



# Fractionally charged magneto-excitons

Arkadiusz Wójs<sup>a,b,\*</sup>, John J. Quinn<sup>a</sup>

<sup>a</sup>Department of Physics, University of Tennessee, Knoxville, Tennessee 37996, USA

<sup>b</sup>Institute of Physics, Wrocław University of Technology, Wrocław 50-370, Poland

Received 27 February 2001; accepted 1 March 2001 by M. Cardona

## Abstract

The photoluminescence (PL) spectrum of a two-dimensional electron gas (2DEG) in the fractional quantum Hall regime is studied. The response of the 2DEG to an optically injected valence hole depends on the separation  $d$  between the electron and hole layers. At  $d$  smaller than the magnetic length  $\lambda$ , the PL spectrum shows recombination of neutral ( $X$ ) and charged ( $X^-$ ) excitons. At  $d > \lambda$ , the hole binds one or two Laughlin quasielectrons (QE) of the 2DEG to form fractionally charged excitons (FCX),  $hQE$  or  $hQE_2$ . Different FCX states have different optical properties, and their stability depends critically on the presence of QE's in the 2DEG. This explains discontinuities observed in the PL spectrum at such (Laughlin) filling factors as  $\nu = 1/3$  or  $2/3$ . © 2001 Elsevier Science Ltd. All rights reserved.

PACS: 71.35.Ji; 71.35.Ee; 73.20.Dx

Keywords: A. Semiconductor; A. Nanostructures; D. Fractional Quantum Hall Effect

The photoluminescence (PL) spectrum of a quasi-two-dimensional electron gas (2DEG) measures the one-particle Green's function describing removal of an electron ( $e$ ) at the position of the valence hole ( $h$ ) [1–21]. Thus, PL probes the electron correlations in the close vicinity of the (optically injected) hole, which, depending on the type of response of the 2DEG to the perturbation associated with this hole, may or may not resemble the original correlations of an unperturbed 2DEG. In an unperturbed 2DEG in a sufficiently high magnetic field  $B$ , the electrons fill a fraction  $\nu$  of the degenerate lowest Landau level (LL). The short range of the Coulomb  $e$ – $e$  repulsion in this LL causes Laughlin correlations [22] (i.e., avoiding pair states of the smallest relative pair angular momentum [23–26]) which lead to the incompressible-fluid Laughlin–Jain ground states [22,27,28] and the fractional quantum Hall (FQH) effect [29] at a series of fractional LL fillings,  $\nu = 1/3, 2/3, 2/5$ , etc. Two effective parameters describe perturbation introduced by the hole: strength  $U$  and range  $D$  of its Coulomb potential  $V_{UD}$ . Although the hole's density profile in the plane parallel to the 2DEG is determined by its confinement

to the lowest LL, potential  $V_{UD}$  can still be varied by placing the hole at a finite distance  $d$  from the 2DEG. This is realized experimentally in asymmetric quantum wells or heterojunctions [1–5], where an electric field perpendicular to the 2DEG causes spatial displacement of electron and hole layers. Depending on the relation between  $D$  and  $U$  and the length and energy scales of the 2DEG (magnetic length  $\lambda$  and the energy gap  $\delta = \varepsilon_{QE} + \varepsilon_{QH}$  to create a QE–QH pair), two distinct types of response occur:

- (i) In the “strong-coupling” regime, the hole binds one or two electrons to form neutral ( $X$ ) or charged ( $X^-$ ) excitons [8–14,30–33]. Although for different reasons (“hidden symmetry” for  $X$  [34,35] and Laughlin  $e$ – $X^-$  correlations for  $X^-$  [13,14,32,33]), both quasiparticles (QP) are weakly coupled to the remaining electrons, and PL probes *their* binding energy  $\Delta$  and the optical properties (oscillator strength  $\tau^{-1}$  and emission energy  $\omega$ ) rather than the original correlations of the unperturbed 2DEG [15].
- (ii) In the “weak-coupling” regime, the distant hole interacts (weakly) with Laughlin excitations of the 2DEG (attracts quasielectrons QE and repels quasiholes QH), and the dynamics and PL of the system can be understood in terms of these three ( $h$ , QE, and QH) QP's [16–21].

In this Letter, we study the crossover between these two

\* Corresponding author. Tel.: +1-865-974-0771; fax: +1-865-974-6378.

