Interaction and dynamical binding of spin waves or excitons in quantum Hall systems

A. Wójs, A. Gładysiewicz, D. Wodziński, and J.J. Quinn

Abstract: Interaction between spin waves (or excitons) moving in the lowest Landau level is studied using numerical diagonalization. Because of complicated statistics obeyed by these composite particles, their effective interaction is completely different from the dipole–dipole interaction predicted in the model of independent (bosonic) waves. In particular, spin waves moving in the same direction attract one another, which leads to their dynamical binding. Effective interaction pseudopotentials $V_{\uparrow\uparrow}(k)$ and $V_{\uparrow\downarrow}(k)$ for two spin waves with equal wave vectors k and moving in the same or opposite directions are calculated and shown to obey power laws $V(k) \propto k^{\alpha}$ at small k. A high value of $\alpha_{\uparrow\uparrow} \approx 4$ explains the occurrence of linear bands in spin excitation spectra of quantum Hall droplets.

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Résumé : Nous étudions par diagonalisation numérique l'interaction entre les ondes de spin (excitons) dans le niveau de Landau le plus bas. À cause de la statistique compliquée de ces particules composites, leur interaction efficace est complètement différente de l'interaction dipôle–dipôle prédite par le modèle des ondes indépendantes (bosons). En particulier, les ondes de spin se déplaçant dans la même direction s'attirent, menant à une liaison dynamique. Nous calculons les pseudo-potentiels $V_{\uparrow\uparrow}(k)$ et $V_{\uparrow\downarrow}(k)$ pour deux ondes de spin de même vecteur d'onde et se déplaçant dans la même direction ou en direction opposée et observons qu'ils obéissent à des lois de puissance $V(k) \propto k^{\alpha}$. Une haute valeur de $\alpha_{\uparrow\uparrow} \approx 4$ explique l'apparition de bandes linéaires dans le spectre d'excitation de spin de l'effet Hall quantique.

[Traduit par la Rédaction]

1. Introduction

Description of interactions and correlations between excitons [1] (electron–hole pairs, $X = \mathrm{e} + \mathrm{h}$) is somewhat problematic because of their complicated statistics. Being pairs of fermions, the excitons obey Bose statistics under a "full" exchange and, consequently, condense into a Bose–Einsetin ground state at sufficiently low density [2]. However, their composite nature comes into play when the excitons overlap and "partial" exchanges (of only a pair of electrons or holes) can occur. And, unlike charged

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complexes (such as trions, $X^- = 2e + h$) naturally separated by the Coulomb repulsion, the overlaps between neutral excitons can often be significant.

In the absence of a magnetic field B, exciton correlations have been discussed [3] in connection with four-wave mixing experiments that involve two-photon absorption [4–7]. Here, we will consider two-dimensional systems in the high-B limit, so-called "quantum Hall systems" [8]. While the bosonization scheme for excitons confined to the lowest Landau level (LL_0) has recently been proposed [9], we will concentrate on the numerical results for the X–X interaction pseudopotential $V(k)$.

In analogy with the Haldane pseudopotential [10], $V(k)$ will mean the X–X interaction energy in the specific eigenstates of a pair of excitons (lowest eigenstates at a given total wave vector q, and all eigenstates at $q = 0$), that nevertheless can be (approximately) attributed to a single-exciton wave vector k. The pseudopotential $V(k)$ is not equivalent to the X–X scattering matrix element [11] $v(k_1, k_2; \Delta) = \langle k_1, k_2 | V | k_1 - \Delta, k_2 + \Delta \rangle$, which for $\Delta = 0$ describes interaction of excitons with well-defined incoming wave vectors. The different behavior of $V(k)$ and $v(k, \pm k; 0)$ at small k is the subject of this paper.

In LL_0, a well-known statistics and (or) correlation effect is the decoupling and condensation of $k = 0$ excitons in the ground state of interacting electrons and holes [12]. It can be interpreted in terms of an inter-exciton (X–X) exchange attraction exactly compensating for a decrease in the intra-exciton (e–h) attraction due to the phase space blocking for the coexisting identical constituent fermions.

The exciton condensation in LL_0 results from the mapping of an e–h system onto a two-spin system with spin-symmetric interactions [13]. The "hidden" e–h symmetry corresponding to the conservation of the total spin and responsible for exciton condensation holds in LL_0 because there the electron and hole orbitals are identical despite different effective masses (in experimental systems with finite width, this also requires symmetric doping to avoid a normal electric field that would split the e and h layers).

The mapping between e–h and two-spin systems makes inter-band excitons in an empty LL_0 equivalent to spin waves (SWs) in a filled LL_0, i.e., in the quantum Hall ground state with the filling factor $\nu = 1$. A SW (or spin exciton) consists of a hole in the spin-polarized LL_0 and a reversed-spin electron in the same LL_0. Although excitons and SWs in LL_0 are formally equivalent and the conclusions of ref. 9 and ours apply to both complexes, they are relevant for two different types of experiments (photoluminescence and spin relaxation).

