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Incompressible composite fermion liquids

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Abstract

The "second generation" fractional quantum Hall states containing a partially filled composite fermion (CF) Landau level (LL) are studied theoretically. The role of the unique form of the CF–CF interaction in the incompressibility of the underlying quantum electron liquid is explained. In particular, the two- and three-body CF correlation functions for these liquids are determined from exact diagonalization on a Haldane sphere. They are used to show that the CFs form a paired state (rather than a Laughlin liquid) when filling $\nu = \frac{1}{3}$ of their second LL. Similarly, at $\nu = \frac{1}{2}$ the CFs do not appear to form a Moore–Read paired state but tend to group into larger clusters instead. The spin polarization of the interacting CFs is also investigated, and a transition to the partially unpolarized ground state is predicted in realistic conditions.

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1. Introduction

Quite recently Pan et al. [1] discovered fractional quantum Hall (FQH) effect in a spin polarized two-dimensional (2D) electron gas, at a new series of Landau level (LL) filling factors ν_e. These new fractions lied outside of the Jain sequence [2] of states $\nu_e = n/(2pn \pm 1)$ defined by the complete filling of n shells by the composite fermions (CFs) carrying $2p$ magnetic flux quanta. The novel FQH states occur at $\nu_e = \frac{4}{11}$ and $\frac{3}{8}$, corresponding to *fractional* fillings $\nu = \frac{1}{3}$ and $\frac{1}{2}$ of the second CF LL. Evidently, incompressibility of the electron liquids formed at these values of ν_e must depend on the interactions and correlations among the CFs. Therefore, in contrast to the Laughlin and Jain states whose understanding within the CF model invokes only the emergence of a quasicyclotron gap in the single-CF spectrum, the new liquids have been called the "second generation" FQH states [3–5].

Familiar values of $\nu = \frac{1}{3}$ and $\frac{1}{2}$ immediately suggested similarity between partially filled electron and CF LLs [6]. For $\nu_e = \frac{4}{11}$, it revived the "quasiparticle hierarchy" [7]. Its CF formulation consists of the CF → electron mapping followed by the reapplication of the CF picture in the second CF LL [8], leading to a "second generation" of CFs [3–5]. However, this idea ignored the requirement of a strong short-range repulsion [9–11]. Indeed, we show in the following that it is precluded by exact-diagonalization studies [12], in which a different series of finite-size $\nu_e = \frac{4}{11}$ liquids with larger gaps is identified. On the other hand, Moore–Read liquid [13] of paired CFs was tested [14] for $\nu_e = \frac{3}{8}$, but it was eventually ruled out in favor of the stripe order [15,16].

In this paper we often review the ideas published in a series of our earlier papers on the problem of CF–CF interaction [11,12,17–19]. We begin by recalling the CF picture of the $\nu_e = \frac{4}{11}$ and $\frac{3}{8}$ states, explaining how the CF wave functions and Haldane interaction pseudopotentials [20] can be extracted from N_e-electron exact-diagonalization calculations [11], and justifying the use of such effective CF pair pseudopotential for the description of electron dynamics at $\nu = \frac{1}{3}$ or $\frac{1}{2}$. Then we move to the

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description of numerical calculations for N interacting CFs, in which we identified the finite-size series of nondegenerate, gapped ground states extrapolating to $\nu = \frac{1}{3}$ or $\frac{1}{2}$ in the limit of large N [12]. Next, we analyze the wave functions of these N-CF ground states and, from the form of two- and three-body correlation functions, demonstrate that the $\nu_e = \frac{4}{11}$ state is a paired state of CFs [17,18]. The pair–pair correlations are not established, but a Laughlin form (the maximum avoidance of the relative two-pair angular momentum) [21] is excluded. Finally, we consider partially unpolarized "second generation" states, construct the spin phase diagram at $\nu_e = \frac{4}{11}$, and predict a spin transition under realistic (though yet unexplored) realistic conditions [19].