E-mail address: awojs@utk.edu (A. Wójs).

phases and identify another, “intermediate-coupling” regime. In this regime, as in (ii), the Laughlin  $e-e$  correlations dominate over the  $e-h$  correlations, and neither  $X$  nor  $X^-$  states form. However, in contrast to (ii), the  $h$ -QE attraction is sufficiently strong to cause binding of one or two QE’s to a hole to form fractionally charged excitons (FCX)  $h$ QE or  $h$ QE<sub>2</sub>. The FCX’s, together with a “decoupled hole” state  $h$ , are the relevant QP’s whose interaction with one another and with the underlying Laughlin state is weak and can be treated perturbatively. The single-particle properties of these QP’s determine the low-energy dynamics and PL of the 2DEG at  $d$  larger than but comparable to  $\lambda$ . The critical dependence of the stability of different FCX’s on the presence of QE’s in the 2DEG causes discontinuities observed [1–5] in the PL spectrum at  $\nu = 1/3$  or  $2/3$ , where the type of free Laughlin QP’s changes (and the FQH effect [29] occurs). These discontinuities resemble those found at  $\nu = 1, 2, \dots$  [6,7], although the reconstruction of radiative QP’s at integer  $\nu$  is driven by the competition between  $V_{UD}$  and the (single-particle) cyclotron energy, while the effect discussed here depends on the special form of many-body correlations in the lowest LL. Let us also note that our results do not support the earlier theory [16–18] which involved neutral “anyon excitons”  $h$ QE<sub>3</sub> (we find that these complexes are neither stable nor radiative at any  $d$  or  $\nu$ ).

An infinite, spin-polarized 2DEG at a fractional filling factor  $\nu$  is modeled by a finite system of  $N$  electrons confined to the lowest shell (LL) of a Haldane sphere of radius  $R$  [23]. The angular momentum of this shell is  $l = S$ , half the strength of Dirac’s magnetic monopole (defined in the units of elementary flux, so that  $4\pi R^2 B = 2S hc/e$  and  $\lambda = R/\sqrt{S}$ ). The electrons interact with one another and with a valence hole of the same angular momentum  $l_h = S$ , moving in the lowest LL of a parallel 2D layer, separated from the 2DEG by a distance  $d$ . After the constant cyclotron and Zeeman terms are omitted, the reduced  $Ne-h$  Hamiltonian contains only the  $e-e$  and  $e-h$  interactions defined by a pair of 2D potentials,  $V_{ee}(r) = e^2/r$  and  $V_{eh}(r) = -e^2/\sqrt{r^2 + d^2}$ .

Using Lanczos-based algorithms we were able to diagonalize Hamiltonians of up to nine electrons and a hole at  $2S \approx 3(N - 1)$  corresponding to  $\nu \approx 1/3$ . Obtained energies and wavefunctions of low-lying  $Ne-h$  states were labeled by total angular momentum  $L$  and its projection  $M$ . From the energy spectra, all bound states of a 2DEG coupled to a hole (depending on  $d$  and  $\nu$ ) were identified, and their binding energies  $\Delta$  were calculated. From the wavefunctions, the PL energy  $\omega$  and oscillator strength  $\tau^{-1}$  of each QP were calculated. The total QP angular momenta  $L$  result from addition of angular momenta of their constituent particles ( $h$ ,  $e$ , and QE).

Conservation of both orbital quantum numbers in a finite-size calculation is a major advantage of using Haldane’s (spherical) geometry to model an infinite (planar) 2DEG with a full 2D translational invariance. Conversion of the results between the two geometries follows from the exact mapping [13,14] between the two spherical numbers,  $L$  and  $M$ , and two conserved quantities on a plane: total angular momentum projection  $\mathcal{M}$  and an additional quantum number  $\mathcal{K}$  associated with partial decoupling of the center-of-mass motion in a homogeneous magnetic field [36,37]. Resolution of both  $\mathcal{M}$  and  $\mathcal{K}$  (or  $L$  and  $M$ ) of the QP’s is essential for identification of the optical selection rule:  $\Delta\mathcal{M} = \Delta\mathcal{K} = 0$  (or  $\Delta L = \Delta M = 0$ ) which results from the commutation of both  $\mathcal{M}$  and  $\mathcal{K}$  (or  $L$  and  $M$ ) with the PL operator [13,14].

