_{\mathrm{QE}}=1$ state would fall in the continuum and could not be identified in the energy spectra. In Fig. 1(c), on top of the 11-electron spectrum at Dirac's monopole strength (the number of magnetic flux quanta piercing the Haldane sphere[9,4,17]) $2S=28$, marked with full dots, in which the lowest-energy states contain two QE's at $\nu = \frac{1}{3}$, with open circles we have marked another spectrum calculated for the same $N=11$ but at $2S=32$, whose lowest-energy band contains two QH's. The second spectrum is vertically shifted so that the energies of the QE and QH pair states coincide at $L=1$ (i.e. at $\mathcal{R}=11$) at which V_{QE} and V_{QH} are both negligible, but the energy units (e^2/λ) are the same. Since the Laughlin gap Δ to the continuum of states with additional QE–QH pairs involves the sum of QE and QH energies, it is roughly the same in both spectra. However, the minima and maxima in $V_{\mathrm{QE}}(\mathcal{R})$ are stronger than those in $V_{\mathrm{QH}}(\mathcal{R})$, and the difference $|V_{\mathrm{QE}} - V_{\mathrm{QH}}|$ increases at larger L. While it is hardly possible to rescale V_{QH} so as to reproduce V_{QE} at $L \leqslant 9$ and convincingly predict its value at $L=11$ ($\mathcal{R}_{\mathrm{QE}}=11$), it seems likely that $V_{\mathrm{QE}}(11)$ is indeed larger than Δ, which would explain the absence of the $\mathcal{R}_{\mathrm{QE}}=11$ state below the continuum. An example of such ''rescaling'' procedure is shown in Fig. 1(c) with the line obtained by stretching V_{QH} so that it crosses $V_{\mathrm{QE}}(5)$ and $V_{\mathrm{QE}}(3)$ at $L=7$ and 9, respectively. Similar lines are shown in Fig. 1(d) for the 12-electron spectrum corresponding to two QE's in the lowest band ($2S=31$). Certainly, this procedure, based on the assumption that $V_{\mathrm{QE}}(3)$ and $V_{\mathrm{QH}}(3)$ are small and that $V(1)$ is proportional to $V(5)$, is not accurate. Nevertheless, having in mind the similarities of V_{QE} and V_{QH} in Figs. 1 and 2, and in the absence of any physical reason why the $\mathcal{R}_{\mathrm{QE}}=1$ state might not exist while the $\mathcal{R}_{\mathrm{QH}}=1$ state does, we believe that it is more reasonable to assume that $V_{\mathrm{QE}}(1)$ is finite, although larger than Δ. The fact that the $\mathcal{R}_{\mathrm{QE}}=1$ state is pushed into the 3QE+QH continuum simply means that it is unstable toward spontaneous creation of a low-energy QE–QH pair with finite angular momentum (magnetoroton).

The assumption that $\Delta < V_{\mathrm{QE}}(1) < \infty$ restores the elegant symmetry of Haldane's picture of QP's ''placed'' between electrons.[9] It replaces the problem of explaining the QE hard core[16] by a question of why V_{QE} is larger than V_{QH} at a short distance (e.g., at $\mathcal{R}=1$ and 5; see Fig. 1). But the fact that V_{QE} and V_{QH} are not equal at short distances is by no means surprising since the QE and QH have different wave functions.

In conclusion, we have calculated the pseudopotentials $V_{\mathrm{QP},\nu}(\mathcal{R})$ describing interaction of QE's and QH's in Laughlin $\nu=(2p+1)^{-1}$ states of an infinite 2DEG. These pseudopotentials are all similar, showing strong repulsion at $\mathcal{R}=1$ and 5, and virtually no interaction at $\mathcal{R}=3$. The unexpected QE–QE and QH–QH attraction that results in few-electron calculations disappears in the limit of an infinite system. Because the QP charge at $\nu=(2p+1)^{-1}$ decreases with increasing p, the QP interaction at $\nu=\frac{1}{3}$ is stronger than at $\nu=\frac{1}{5}$. Because of different QE and QH wave functions, V_{QE} is larger than V_{QH} at small $\mathcal{R}$ (short distance). The coefficient $V_{\mathrm{QE}}(1)$ exceeds the Laughlin gap Δ to create an additional QE–QH pair, which makes the QE pair state at $\mathcal{R}=1$ unstable. This instability, rather than a mysterious QE hard core or an inherent asymmetry between the QE and QH angular momenta, is the reason for the overcounting of few-QE states when, following Haldane, QE's are treated as bosons with $\tilde{l}=\frac{1}{2}N$. In particular, it explains the absence of the $L=N$ multiplet in the low-energy band of states in the N-electron numerical spectra at the values of $2S=(2p+1)(N-1)-2$, corresponding to two QE's in the Laughlin $\nu=(2p+1)^{-1}$ state. The (near) vanishing of $V_{\mathrm{QP}}(3)$ is the reason why no hierarchy states other than those from Jain's $\nu=n(2pn\pm1)^{-1}$ sequence are stable. It is also the reason for the strong binding of the fractionally charged exciton $h\mathrm{QE}_2$.

The author gratefully acknowledges discussions with John J. Quinn, Pawel Hawrylak, Lucjan Jacak, Izabela Szlufarska, and Piotr Sitko, and support from the Polish State Committee for Scientific Research (KBN) Grant No. 2P03B11118, and thanks the Joint Institute for Computational Science at the University of Tennessee for providing access to the IBM SP2 supercomputer and for user support.

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