2. Interaction of Laughlin quasielectrons (QEs)

In the (admittedly, somewhat trivialized) description of the CF model, electrons filling a fraction ν_e of the lowest LL capture part of the external magnetic field B in form of quantized flux tubes of strength $2p\phi_0$ (here, $\phi_0 = hc/e$ is the flux quantum and p is an integer). In this way (by binding magnetic flux tubes) electrons are converted into CFs which experience reduced magnetic field B^*. The (real) electron and (effective) CF filling factors are related to each other through $\nu_{CF}^{-1} = \nu_e^{-1} - 2p$. For $\frac{1}{3} < \nu_e < \frac{2}{5}$ the choice of $2p = 2$ yields $1 < \nu_{CF} < 2$ and a fractional filling $\nu \equiv \nu_{CF} - 1 < 1$ of the second CF LL.

In particular, in the CF picture of the $\nu_e = \frac{4}{11}$ FQH state of electrons (assuming complete spin polarization), the CFs fill their entire lowest LL (CF-LL$_0$) and a fraction $\nu_{QE} \equiv \nu = \frac{1}{3}$ of their second LL (CF-LL$_1$). Similarly, $\nu_e = \frac{3}{8}$ corresponds to $\nu = \frac{1}{2}$ in CF-LL$_1$. The CFs in the partially filled CF-LL$_1$ represent QEs of the underlying incompressible $\nu_e = \frac{1}{3}$ Laughlin liquid [22] (represented by the completely filled CF-LL$_0$). This CF$\leftrightarrow$QE equivalence is exact at $\nu \ll 1$, but we will show later (see Fig. 2) that it appears valid at higher values of ν as well. Note also that we adopt here fermionic description of the QEs (bosonic [7] or anyonic [23] descriptions being equivalent as long as the CF-LL degeneracy and the QE–QE interaction are adjusted appropriately).

To study CF–CF correlations in CF-LL$_1$ (QE–QE correlations) in the incompressible electron liquids at $\nu_e = \frac{4}{11}$ or $\frac{3}{8}$ we must first determine the form of QE–QE interactions. Interaction within the Hilbert space restricted to an isolated LL (here, CF-LL$_1$) is conveniently defined by its Haldane pseudopotential $V(\mathcal{R})$ [20], i.e., dependence of the pair interaction energy V on the relative angular momentum $\mathcal{R}$ (for identical fermions, $\mathcal{R} = 1, 3, 5, \ldots$). The pseudopotential can be extracted from exact diagonalization of N_e electrons (with the Coulomb interaction) on a Haldane sphere [7], with the angular momentum of the LL shell equal to $l = 3(N-1)/2 - 1$ (on a sphere, $2l+1$ is the LL degeneracy), such as that for $N_e = 11$ in Fig. 1(d). In these spectra, the lowest band contains states of two QEs, and the dependence of N_e-electron energy E on total angular momentum L is (up to a constant) the QE–QE pseudopotential. The constant can be determined from the asymptotic long-range behavior for two particles of known charge $-e/3$, and $\mathcal{R} = 2l - L$.

The result is plotted in Fig. 1(c), where the QE–QE pseudopotential is compared to those for other Laughlin quasiparticles: quasiholes (QHs) and reversed-spin quasielectrons (QE$_R$). The key feature of the QE–QE pseudopotential is the weak repulsion at the smallest allowed relative angular momentum, $\mathcal{R} = 1$. This is very different from the CF pseudopotentials in other CF-LLs (i.e., from V_{QH} or V_{QER}) or from the electron pseudopotential in LL$_0$ or LL$_1$ (not shown). The reason is the ring-like charge distribution profile of the QEs shown in Fig. 1(a), very different from other CFs or from electrons in other LLs.