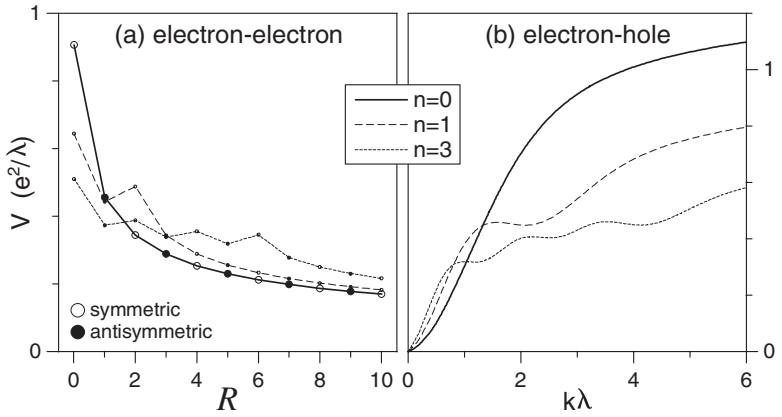
Being charge-neutral, excitons move along straight lines and carry a linear wave vector k even in a magnetic field B. The origin of their (continuous) dispersion [14] $\varepsilon(k)$ in LL_0 is not the (constant) e or h kinetic energy, but the dependence of an average e–h separation on k. A moving exciton carries an electric dipole moment d, proportional and orthogonal to both k and B.

For a pair of moving excitons, one could think that the dominant contribution to their interaction $V(\boldsymbol{k}_1, \boldsymbol{k}_2)$ would be the dipole–dipole term [15], specially at small values of k_1 and k_2, when this term is too weak on the scale of $\varepsilon(k)$ to cause a significant polarization of the X wave functions. Such assumption would lead to the repulsion between excitons moving in the same direction.

However, we show that this assumption is completely false because of the required (anti)symmetry of the wave function of overlapping excitons under exchange of individual constituent electrons or holes. This statistics and (or) correlation effect is significant even at small k, and it reverses the sign of the X–X interaction, compared to the dipole–dipole term. Specifically, excitons moving in the same direction attract one another, and the ground state of a pair of excitons carrying a total wave vector $\boldsymbol{q}$ is a (dynamically) bound state with $\boldsymbol{k}_1 = \boldsymbol{k}_2 = \frac{1}{2}\boldsymbol{q}$. More precisely, *the lowest state of a pair of interacting excitons has lower energy than any pair of noninteracting excitons with the same* $\boldsymbol{q}$.

The X–X interaction pseudopotential is calculated numerically for two special cases: $\boldsymbol{k}_1 = \pm\boldsymbol{k}_2$, corresponding to a pair of excitons moving with equal wave vectors $k_1 = k_2 \equiv k$ in the same ($\uparrow\uparrow$) and opposite ($\uparrow\downarrow$) direction. In addition to the sign reversal, we find that the inclusion of the statistics' effects leads to the significant weakening of the X–X interaction, specially at small k (e.g., for the $\uparrow\uparrow$ configuration, we find a $V \propto k^4$ power-law behavior).

Fig. 1. (*a*) Electron–electron and (*b*) electron–hole pseudopotentials in the *n*th ($n = 0$, 1, and 3) LL. V is the pair interaction energy, $\mathcal{R}$ is the relative pair angular momentum, k is the total pair wave vector, and λ is the magnetic length.



The near vanishing of the interaction between excitons moving in the same direction explains the occurrence of nearly linear multiexciton bands found numerically in the spin-excitation spectra of finite-size quantum Hall droplets [16, 17] and of extended quantum Hall systems [18]. The attractive character of this interaction explains the slightly convex shape of these bands, which for a confined droplet leads to the oscillations of the total spin as a function of the magnetic field [16, 17].

2. Model

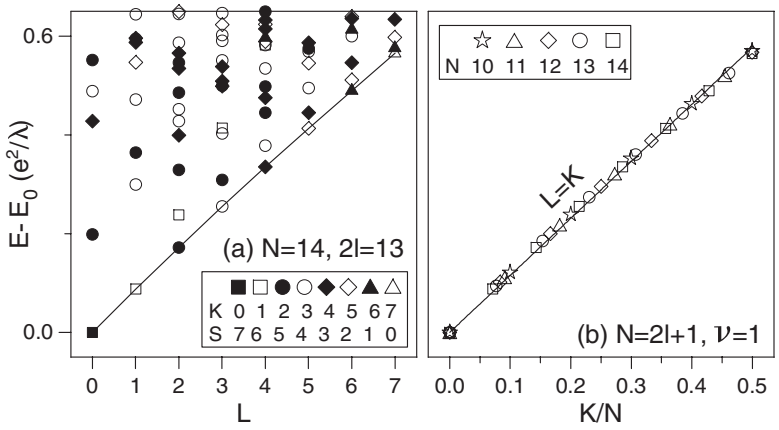
We consider spin excitations at the filling factor $\nu = 1$, i.e., in a system of N electrons half-filling the lowest Landau level (LL_0) single-particle angular momentum (l) shell with two-fold spin degeneracy and the orbital degeneracy $g \equiv 2l + 1 = N$. The interaction among the electrons in the Hilbert space restricted to LL_0 is entirely determined by the Haldane pseudopotential [10] defined as pair interaction energy V_{ee} as a function of relative pair angular momentum $\mathcal{R}$ and plotted in Fig. 1(a).

The even and odd values of $\mathcal{R}$ correspond to symmetric and antisymmetric pair wave functions, i.e., to the singlet and triplet pair spin state, respectively. Assuming large cyclotron gap $\hbar\omega_c$ between LL's (compared to the Zeeman gap E_Z and the interaction energy scale e^2/λ, where $\lambda = \sqrt{hc/eB}$ is the magnetic length), similar low-energy excitations of electrons at larger odd-integral values of $\nu = 2n+1$ occur only in the half-filled LL_n, and the only difference compared with the $\nu = 1$ case is a different form of $V(\mathcal{R})$, as shown in Fig. 1(*a*) for $n = 1$ and 3.