The bound states of a hole and either electrons or QE’s can be determined by studying the interactions of the constituent particles. A two-body interaction is defined by a pseudopotential, the pair interaction energy  $V$  as a function of an appropriate pair orbital quantum number. The pseudopotentials most important for our analysis are shown in Fig. 1 for a few values of  $d$ . Different bound states are marked on the curves for  $d = 0$ . The pseudopotentials of  $h$ -QE (a) and  $h$ -QE<sub>2</sub> (b) pairs were calculated in a  $7e-h$  system at  $2S = 17$  and  $16$ , respectively. The allowed values of pair angular momentum  $L$  result from addition of  $l_h = S$  and either  $l_{QE} = S - N + 2$  or  $l_{QE_2} = 2l_{QE} - 1$  (note that we use fermionic description of QE’s [38];  $l_{QE}$  is equal to angular momentum

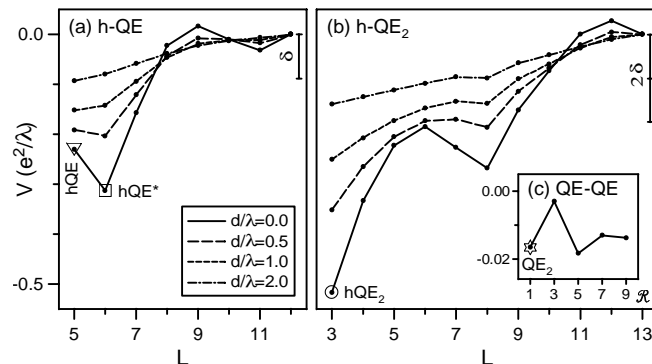


Fig. 1. Interaction pseudopotentials  $V_{h-QE}$  (a) and  $V_{h-QE_2}$  (b) calculated in the  $7e-h$  system, and  $V_{QE-QE}$  (c) calculated in the  $11e$  system.  $L$  is the pair angular momentum,  $\mathcal{R}$  is the relative pair angular momentum,  $\lambda$  is the magnetic length, and  $\delta = \epsilon_{QE} + \epsilon_{QH}$  is the Laughlin gap.

of the excited shell of Jain’s composite fermions (CF) [27,28]). To assure that exactly one or two QE’s interact with a hole at any  $d$  (and no spontaneously created QE–QH pairs “dress” the FCX), the  $e$ – $h$  interaction used to calculate the  $h$ –QE $_n$  wavefunctions was multiplied by  $\epsilon^{-1} \ll 1$ . The pseudopotentials were calculated by subtracting from the eigenvalues obtained with this reduced potential the appropriate energy of the 2DEG together with the interaction energy of the hole and uniform density  $\nu = 1/3$  fluid, and then multiplying by  $\epsilon$ . Both pseudopotentials have been shifted so that  $V_{h\text{--}QE_n} = 0$  at the maximum  $L$  (maximum average  $h$ –QE $_n$  separation  $\langle r \rangle$ ). In addition to the  $h$ QE and  $h$ QE $_2$  states, we also marked the  $h$ QE\* state which later will be important in the analysis of PL. The  $V_{QE\text{--}QE}$  of an unperturbed 2DEG (c) was calculated for 11 electrons at  $2S = 28$ . For identical fermions, the pair quantum number increasing with increasing  $\langle r \rangle$  is the relative angular momentum  $\mathcal{R} = 2l - L$  (an odd integer) [24–26]. Remarkably, despite QE’s being charge excitations, the QE–QE interaction is not generally repulsive [38], and the low energy of the QE $_2$  molecule at  $\mathcal{R} = 1$  additionally stabilizes the  $h$ QE $_2$ .

Occurrence of  $h$ QE $_n$  states of Fig. 1(ab) requires the presence of free or induced QE’s in the 2DEG, and thus depends on both  $\nu$  and the strength of  $h$ –QE $_n$  attraction relative to  $\delta = \epsilon_{QE} + \epsilon_{QH}$  (marked in each frame). At  $d < 2\lambda$ , the  $h$ –QE attraction in both  $h$ QE and  $h$ QE\* exceeds  $\delta$ , and these QP’s will occur at any value of  $\nu$  close to  $1/3$ . However, at  $d > 2\lambda$ , the  $h$ –2DEG coupling is too weak to induce QE–QH pairs, the hole can only bind existing QE’s, and  $h$ QE and  $h$ QE\* will only occur at  $\nu > 1/3$  (at  $\nu < 1/3$ , holes repel free QH’s and cause no local response of the

2DEG). Similarly, the comparison of  $V_{h\text{--}QE_2}$  and  $2\delta$  in Fig. 1(b) shows that at sufficiently small  $d$  the hole induces two QE–QH pairs, and the most stable QP at any value of  $\nu$  close to  $1/3$  is  $h$ QE $_2$ .