Let us stress two things. (i) The weak short-range QE–QE repulsion is evident from different numerical calculations (exact diagonalization [11,18] and Monte Carlo [15]) done for the quasi-2D electrons interacting through the Coulomb potential (actually, through any short-range repulsion). Therefore, it does not depend on any a priori assumptions on the nature of QEs themselves and it need be considered a "numerical–experimental" fact. (ii) This peculiar short-range behavior of $V_{QE}(\mathcal{R})$ invalidates analogy between electron and QE systems at the same ν. In particular, it is responsible for the lack of Laughlin

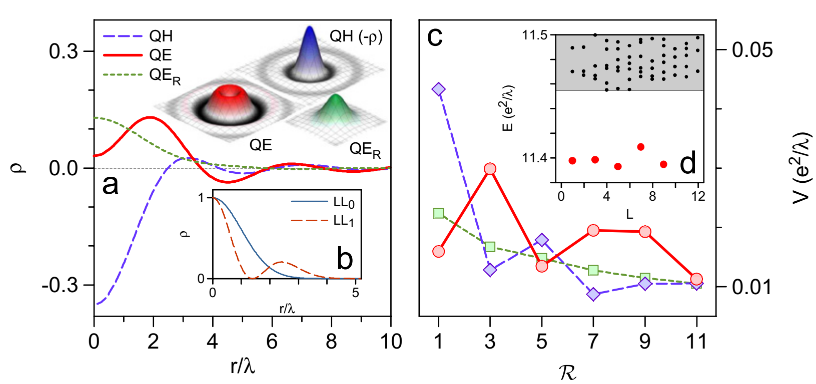

Fig. 1. (a) Radial charge distributions of different CFs (QE, QH, and QE$_R$) obtained from 10-electron diagonalization. (b) Same for electrons in two lowest LLs. (c) CF interaction pseudopotentials. (d) Example of electron spectrum used to obtain V_{QE}. λ is the magnetic length ($N_e = 11$, $2q = 28$).

correlations among QEs at $\nu = \frac{1}{3}$ or Moore–Read pairing at $\nu = \frac{1}{2}$.

Why cannot $V_{QE}(\mathcal{R})$ support Laughlin correlations? This results from the unique effect of "harmonic" interaction pseudopotential $V_H(\mathcal{R})$, defined as being proportional to the average squared distance $\langle r^2 \rangle$. On a sphere, $V_H(\mathcal{R}) = \alpha + \beta \cdot L(L+1)$, with constant α and β, and $\mathcal{R} = 2l - L$. For large $2l$ (or on a plane) this translates into $V_H \propto \mathcal{R}$ at $\mathcal{R} \ll 2l$ (at short range). The following operator identity, $\sum_{ij} \hat{L}_{ij}^2 = \hat{L}^2 + N(N-2)\hat{l}^2$ [10], connects the total angular momentum L of N single-particle angular momenta l with the pair angular momenta L_{ij}. It can be used to show that V_H induces no correlations (all many-body multiplets at the same L have the same energy). In other words, the relative occupation of different pair states (labeled by $\mathcal{R}$) in a many-body state has no effect on its total interaction energy. This changes when V is not harmonic. Any superharmonic contribution to V causes avoidance of the corresponding pair state in the low-energy many-body states. In particular, the dominant anharmonic repulsion at $\mathcal{R} = 1$ leads to Laughlin correlations (and the Laughlin ground state at $\nu = \frac{1}{3}$). Since pseudopotentials in LL_0, LL_1, and CF-LL_1 are all qualitatively different in comparison to V_H (being strongly superharmonic, roughly harmonic, and strongly subharmonic at short range, respectively), also the correlations induced by the respective $V(\mathcal{R})$ are all different.

The key questions now become: can $V_{QE}(\mathcal{R})$ lead to incompressibility? At what ν? And guided by Pan's experiment: what are the QE–QE correlations? What is the many-QE wave function at $\nu = \frac{1}{3}$ and $\frac{1}{2}$?