The two-spin system of $N = N_\downarrow + N_\uparrow$ electrons can be mapped onto that of $K_e = N_\uparrow$ spin-$\uparrow$ electrons and $K_h = N - N_\downarrow$ of spin-$\downarrow$ holes [13]. At $\nu = 1$, $K_e = K_h \equiv K$. The electrons and holes obtained through such mapping are both spin-polarized, and their (equal) e–e and h–h interactions are determined by the pseudopotential parameters $V_{ee}(\mathcal{R})$ corresponding only to odd values of $\mathcal{R}$. The effective e–h interaction depends on $V_{ee}(\mathcal{R})$ at both even and odd values of $\mathcal{R}$, but it can be described more directly by an e–h pseudopotential (pair e–h energy V_{eh} as a function of pair wave vector k) plotted in Fig. 1(*b*). In LL_0, both e–e and e–h pseudopotentials are monotonic, while in higher LL's they have oscillations reflecting additional nodes of the single-particle wave functions.

Because of the exact mapping between two-spin and two-charge systems, all results discussed here are, in principle, applicable to systems of conduction electrons and valence holes. This equivalence is true for ideal systems (with zero layer width w and no LL mixing) considered here. However, in realistic interband systems (realized, e.g., by optical excitation of an electron gas) the e and h wave functions are different both in the plane of motion (because of mass-dependent LL mixing) and in the normal direction (because of mass-dependent density profiles $\varrho(z)$ and an additional separation of e and h planes induced

Fig. 2. (*a*) Energy spectrum (interaction energy E versus total angular momentum L) of $N = 14$ electrons calculated on a sphere for $2l + 1 = N$ (at filling factor $\nu = 1$). S is the total spin, $K = \frac{1}{2}N - S$, and λ is the magnetic length. (*b*) Low-energy $L = K$ band for different N as a function of $\zeta = K/N$.



by a charged doping layer). Therefore, the "hidden symmetry" is broken in experimental e–h systems, while the equivalent conservation of the total spin S is easily realized in the corresponding two-spin systems.

3. Spin-excitation spectrum at $\nu = 1$

An intriguing feature known to occur in the spin-excitation spectrum at $\nu = 1$ is the low-energy band that is linear in spin and angular momentum. It was first identified in finite-size quantum Hall droplets [16], and later discussed [17] in Haldane spherical geometry [19], convenient in modeling infinite, translationally invariant systems.

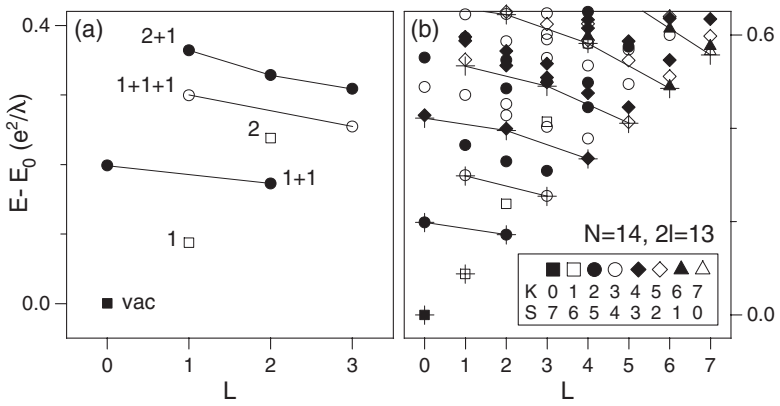
The energy spectrum obtained for $N = 14$ electrons on a sphere is shown in Fig. 2*a*. The energy E is counted from the energy E_0 of the spin-polarized $\nu = 1$ ground state, and it is plotted as a function of total angular momentum L. Different values of the total spin S (or of the number $K = \frac{1}{2}N - S$ of spin flips relative to the polarized ground state) are indicated by different symbols. Each point (E, L) represents a $(2L+1)(2S+1)$-fold degenerate multiplet of states distinguished by L_z and/or S_z. The orbital degeneracy, on a sphere associated with different orientations of the vector $\boldsymbol{L}$, on a plane corresponds to different directions of the wave vector $\boldsymbol{k}$ of finite length $k = L/R$ (where R is the sphere radius).

It is clear in Fig. 2*a* that the lowest state at each L has $K = L$. This band is nearly linear in L and, thus, it can be interpreted as containing states of K ordered and noninteracting SWs, each carrying angular momentum $\ell = 1$ and energy $\varepsilon_\ell = V_{\rm eh}(k_\ell)$, where $k_\ell = \ell/R$. Ordering means here that the angular momentum vectors of the K SWs are all parallel to give a total $L = K\ell$, i.e., that all SWs move in the same direction along the same great circle of the sphere. On a plane (corresponding to $R \to \infty$), this corresponds to K SWs moving in parallel along a straight line, each with an infinitesimal wave vector k_ℓ.