Knowing the QP’s and their angular momenta allows understanding of the low-energy spectra of  $Ne$ – $h$  systems at any values of  $d$  and  $\nu$ . A few spectra for  $N = 9$  are shown in Fig. 2. At smaller  $d$ , the QP’s are  $e$ ,  $X$ , and  $X^-$  with  $l = S, 0$ , and  $S - 1$ , respectively. The low-lying states describe either a Laughlin-correlated two-component  $(N - 2)e$ – $X^-$  fluid or an  $X$  (nearly) decoupled from a (also Laughlin-correlated)  $(N - 1)e$  state. A system containing electrons and charged excitons can contain four types of Laughlin QP excitations: QE $_e$ , QH $_e$ , QE $_{X^-}$ , or QH $_{X^-}$ , whose angular momenta can be found from a generalized CF model [32,33]. In Fig. 2(a), a standard CF-type analysis predicts that the ground state at  $L = 2$  contains a single QE $_{X^-}$ . In Fig. 2(b), a band of QE $_{X^-}$ –QH $_e$  pair states with  $1 \leq L \leq 6$  starts at the energy close to the lowest  $(N - 1)e$ – $X$  state at  $L = 0$  ( $X$  decoupled from the Laughlin  $8e$  state). Note that the present interpretation of Fig. 2(b) invalidates the concept of a “dressed” exciton  $X^*$  [39,40] which turns out to be unstable toward the formation of an  $X^-$  (coupling of an  $X$  with  $k \neq 0$  to the 2DEG is too strong to be treated perturbatively). At larger  $d$ , the QP’s are  $h$ ,  $h$ QE, and  $h$ QE $_2$  with  $l = S, N - 2$ , and  $1/2(N - 1)$ , respectively. An isolated  $h$ QE $_2$  is the ground state at  $L = 4$  in Fig. 2(c). In Fig. 2(d), two low-energy bands occur: at  $L \leq 6$  the  $h$ QE $_2$  (weakly) interacts with a remaining QH, and the band at  $L \geq 7$  describes the  $h$ –QE dispersion of Fig. 1(a), with  $h$ QE and  $h$ QE\* being the two lowest states.

The recombination of QP’s must *locally* conserve  $\mathcal{M}$  and

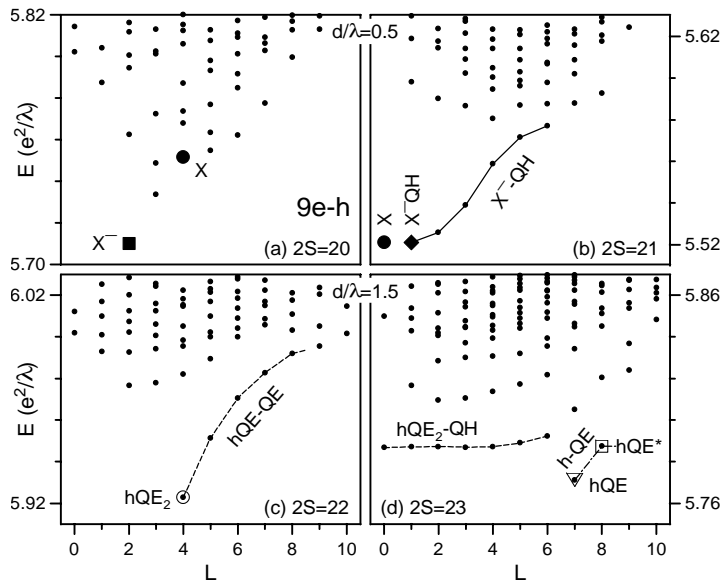


Fig. 2. Energy spectra (energy  $E$  vs. angular momentum  $L$ ) of the  $9e$ – $h$  system on a Haldane sphere. Symbols and lines mark different quasiparticles.  $2S$  is the monopole strength,  $d$  is the layer separation, and  $\lambda$  is the magnetic length.