3. Incompressible QE liquids

Knowing $V_{QE}(\mathcal{R})$ one can search for QE incompressibility by exact diagonalization of N-QE interaction Hamiltonians at the values of $2l$ (CF-LL_1 degeneracy) given approximately by $2l = N/\nu$ (for the finite systems on a sphere, the exact relation between N and $2l$ depends on the form of wave function and is not known a priori; only for large N is the ratio $N/2l \to \nu$ restored). However, several troubling questions need first be answered to justify this approach: Is two-body pseudopotential sufficient to describe interaction among many QEs? Can the pseudopotential determined at $\nu \ll 1$ be used for $\nu = \frac{1}{3}$ or $\frac{1}{2}$? (Or, does the nature of Laughlin quasiparticles represented by the CFs remain unchanged when CF-LL_1 is filled from $\nu = 0$ to $\frac{1}{2}$?) Can the QE–QH excitations be neglected? (Or, does QE–QE interaction conserve the QE number?)

In the absence of rigorous analysis of these assumptions/approximations let us present an example of how accurate actually is the mapping of an N_e electron system with Coulomb interaction at $\frac{1}{3} < \nu_e < \frac{2}{5}$ onto a corresponding N-QE system with $V_{QE}(\mathcal{R})$. In Fig. 2 we compare the 12-electron spectrum at $2l = 29$ with the corresponding 4-QE spectrum at $2l = 9$. Indeed, except for the different reference vacuum energies, the QE spectrum reproduces the bottom of the electron spectrum rather well (higher electron states involve additional QE–QH pairs), justifying the CF mapping.

Two examples of N-QE energy spectra obtained by exact diagonalization of $V_{QE}(\mathcal{R})$ and showing nondegenerate ($L = 0$) ground states with a gap are presented in Fig. 3. We identified a series of such ground states at $2l = 3N - 7$, different from Laughlin's relation $2l = 3N - 3$ but also extrapolating to $\nu = \frac{1}{3}$. We also found a gapped ground state for $(N, 2l) = (14, 25)$, coincident with the $2l = 2N - 3$ series of the Moore–Read electron liquid in LL_1, but the assignment of $\nu = \frac{1}{2}$ to this state is less certain. At other $(N, 2l)$ we found that either the ground state is degenerate ($L \neq 0$) or the excitation gap is marginal.

To build confidence that the $2l = 3N - 7$ (and the more problematic $2l = 2N - 3$) series of states represent the extended $\nu = \frac{1}{3}$ (and $\nu = \frac{1}{2}$) incompressible QE liquids, in Fig. 4(a) we plot the energy gap Δ as a function of N. Indeed, it seems plausible that Δ will survive in large systems. Moreover, in Fig. 4(b) we show that the pair correlation functions $g(r)$ for the $N = 11$ and 12 states of the $2l = 3N - 7$ series are essentially identical, at the same being very different from both the curve for 14 QEs at

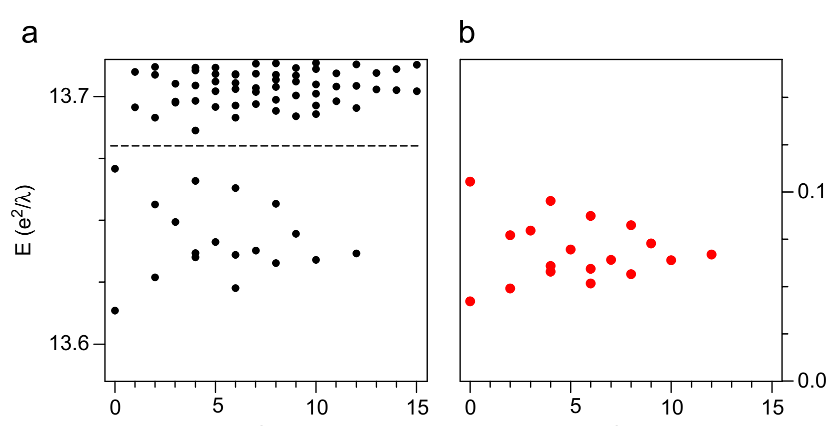

Fig. 2. Energy spectra (energy E vs. angular momentum L) calculated on a sphere for 12 electrons in LL_0 with $2l = 29$ (a) and for QEs interacting through V_{QE} in CF-LL_1 with $2l = 9$ (b). Energy scale is the same in both frames.