Scaling of this $L = K$ band with the size of the system is shown in Fig. 2*b*, where we overlay the data for different $N \leq 14$. The excitation energy E appears be a (nearly size-independent) linear function of "spin polarization" $\zeta = K/N$. Assuming exact decoupling of the SWs in this band, $E(\zeta) \equiv K\varepsilon_\ell$ can be extrapolated to the planar geometry, where the SW dispersion is [14]

$$V_{\rm eh}(k) = \sqrt{\frac{\pi}{2}}\left(1 - e^{-\kappa^2} I_0\left(\kappa^2\right)\right)\frac{e^2}{\lambda} \tag{1}$$

Fig. 3. (*a*) Low-energy part of Fig. 2(*a*). Labels indicate angular momenta ℓ of the (interacting) SW's in each 14-electron state. (*b*) Approximate energies ("+") of 14-electron states containing a number of interacting SW's each with $\ell = 1$ compared with the exact spectrum of Fig. 2(*a*).



with $\kappa = \frac{1}{2}k\lambda$ and I_0 being the modified Bessel function of the first kind. For small k_ℓ,

$$\varepsilon_\ell \equiv V_{\rm eh}\,(k_\ell) \approx \sqrt{\frac{\pi}{2}}\,\kappa_\ell^2\,\frac{e^2}{\lambda} \tag{2}$$

Substituting $k_\ell\lambda = \ell/R$, $R = \sqrt{Q}\lambda$ (where $2Q$ is the magnetic monopole strength; $2Q \cdot hc/e = 4\pi R^2 B$), $l = Q$ for the lowest electron shell (LL), and, at $\nu = 1$, $N = g \equiv 2l + 1$, we have $k_\ell\lambda = \sqrt{2/N}$, and finally

$$E(\zeta) = \zeta\sqrt{\frac{\pi}{8}}\,\frac{e^2}{\lambda} \tag{3}$$

This slope is much smaller from that in Fig. 2*b* due to finite-size and (or) curvature errors on a sphere, particularly significant at small k_ℓ. The total wave vector $k = L/R = Kk_\ell$ for the $L = K$ band scales as

$$k\lambda = \sqrt{2N}\zeta \tag{4}$$

i.e., on a plane is it divergent. Therefore, $E(\zeta)$ is a lower bound for the actual excitations at a given ζ that will have large but finite k.

4. Effective SW–SW interaction

Let us disregard the divergence of k in (4) and concentrate on the fact that the (nearly) linear behavior of $E(K)$ suggests decoupling of SW's in the $L = K$ band. This invokes a more general question of interaction between SW's in the lowest (or higher) LL's. Unlike their number $K = \frac{1}{2}N - S$, the individual angular momenta of interacting SW's are not conserved. For example, a pair of SW's both with $\ell = 1$ and with the total angular momentum $L = 2$ are coupled to a pair with the same L but with different $\ell = 1$ and 2 (the vector addition of ℓ and ℓ' gives L going from $|\ell - \ell'|$ to $\ell + \ell'$); these two configurations being denoted as $|1 + 1; 2\rangle$ and $|1 + 2; 2\rangle$. However, unless the single-SW energies $\mathcal{E}$ of such coupled configurations (here, $\mathcal{E} = 2\varepsilon_1$ and $\varepsilon_1 + \varepsilon_2$) are close, this coupling can be effectively incorporated into the SW–SW interaction. In Fig. 3(*a*) we have made such assignment for the lowest excitations of the 14-electron spectrum.

Following this assignment, we can extract not only the (exact) single-SW energies, $\varepsilon_L = E[L] - E_0$, but also the parameters of an effective SW–SW interaction pseudopotential, $V[\ell + \ell'; L] = E[\ell +$

$\ell'; L] - \varepsilon_\ell - \varepsilon_{\ell'} - E_0$. Using these two-SW interaction parameters one can describe interactions in the states of more than two SW's.

Let us demonstrate it on a simple example of K SW's each with $\ell = 1$. In this case, there are only two pair-SW states, at $L = 0$ and 2, corresponding to the relative (with respect to the center of mass of the two SW's) angular momenta $\mathcal{R} \equiv 2\ell - L = 2$ and 0 (SW's are pairs of fermions, and thus for two SW's with equal ℓ, $\mathcal{R}$ must be even as for two identical bosons). Thus, there are only two interaction parameters, in a 14-electron system equal to $V_2 \equiv V[1+1;0] = 0.0236\, e^2/\lambda$ and $V_0 \equiv V[1+1;2] = -0.0026\, e^2/\lambda$ (for the subscripts in V_0 and V_2 we use notation $V_\mathcal{R}$ and not V_L).

The total energy of the state Ψ of K SW's, $E = E_0 + K\varepsilon_\ell + U$, contains the inter-SW interaction energy that can be expressed as

$$U = \binom{K}{2} \sum_\mathcal{R} \mathcal{G}_\mathcal{R} V_\mathcal{R} \tag{5}$$

Here, $\mathcal{G}_\mathcal{R}$ are the pair amplitudes [10, 20] (pair-correlation functions) that measure the number of SW pairs with a given $\mathcal{R}$ (for brevity, we omit index Ψ in E, U, and $\mathcal{G}_\mathcal{R}$). They are normalized, $\sum_\mathcal{R} \mathcal{G}_\mathcal{R} = 1$, and satisfy an additional sum rule that on a sphere has the form [21]

$$L(L+1) + K(K-2)\,\ell(\ell+1) = \binom{K}{2} \sum_\mathcal{R} \mathcal{G}_\mathcal{R}\, \mathcal{L}(\mathcal{L}+1), \tag{6}$$

where L and $\mathcal{L} \equiv 2\ell - \mathcal{R}$ are the total and pair SW angular momenta, respectively.