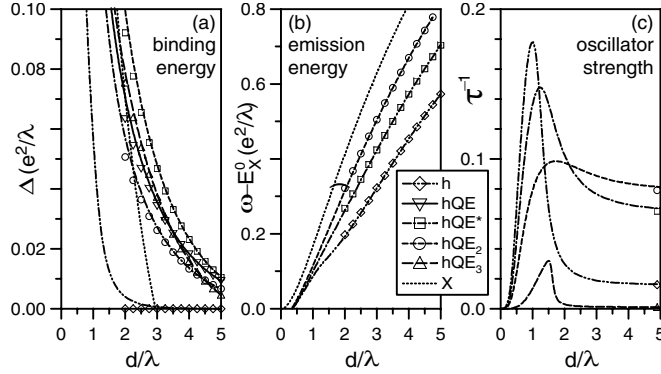


Fig. 3. Binding energy  $\Delta$  (a), recombination energy  $\omega$  (b), and oscillator strength  $\tau^{-1}$  (c) of different quasiparticles as a function of layer separation  $d$ , calculated for the  $8e-h$  system. Lines (symbols) mark the exact (approximate;  $\epsilon^{-1} \ll 1$ ) calculation (see text).  $E_X^0$  is the exciton energy in the absence of the electron gas and  $\lambda$  is the magnetic length.

$\mathcal{K}$ . The associated selection rules can be more easily obtained in the spherical geometry, where an area containing an isolated QP is represented by a whole (finite) system with appropriate  $2S$ , and the two conserved quantities are  $L$  and  $M$ . At smaller  $d$ , the only radiative state is  $X$ . The  $X^-$  is dark because  $l_{X^-} = S - 1$  is different from  $l = S$  of an electron left behind in the final state [30–33]. At larger  $d$ , efficient  $hQE_n$  recombination must occur with the minimum number of QE's and QH's involved [19–21]. For  $\nu \approx 1/3$ , such processes are  $hQE_n \rightarrow (3-n)QH + \gamma$ , where  $n \leq 3$  and  $\gamma$  denotes emitted photon. To find the  $hQE_n$  and  $(3-n)QH$  angular momenta we use  $l_h = S$  and  $l_{QE} = l_{QH} = S - N + 2$ , where  $2S = 3(N-1) - n$ . For the initial states, the result is:  $l_h = 3/2(N-1)$ ,  $l_{hQE} = N-2$ ,  $l_{hQE^*} = N-1$ ,  $l_{hQE_2} = 1/2(N-1)$ , and  $l_{hQE_3} = 3$ . For the final states:  $L_{3QH} \geq l_{QH_3} = 3/2(N-1)$ ,  $L_{2QH} = l_{QH_2} - j$  (where  $l_{QH_2} = N-1$  and  $j$  is an even integer), and  $l_{QH} = 1/2(N-1)$ . From the comparison of these values, it is clear that the only allowed radiative processes are:

$$h \rightarrow QH_3 + \gamma,$$

$$hQE^* \rightarrow QH_2 + \gamma,$$

$$hQE_2 \rightarrow QH + \gamma.$$

Because  $\tau^{-1}$  is proportional to the overlap between annihilated  $e$  and  $h$  charges, and  $h$  is smaller than QH, the following ordering of PL oscillator strengths is expected:  $\tau_{hQE_2}^{-1} > \tau_{hQE^*}^{-1} > \tau_h^{-1}$ . Because of the high energy of the QH<sub>2</sub> and QH<sub>3</sub> molecules ( $V_{QH-QH}$  is strongly repulsive at  $\mathcal{R} = 1$  [38]), we also predict:  $\omega_{hQE_2} > \omega_{hQE^*} > \omega_h$ .

The values of  $\Delta$ ,  $\omega$ , and  $\tau^{-1}$ , calculated from the exact wavefunctions of the  $hQE_n$  states identified in the  $8e-h$  spectra similar to those of Fig. 2, are plotted as a function of  $d$  in Fig. 3.  $\Delta$  is the total interaction energy of the hole and either electrons or QE's, and does not include the attraction of the hole to the underlying Laughlin state. The results of exact calculations shown in Fig. 3 confirm our predictions based on the analysis of the pseudopotentials of Fig. 1. In particular,  $hQE_2$  is the most strongly bound and most strongly radiative complex in the entire range of  $d$  in which the FCX's form. The change of correlations (replacing of  $X$ 's and  $X^-$ 's by  $hQE_n$ 's) between  $d = \lambda$  and  $2\lambda$  is best seen in the  $\tau^{-1}$  curves in Fig. 3(c). Good agreement of the exact (lines) and approximate ( $\epsilon^{-1} \ll 1$ ; symbols) calculation at  $d > 2\lambda$  indicates that the  $hQE_n$ 's are virtually uncoupled from the charge excitations of the 2DEG (and thus have well defined  $N$  and  $d$ -independent wavefunctions).