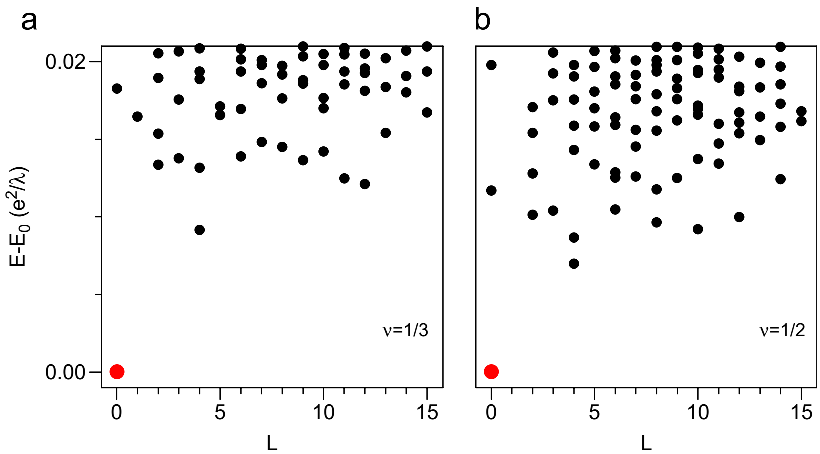

Fig. 3. Sample N-QE excitation spectra (energy E vs. angular momentum L; E_0 is ground state energy) corresponding to fractional fillings $\nu = \frac{1}{3}$ and $\frac{1}{2}$ of CF-LL$_1$ (a) $N = 12,\ 2l = 2a$; (b) $N = 14,\ 2l = 25$.

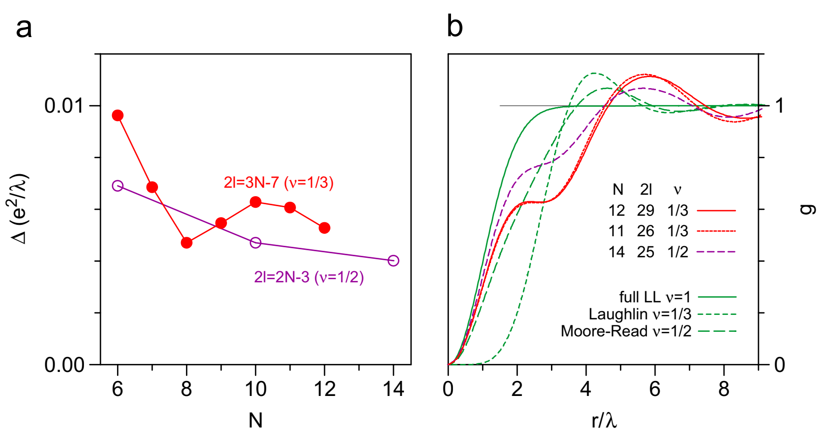

Fig. 4. (a) Excitation gaps Δ for the series of N-QE ground states at $2l = 3N - 7$ ($\nu = \frac{1}{3}$) and $2l = 2N - 3$ ($\nu = \frac{1}{2}$), as a function N. (b) QE pair-distribution functions $g(r)$ of those ground states, compared to known states of electrons.

$2l = 25$, and several curves for known FQH states of electrons [17].