For $\ell = 1$, there are only two pair amplitudes, $\mathcal{G}_0$ and $\mathcal{G}_2$, and hence they are independent of the SW–SW interaction and can be completely determined from (6). This allows expression of $\mathcal{G}_\mathcal{R}$ and, using the values of $V_\mathcal{R}$ and (5), of U and E as a function of K and L

$$U = \frac{L(L+1) + 2K(K-2)}{6}\,(V_0 - V_2) + \frac{K(K-1)}{2} V_2 \tag{7}$$

For $L = K$ this gives $\mathcal{G}_2 = 0$ and $U = \frac{1}{2}K(K-1)V_0$, i.e., the linearity of $E(K)$ depends on the vanishing of V_0. Energies $E(K, L)$ obtained from (7) for all combinations of L and K are compared with the exact 14-electron energies in Fig. 3(*b*). Good agreement, especially for the $L = K$ band, justifies interpretation of the actual spin excitations in terms of K SW's with well-defined ℓ, interacting through the effective SW–SW pseudopotentials.

5. SW–SW pseudopotential

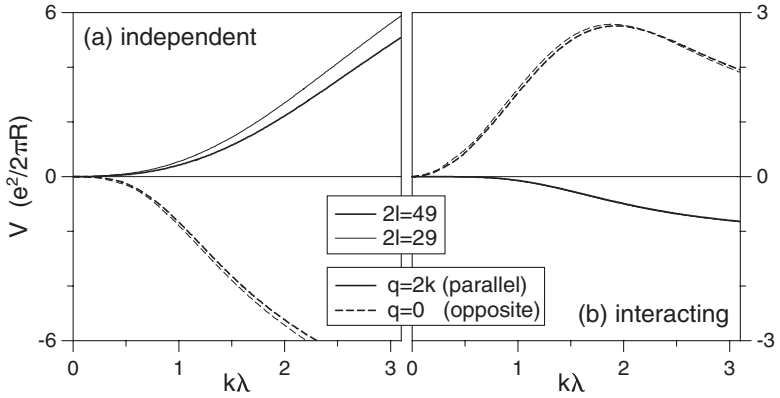
This brings up the question of why are the SW's in the $L = K$ band (nearly) noninteracting (i.e., why is V_0 so small compared to V_2 or ε_1). And a more general one, what is the pseudopotential describing interaction between the SW's. The SW–SW pseudopotential V depends on the pair of wavevectors, $\boldsymbol{k}$ and $\boldsymbol{k}'$. However, in extension of V_0 and V_2 in (7), we will only consider two special cases: $V_{\uparrow\uparrow}(k)$ and $V_{\uparrow\downarrow}(k)$, corresponding to two SW's with equal wavevectors k moving in the same and opposite directions, respectively.

5.1. Independent SW's

A moving SW carries [14] an in-plane dipole electric moment $\boldsymbol{d}$, with magnitude d proportional to k and oriented orthogonally to the direction of $\boldsymbol{k}$. For a pair of uncorrelated SW's this implies simple dipole–dipole interaction, repulsive for the $\uparrow\uparrow$ configuration, and attractive for $\uparrow\downarrow$. Indeed, in Fig. 4(*a*) we plot $V_{\uparrow\uparrow}(k)$ and $V_{\uparrow\downarrow}(k)$ showing such behavior. Moreover, at small k we find a very regular power-law dependence

$$V_{\uparrow\uparrow}(k) \sim 0.42(k\lambda)^{5/2} \frac{e^2}{2\pi R} \tag{8}$$

Fig. 4. SW–SW pseudopotentials (two-SW interaction energy V vs. single-SW wavevector k) for the independent (a) and correlated (b) SW's moving in the same or opposite direction (total pair wavevector $q = 2k$ or 0, respectively). λ is the magnetic length and R is the sphere radius.



The curves in Fig. 4(a) have been calculated as an expectation value of the Coulomb interaction in a trial state $|k, k; q\rangle$ describing two uncorrelated (independent) SW's, each with the wavevector k and with the total wavevector $q = 2k$ ($\uparrow\uparrow$) and $q = 0$ ($\uparrow\downarrow$), and thus they are equivalent to the SW–SW scattering matrix element with zero wavevector transfer, $v(k, \pm k; 0)$.

The trial states have been constructed on a sphere in the $2e+2h$ basis in a lowest LL with $l = Q$. The two electrons (and two holes) are distinguished by different isospins $\sigma = \pm\frac{1}{2}$. A pairing hamiltonian H_ℓ is introduced with the e–h pseudopotential in the form

$$V_{eh}^{(\ell)}\left(\sigma_e, \sigma_h, \ell'\right) = -\delta_{\sigma_e\sigma_h}\delta_{\ell\ell'} \tag{9}$$

and the e–e and h–h interactions set to zero. At each total angular momentum L, there is exactly one eigenstate of H_ℓ corresponding to the eigenvalue -2. It describes two independent e–h pairs (i.e., excitons or SW's), one with $\sigma_e = \sigma_h = \frac{1}{2}$ and one with $\sigma_e = \sigma_h = -\frac{1}{2}$, each in an eigenstate of pair angular momentum ℓ corresponding to the pair wavevector $k_\ell = \ell/R$ (on a sphere, describing motion of a charge-neutral pair along a great circle). The total angular momentum L of two pairs can also be converted into the total wavevector, $q = L/R$.