Since the stability of different  $hQE_n$  QP's depends critically on the presence of free QE's, their most striking

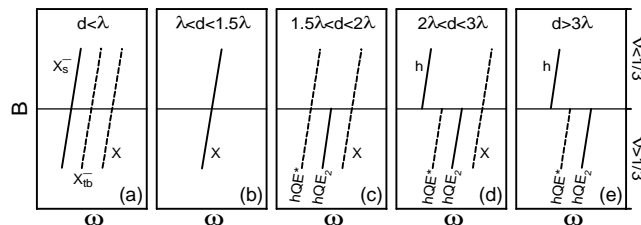


Fig. 4. Schematic PL spectra (recombination energy  $\omega$  vs. magnetic field  $B$ ) at  $\nu \approx 1/3$  and at different layer separations  $d$ . Solid and dashed lines mark recombination from ground and excited states, respectively.  $\lambda$  is the magnetic length.

signature in PL experiments should be discontinuities at Laughlin filling factors, such as  $\nu = 1/3$  or  $2/3$ . Schematic PL spectra at  $\nu \approx 1/3$  predicted by our model are shown in Fig. 4. Consecutive frames correspond to increasing  $d$  (although the actual critical values may change considerably in a more realistic model): (a) The  $X$  and *radiative* (not discussed here)  $X^-$  states are observed [13,14], the lowest being the bright spin-singlet  $X^-$  (the singlet  $X^-$  does not occur in the very high magnetic field limit,  $\hbar\omega_c \gg e^2/\lambda$ , emphasized in this paper). (b) The  $X^-$ 's unbind but the  $X$ 's still exist and dominate the PL spectrum (FCX's have smaller  $\Delta$  and  $\tau^{-1}$ ). (c) The FCX's become visible due to their increasing  $\Delta$ ;  $hQE$ 's occur (and give rise to the  $hQE^*$  recombination) at any  $\nu$  because of the spontaneous creation of one QE–QH pair in response to the hole charge; the (more strongly bound)  $hQE_2$ 's occur only at  $\nu > 1/3$ . (d) Both  $hQE$ 's and  $hQE_2$ 's exist only at  $\nu > 1/3$ ; at  $\nu < 1/3$ , the hole causes no (local) response of the 2DEG, and the recombination occurs from the “decoupled hole” state  $h$ . (e) The  $X$ 's unbind.

In conclusion, we have studied PL from a 2DEG in the FQH regime as a function of the  $e$ – $h$  layer separation  $d$ . The QP's of this system have been identified, which (depending on  $d$ ) consist of either one or two electrons or up to two Laughlin QE's bound to a hole. The angular momenta, binding and recombination energies, and oscillator strengths of the QP's have been calculated and used to explain the numerical energy spectra and the experimental PL spectra of the 2DEG at an arbitrary  $d$ . The discontinuities in the PL spectra of spatially separated systems at  $\nu = 1/3$  or  $2/3$  have been understood.

The authors acknowledge support by the Materials Research Program of Basic Energy Sciences, US Dept. of Energy and helpful discussions with P. Hawrylak (NRC, Ottawa) and M. Potemski (HMFL, Grenoble). AW acknowledges support from the KBN grant 2P03B05518.