4. Pairs and clusters of QEs

In order to capture the nature of gapped N-QE ground states identified in exact diagonalization, we calculated their pair and triplet Haldane amplitudes (i.e., the discrete two- and three-body correlation functions) [18]. These amplitudes, $\mathcal{G}_2(\mathcal{R})$ and $\mathcal{G}_3(\mathcal{T})$, count the fraction of pairs or triplets as a function of two- and three-body relative angular momentum, respectively. They are calculated as the expectation values of the short-range two- and three-body interaction pseudopotentials, $V_{\mathcal{R}}(\mathcal{R}') = \delta(\mathcal{R}, \mathcal{R}')$ and $W_{\mathcal{T}}(\mathcal{T}') = \delta(\mathcal{T}, \mathcal{T}')$. Two-body matrix elements $\langle i,j|V_{\mathcal{R}}|k,l\rangle = \langle i,j|L\rangle\langle L|k,l\rangle\delta(L, 2l - \mathcal{R})$ are products of the appropriate Clebsh–Gordan coefficients. Analogously, $\langle i,j,k|W_{\mathcal{T}}|l,m,n\rangle = \langle i,j,k|L\rangle\langle L|l,m,n\rangle\delta(L, 3l - \mathcal{T})$ involves three-body expansion parameters related to Racah coefficients (for simplicity we ignore here the angular-momentum degeneracy at $\mathcal{T} \geqslant 9$).

From the amplitudes corresponding to the minimum allowed values of $\mathcal{R}_{\min} = 1$ and $\mathcal{T}_{\min} = 3$ we were able to calculate the average number of "compact" pairs or triplets, $\mathcal{N}_2 = \binom{N}{2}\mathcal{G}_2(\mathcal{R}_{\min})$ and $\mathcal{N}_3 = \binom{N}{3}\mathcal{G}_3(\mathcal{T}_{\min})$. In Fig. 5(a) we plot $\mathcal{N}_2/N$ as a function of $N/2l$ for the ground states of N particles at different values of $2l$. The data for $N = 10$ and 12 nearly overlap, while the difference between QEs in CF-LL$_1$ and electrons in LL$_0$ or LL$_1$ is noticed immediately. While $N/2l \approx \nu$, the exact values of ν assigned to the particular 12-particle incompressible ground states are indicated next to the filled symbols. The well-known property of the Laughlin state—the complete avoidance of the $\mathcal{R} = 1$ pair state—is clearly visible in Fig. 5(a) as the vanishing of $\mathcal{N}_2$ at the $\nu = \frac{1}{3}$ filling of LL$_0$. Evidently, the $\nu = \frac{1}{3}$ state of QEs does not have this property.

In Fig. 5(b) we show a matching plot of $\mathcal{N}_3/N$. The most striking result is the vanishing of $\mathcal{N}_3$ of the QEs at the $\nu = \frac{1}{3}$ filling of CF-LL$_1$. Combined with the value of $\mathcal{N}_2 \approx N/2$, this proves the paired character of the $\nu = \frac{1}{3}$ QE liquid. On the other hand, $\mathcal{N}_3/N \sim 0.4$ for the QE