We have concentrated on the trial states with $L = 2\ell$ and 0 (i.e., with $q = 2k_\ell$ and 0), denoted as $|k_\ell, k_\ell; 2k_\ell\rangle$ and $|k_\ell, k_\ell; 0\rangle$. They describe two pairs each with the same k_ℓ and moving in the same and opposite directions, respectively. Discrete SW–SW pseudopotentials $V_{\uparrow\uparrow}(k_\ell)$ and $V_{\uparrow\downarrow}(k_\ell)$ on a sphere have been calculated as the expectation value of the inter-SW Coulomb interaction (i.e., the total Coulomb energy of the $2e + 2h$ state minus the intra-SW e–h attraction $2\varepsilon_\ell$). When the sphere curvature $R/\lambda = \sqrt{(Q)}$ decreases, the discrete values quickly converge to the continuous curves $V_{\uparrow\uparrow}(k)$ and $V_{\uparrow\downarrow}(k)$ appropriate for a planar system. The interpolated curves for the LL degeneracy $2l + 1 \equiv 2Q + 1 = 30$ and 50 are compared in Fig. 4(a). Note that V is plotted as a function of $e^2/2\pi R$ (rather than e^2/λ) what reflects the fact that SW's are extended objects confined to a great circle of length $2\pi R$ (in contrast to electrons or holes that are confined to cyclotron orbits of radius $\sim \lambda$).

5.2. Coupled SW's

The SW–SW pseudopotentials obtained above describe interaction between independent SW's (distinguished by isospins σ_e and σ_h). However, the following two correlation effects must be incorporated into the effective SW–SW interaction to describe the actual spin excitations at $\nu \sim 1$ (i.e., the interacting e–h systems).

First, the Coulomb (charge–charge) interaction between the SW's breaks the conservation of ℓ and causes relaxation of the individual SW wavefunctions and their energies ε_ℓ. This perturbation effect mixes the SW states within the energy range $\Delta\varepsilon \sim V$, so it becomes negligible when V is small, i.e., at small k. In particular, it does not affect the behavior of $V_{\uparrow\uparrow}(k)$ at small k, responsible for the linearity of the $L = K$ band.

Second, strictly speaking, the SW's are not bosons but pairs of fermions, and a wavefunction of two SW's must not only be symmetric under interchange of the entire SW's, but also antisymmetric under interchange of two constituent electrons or holes. The trial paired states $|k, k; q\rangle$ with $H_\ell = -2$ do not obey these symmetry requirements, because H_ℓ is isospin-asymmetric and hence it does not commute with pair e or h isospins, Σ_e and Σ_h. Therefore, the trial eigenstates of $H_\ell = -2$ are different from the properly symmetrized eigenstates of $\Sigma_e = \Sigma_h = 1$. This statistics effect is generally weak for spatially separated composite particles, but for the SW's moving along the same line (or great circle) it is large and cannot be treated perturbatively (even at small k when the Coulomb SW–SW interaction is negligible). At each L, the exact form of the ground state in the $\Sigma_e = \Sigma_h = 1$ subspace depends on ℓ and on the details of the actual (Coulomb) hamiltonian, and so does the average value of H_ℓ (measuring the actual "degree of pairing"). However, as a reasonable approximation one can introduce the "maximally paired" states, defined at each L as the lowest-energy state of the pairing interaction hamiltonian $V_{eh}^{(\ell)}$ within the $\Sigma_e = \Sigma_h = 1$ subspace.

The relaxation of the wavefunctions of the overlapping SW's is evident from the analysis of the e–e and h–h pair amplitudes $\mathcal{G}(\mathcal{R})$. For a pair of different particles, such as electrons or holes distinguished by isospin σ in the trial state $|k, k; q\rangle$, $\mathcal{R}$ can be any integer. Therefore, $\mathcal{G}_{ee}(\mathcal{R})$ and $\mathcal{G}_{eh}(\mathcal{R})$ calculated for the independent SW's are positive at both even and odd $\mathcal{R}$ (in fact, there is no obvious correlation whatsoever between the parity of $\mathcal{R}$ and the value of $\mathcal{G}_{ee}$ or $\mathcal{G}_{eh}$). In contrast, for a pair of identical fermions, such as electrons or holes in an actual, interacting state of two SW's, $\mathcal{G}_{ee}(\mathcal{R})$ and $\mathcal{G}_{eh}(\mathcal{R})$ vanish exactly at all even values of $\mathcal{R}$. The change of pair amplitudes when going from the trial states $|k, k; q\rangle$ to the actual Coulomb ground states is quite dramatic, precluding adequacy of the pseudopotentials of Fig. 4(a) for the description of many-SW systems.

Because of the above relaxation effects, interaction between the SW's is not purely a two-body interaction, and thus it cannot be completely described by a (pair) pseudopotential $V(k)$. In other words, a SW–SW pseudopotential taking these effects into account is not rigorously defined. However, as demonstrated in Fig. 2(b), many-SW spectra can be reasonably well approximated using an effective pseudopotential obtained for only two SW's.