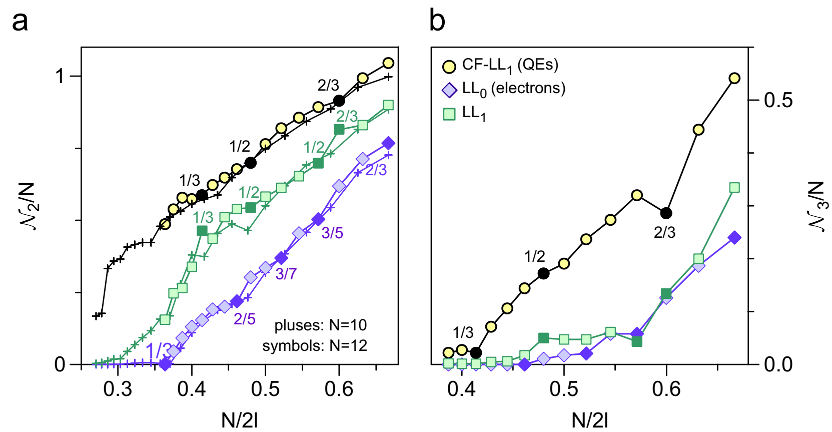

Fig. 5. Number of pairs $\mathcal{N}_2$ (a) and triplets $\mathcal{N}_3$ (b) with the minimum relative angular momentum, calculated for N electrons (in LL_0 or LL_1) or QEs (in CF-LL_1) as a function of $N/2l \approx \nu$. Incompressible states are labeled by ν.

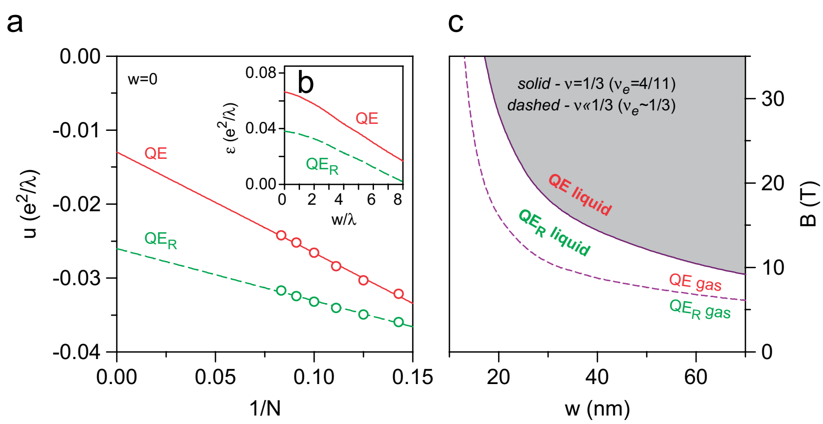

Fig. 6. (a) Correlation energies u in $\nu = \frac{1}{3}$ liquids of QEs or QE_Rs, as a function of their inverse number N^{-1}. (b) QE and QE_R Coulomb energies ε as a function of electron layer width w. (c) Phase diagram (critical layer width w vs. magnetic field B) for QE–QE_R spin transition at $\nu = \frac{1}{3}$ (i.e., at $\nu_e = \frac{4}{11}$). Thin dashed line is for uncorrelated QEs or QE_Rs (at $\nu \ll \frac{1}{3}$).

filling $\nu = \frac{1}{2}$ suggests formation of some triplets (or even larger clusters) in this state. This is in contrast to the known vanishing of $\mathcal{N}_3$ in the Moore–Read state describing the half-filled electron LL_1.

5. Corresponding QE and QH states

Pan's experiment revealed more new FQH states besides $\nu_e = \frac{4}{11}$, $\frac{3}{8}$, and $\frac{5}{13}$ (the latter corresponding to $\nu = \frac{2}{3}$ and related to $\nu_e = \frac{4}{11}$ through the approximate particle–hole symmetry in CF-LL_1). The other family contains $\nu_e = \frac{4}{13}$, $\frac{3}{10}$, and $\frac{5}{17}$, corresponding to $\nu_{CF} = \frac{4}{5}$, $\frac{3}{4}$, and $\frac{5}{7}$, or to the QH filling factors $\nu_{QH} \equiv 1 - \nu_{CF} = \frac{1}{5}$, $\frac{1}{4}$, and $\frac{2}{7}$.