To determine such effective $V_{\uparrow\uparrow}(k)$ and $V_{\uparrow\downarrow}(k)$, we calculate the $2e + 2h$ Coulomb energy spectra similar to the $K \leq 2$ part of Fig. 3(a) and make analogous assignments for the $K = 2$ states. The lowest state at each even value of $L = 2, 4, \ldots$ is interpreted as one of two SW's each with $\ell = \frac{1}{2}L$ and moving in the same direction. Similarly, consecutive states at $L = 0$ contain two SW's each with $\ell = 1, 2, \ldots$ and moving in opposite directions. In both cases, $V(\ell) = E - 2\varepsilon_\ell - E_0$. When ℓ is converted into $k_\ell = \ell/R$ and V is plotted in the units of $e^2/2\pi R$, the discrete pseudopotentials $V(k_\ell)$ fall on the continuous curves $V_{\uparrow\uparrow}(k)$ and $V_{\uparrow\downarrow}(k)$ that very quickly converge to ones appropriate for a planar system when the sphere curvature $R/\lambda = \sqrt{(Q)}$ is decreased. The interpolated curves for $2l + 1 \equiv 2Q + 1 = 30$ and 50 are compared in Fig. 4(b), showing virtually no size dependence. Similar curves were obtained for the "maximally paired" states used instead of actual Coulomb eigenstates.

The justification for the above assignment comes from the observation of distinct bands in the low-energy $K = 2$ spectrum. The values of L within each band are consistent with the addition of angular momenta of two SW's, $|\ell - \ell'| \leq L \leq \ell + \ell'$ (with the additional requirement that $L - 2\ell \equiv \mathcal{R}$ be even for $\ell = \ell'$). In the absence of the SW relaxation, these bands would contain the eigenstates of $\mathcal{E} \equiv \varepsilon_\ell + \varepsilon_{\ell'}$, with the intra-band dispersion reflecting interaction of the independent SW's with ℓ and ℓ'. In the actual spectrum, the bands mix, but remain separated, making the assignment possible. The interband mixing and the resulting changes in the energy spectrum are precisely the relaxation effects,

effectively incorporated into $V(k)$. For $L = 0$ ($\uparrow\downarrow$), the mixing is minimal, because the contributing "independent SW" configurations $\left|\ell, \ell'; L = 0\right\rangle$ must all have $\ell = \ell'$, and thus very different single-SW energies $\mathcal{E}$. For $L = 2\ell$ ($\uparrow\uparrow$), mixing between configurations $|\ell + \delta, \ell - \delta; L = 2\ell\rangle$ with close values of $\mathcal{E}$ can occur, having a stronger effect on the effective $V_{\uparrow\uparrow}(k)$.

The main two findings about the effective SW–SW pseudopotentials shown in Fig. 4(b) are the following. First, the statistics effect turns out so strong as to reverse the sign of interaction. In contrast to the prediction of the model of independent SW's with dipole–dipole interaction, the SW's moving in the same direction decrease their total energy (what can be interpreted as attraction), while the SW's moving in opposite direction increase their energy (i.e., repel one another). Second, the magnitude of the $\uparrow\uparrow$ attraction at small k is greatly reduced compared to (8). It can also be approximated by a power-law dispersion, but with a much higher exponent and a much smaller prefactor,

$$V_{\uparrow\uparrow}(k) \sim -0.069\,(k\lambda)^4 \frac{e^2}{2\pi R} \tag{10}$$

Although the near vanishing of $V_{\uparrow\uparrow}$ at small k was anticipated from the linearity of the $L = K$ band in Fig. 2, the negative sign and large exponent are rather surprising and of a wider consequence. It may be worth stressing that the identified attraction between N SW's (or interband excitons) moving in the same direction is too weak to induce a stable bound ground state, with the total energy lower than N times ground state energy of a single SW/exciton. Therefore, it does not contradict a well-known fact that the ground state of N electrons and N holes in the lowest LL is a multiplicative state [12, 13] of N SW's/excitons each with $k = 0$ (in particular, a biexciton is unstable toward breaking up into two $k = 0$ excitons, while the energy of N SW's is never lower than $N\varepsilon_0 = 0$, and so the $\nu = 1$ ground state is spontaneously polarized). However, for two or more SW's/excitons carrying a conserved total wavevector $q > 0$, the convex shape of $V_{eh}(k)$ causes equal distribution of q among all SW's/excitons, and the SW–SW or X–X attraction binds them together. Such a moving multi-SW/exciton can only break up (into separate SW's/excitons) through an inelastic collision taking away its wavevector. This dynamical binding will affect spin relaxation (for the SW's) or photoluminescence (for the excitons) of an electron gas, but the relevant spectra are yet to be calculated. The effect will have to depend critically on the absence of excess electrons or holes, which would capture the SW's or excitons to form skyrmions at $\nu \neq 1$ or trions at $\nu > 0$.

6. Conclusion

We have studied interaction between moving SW's (excitons) in the lowest LL. For a pair of SW's with equal wavevectors k and moving in the same ($\uparrow\uparrow$) or opposite ($\uparrow\downarrow$) directions, the effective interaction pseudopotentials $V_{\uparrow\uparrow}(k)$ and $V_{\uparrow\downarrow}(k)$ have been calculated numerically. They account for relaxation of overlapping SW's due to the Fermi statistics of constituent (reversed-spin) electrons and (spin-) holes, and differ completely from the prediction for independent SW's interacting through their dipole moments. In particular, the signs of the interactions are reversed and their magnitudes are decreased. The former effect leads to a "dynamical binding" of mobile multiexcitons, and the latter explains the near decoupling of excitons in the linear $L = K$ band in the spin-excitation spectrum at $\nu = 1$.