These three QH states turn out related to the three QE states in the following way. The QH–QH pseudopotential in Fig. 1(c) has strong short-range repulsion at $\mathscr{R} = 1$, causing Laughlin QH–QH correlations that can be modeled by attachment of two flux quanta to each QH (i.e., reapplication of the CF transformation to the vacancies in CF-LL_0). The resulting CF-QHs have an effective filling factor given by $\nu_{CF\,QH}^{-1} = \nu_{QH}^{-1} - 2p$ and the pseudopotential $V_{CF\,QH}(\mathscr{R})$ similar to $V_{QE}(\mathscr{R})$. This similarity leads to the correspondence between QE and QH liquids at filling factors ν_e and μ_e simply connected by $\nu_e^{-1} + \mu_e^{-1} = 6$. This links $\nu_e = \frac{4}{11}$ with $\frac{4}{13}$, $\frac{3}{8}$ with $\frac{3}{10}$, and $\frac{5}{13}$ with $\frac{5}{17}$, and offers equivalent explanation for the incompressibility of the QH family of the second generation liquids.

6. Spin transition of QE liquid

So far we have only considered spin-polarized states. However, it is known [24] that QE_R (the reversed-spin QE; represented by a spin-flip CF in CF-LL_0) has lower Coulomb energy ε than QE, and the latter remains the lowest negatively charged excitation of the Laughlin liquid only when it is additionally favored by a sufficient Zeeman energy, E_Z. This is illustrated in Fig. 6(b), showing the comparison of ε_{QE} and ε_{QER} as a function of width w of the quasi-2D electron layer.

In order to establish whether QEs or QE_R will occur at their filling $\nu = \frac{1}{3}$, their correlation energies per particle u must also be compared [25]. They are defined as $(E + U_{bckg})/N$ where $U_{bckg} = (Ne/3)^2/2R$ accounts for the charge-compensating background (R is the radius of the Haldane sphere). For the QE_Rs, many-body interaction energy E is calculated by exact diagonalization using pseudopotential V_{QER} shown in Fig. 1(c) at $2l = 3N - 3$ (the superharmonic short-range repulsion of V_{QER} supports Laughlin QE_R–QE_R correlations at $\nu = \frac{1}{3}$). The finite-size estimates of u_{QE} and u_{QER} are compared in Fig. 6(a).

The condition for a transition between QE and QE_R liquids at $\nu = \frac{1}{3}$ is $\varepsilon_{QE} + u_{QE} = \varepsilon_{QER} + u_{QER} + E_Z$. Combining the calculated values of ε and u (extrapolated to large N) and the width dependence of electron Landé g-factor [26] we have obtained [19] the spin phase diagram shown in Fig. 6(c). The role of CF–CF interactions in stabilizing the QE_R liquid is evident from comparison with the phase boundary calculated neglecting $u_{QE} - u_{QER}$ (i.e., for a CF gas). It is noteworthy that all FQH experiments so far [1] were done either deep inside the QE phase or close to the predicted phase boundary. Hence, different systems (narrower wells with smaller electron concentrations) should probably be used to observe this spin transition.

7. Conclusion

Combining CF theory with exact numerical diagonalization we have studied "second-generation" incompressible quantum liquids, corresponding to the fractional filling of $CF\text{-}LL_1$. We have shown that the low-energy electron dynamics in these states can be understood in terms of Laughlin QEs interacting through an effective pair pseudopotential whose short-range behavior is strikingly different from that of electrons in LL_0 or LL_1. In consequence, QE–QE correlations in a partially filled $CF\text{-}LL_1$ are also different from electron correlations in a partially filled LL_0 or LL_1. In particular, the $\nu_e = \frac{4}{11}$ state is not a Laughlin state of QEs despite having $\nu_{QE} = \frac{1}{3}$. Instead, we show that it involves QE pairing (similar to the Moore–Read state describing electrons at the half-filling of LL_1). On the other hand, the $\nu_e = \frac{3}{8}$ state is not a paired Moore–Read state of QEs despite having $\nu_{QE} = \frac{1}{2}$. Instead, it seems to involve formation of larger QE clusters. We have also looked at the possible spin transition at $\nu_e = \frac{4}{11}$, corresponding to the crossover between a paired QE state and a Laughlin state of QE_Rs. Predicted phase diagram suggests that this transition could be observable in somewhat narrower quantum wells than used in previous experiments.

Acknowledgments

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