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References

1. H. Haug and S.W. Koch. Quantum Theory of the Optical and Electronic Properties of Semiconductors. World Scientific, Singapore. 1993; S.A. Moskalenko and D.W. Snoke. Bose-Einstein Condensation of Excitons and Biexcitons. Cambridge University Press, Cambridge. 2000.
2. L.V. Keldysh and A.N. Kozlov. Zh. Eksp. Teor. Fiz. **54**, 978 (1968) [Sov. Phys. JETP **27**, 521 (1968)].
3. S. Okumura and T. Ogawa. Phys. Rev. B, **65**, 035105 (2002); M. Combescot and O. Betbeder-Matibet. Europhys. Lett. **59**, 579 (2002).
4. J. Shah. Ultrafast Spectroscopy of Semiconductors and Semiconductor Nanostructures. Springer-Verlag, Berlin. 1996.
5. B.F. Feuerbacher, J. Kuhl, and K. Ploog. Phys. Rev. B, **43**, 2439 (1991); K.H. Pantke, D. Oberhauser, V.G. Lyssenko, J.M. Hvam, and G. Weimann. Phys. Rev. B, **47**, 2413 (1993); E.J. Mayer, G.O. Smith, V. Heuckeroth, J. Kuhl, K. Bott, A. Schulze, T. Meier, D. Bennhardt, S.W. Koch, P. Thomas, R. Hey, and K. Ploog. Phys. Rev. B, **50**, 14730 (1994); J. Ishi, H. Kunugita, K. Ema, T. Ban, and T. Kondo. Phys. Rev. B, **63**, 073303 (2001).
6. T. Baars, W. Braun, M. Bayer, and A. Forchel. Phys. Rev. B, **58**, R1750 (1998); T. Baars, M. Bayer, A.A. Gorbunov, and A. Forchel. Phys. Rev. B, **63**, 153312 (2001).
7. P. Borri, W. Langbein, J. M. Hvam, and F. Martelli. Phys. Rev. B, **60**, 4505 (1999); H.P. Wagner, A. Schätz, W. Langbein, J.M. Hvam, and A.L. Smirl. Phys. Rev. B, **60**, 4454 (1999); W. Langbein and J. M. Hvam. Phys. Rev. B, **61**, 1692 (2000).
8. The Quantum Hall Effect. *Edited by* R.E. Prange and S.M. Girvin. Springer-Verlag, New York. 1987; Perspectives in Quantum Hall Effects. *Edited by* S. Das Sarma and A. Pinczuk. John Wiley & Sons, New York. 1997.
9. R.L. Doretto, A.O. Caldeira, and S.M. Girvin. Phys. Rev. B, **71**, 045339 (2005).
10. F.D.M. Haldane. *In* The Quantum Hall Effect. *Edited by* R.E. Prange and S.M. Girvin. Springer-Verlag, New York. 1987. Chap. 8, pp. 303–352.
11. Yu.A. Bychkov and E.I. Rashba. Zh. Eksp. Teor. Fiz. **85**, 1826 (1983) [Sov. Phys. JETP **58**, 1062 (1983)]; S.T. Chui. Phys. Rev. Lett. **58**, 1355 (1987); T.V. Shahbazyan, N. Primozich, and I.E. Perakis. Phys. Rev. B, **62**, 15925 (2000).
12. I.V. Lerner and Yu.E. Lozovik. Zh. Eksp. Teor. Fiz. **80**, 1488 (1981) [Sov. Phys. JETP **53**, 763 (1981)]; Y.A. Bychkov and E.I. Rashba. Solid State Commun. **48**, 399 (1983); T.M. Rice, D. Paquet, and K. Ueda. Helv. Phys. Acta, **58**, 410 (1985); D. Paquet, T.M. Rice, and K. Ueda. Phys. Rev. B, **32**, 5208 (1985).
13. A.H. MacDonald and E.H. Rezayi. Phys. Rev. B, **42**, 3224 (1990).
14. L.P. Gor'kov and I.E. Dzyaloshinskii. Zh. Eksp. Teor. Fiz. **53**, 717 (1967) [Sov. Phys. JETP **26**, 449 (1968)]. C. Kallin and B.I. Halperin. Phys. Rev. B, **30**, 5655 (1984).
15. Dipole–dipole interaction studied by M.A. Olivares-Robles and S.E. Ulloa. Phys. Rev. B, **64**, 115302 (2001), may become dominant in asymmetric structures (with separated electron and hole planes) in which all excitons carry large dipole moments oriented in the same (normal) direction.
16. J.J. Palacios, L. Martín-Moreno, G. Chiappe, E. Louis, and C. Tejedor. Phys. Rev. B, **50**, 5760 (1994); J.H. Oaknin, L. Martín-Moreno, J.J. Palacios, and C. Tejedor. Phys. Rev. Lett. **74**, 5120 (1995). J.H. Oaknin, L. Martín-Moreno, and C. Tejedor. Phys. Rev. B, **54**, 16850 (1996).
17. A. Wójs and P. Hawrylak. Phys. Rev. B, **56**, 13227 (1997).
18. A. Wójs and J. J. Quinn. Phys. Rev. B, **66**, 045323 (2002); Solid State Commun. **122**, 407 (2002).
19. F.D.M. Haldane. Phys. Rev. Lett. **51**, 605 (1983).
20. A. Wójs and J. J. Quinn. Philos. Mag. B, **80**, 1405 (2000); Acta Phys. Pol. A, **96**, 593 (1999); J.J. Quinn and A. Wójs. J. Phys.: Condens. Matter, **12**, R265 (2000).
21. A. Wójs and J. J. Quinn. Solid State Commun. **110**, 45 (1999); A. Wójs. Phys. Rev. B, **63**, 125312 (2001).