## References

- [1] D. Heiman, B.B. Goldberg, A. Pinczuk, C.W. Tu, A.C. Gossard, J.H. English, Phys. Rev. Lett. 61 (1988) 605.
- [2] A.J. Turberfield, S.R. Haynes, P.A. Wright, R.A. Ford, R.G. Clark, J.F. Ryan, J.J. Harris, C.T. Foxon, Phys. Rev. Lett. 65 (1990) 637.
- [3] B.B. Goldberg, D. Heiman, A. Pinczuk, L.N. Pfeiffer, K. West, Phys. Rev. Lett. 65 (1990) 641.
- [4] E.M. Goldys, S.A. Brown, A.G. Davies, R. Newbury, R.G. Clark, P.E. Simmonds, J.J. Harris, C.T. Foxon, Phys. Rev. B 46 (1992) 7957.
- [5] I.V. Kukushkin, R.J. Haug, K. von Klitzing, K. Eberl, K. Töttemeyer, Phys. Rev. B 50 (1994) 11–259.
- [6] P. Hawrylak, M. Potemski, Phys. Rev. B 56 (1997) 12–386.
- [7] L. Gravier, M. Potemski, P. Hawrylak, B. Etienne, Phys. Rev. Lett. 80 (1998) 3344.
- [8] K. Kheng, R.T. Cox, Y. Merle d'Aubigne, F. Bassani, K. Saminadayar, S. Tatarenko, Phys. Rev. Lett. 71 (1993) 1752.
- [9] H. Buhmann, L. Mansouri, J. Wang, P.H. Beton, N. Mori, M. Heini, M. Potemski, Phys. Rev. B 51 (1995) 7969.
- [10] A.J. Shields, M. Pepper, M.Y. Simmons, D.A. Ritchie, Phys. Rev. B 52 (1995) 7841.
- [11] G. Finkelstein, H. Shtrikman, I. Bar-Joseph, Phys. Rev. Lett. 74 (1995) 976.
- [12] G. Finkelstein, H. Shtrikman, I. Bar-Joseph, Phys. Rev. B 53 (1996) 1709.
- [13] A. Wójs, J.J. Quinn, P. Hawrylak, Physica E 8 (2000) 254.
- [14] A. Wójs, J.J. Quinn, P. Hawrylak, Phys. Rev. B 62 (2000) 4630.
- [15] A.H. MacDonald, E.H. Rezayi, D. Keller, Phys. Rev. Lett. 68 (1992) 1939.
- [16] E.I. Rashba, M.E. Portnoi, Phys. Rev. Lett. 70 (1993) 3315.
- [17] V.M. Apalkov, F.G. Pikus, E.I. Rashba, Phys. Rev. B 52 (1995) 6111.
- [18] M.E. Portnoi, E.I. Rashba, Phys. Rev. B 54 (1996) 13–791.
- [19] X.M. Chen, J.J. Quinn, Phys. Rev. Lett. 70 (1993) 2130.
- [20] X.M. Chen, J.J. Quinn, Phys. Rev. B 50 (1994) 2354.
- [21] X.M. Chen, J.J. Quinn, Phys. Rev. B 51 (1995) 5578.
- [22] R.B. Laughlin, Phys. Rev. Lett. 50 (1983) 1395.
- [23] F.D.M. Haldane, Phys. Rev. Lett. 51 (1983) 605.
- [24] A. Wójs, J.J. Quinn, Acta Phys. Pol. A 96 (1999) 403.
- [25] A. Wójs, J.J. Quinn, Phil. Mag. B 80 (2000) 1405.
- [26] J.J. Quinn, A. Wójs, J. Phys.: Cond. Mat. 12 (2000) 265.
- [27] J. Jain, Phys. Rev. Lett. 63 (1989) 199.
- [28] X.M. Chen, J.J. Quinn, Solid State Commun. 92 (1994) 865.
- [29] D.C. Tsui, H.L. Störmer, A.C. Gossard, Phys. Rev. Lett. 48 (1982) 1559.
- [30] A. Wójs, P. Hawrylak, Phys. Rev. B 51 (1995) 10–880.
- [31] J.J. Palacios, D. Yoshioka, A.H. MacDonald, Phys. Rev. B 54 (1996) 2296.
- [32] A. Wójs, P. Hawrylak, J.J. Quinn, Phys. Rev. B 60 (1999) 11–661.
- [33] A. Wójs, I. Szlufarska, K.S. Yi, J.J. Quinn, Phys. Rev. B 60 (1999) 11–273.
- [34] A. B. Dzyubenko and Yu. E. Lozovik, Fiz. Tverd. Tela, 25, 1519 (1983) [Sov. Phys. Solid State 25, 874 (1983)].
- [35] A.H. MacDonald, E.H. Rezayi, Phys. Rev. B 42 (1990) 3224.
- [36] J.E. Avron, I.W. Herbst, B. Simon, Ann. Phys. 114 (1978) 431.
- [37] A.B. Dzyubenko, Solid State Commun. 113 (2000) 683.
- [38] A. Wójs, J.J. Quinn, Phys. Rev. B 61 (1998) 2846.
- [39] V.M. Apalkov, E.I. Rashba, Phys. Rev. B 46 (1992) 1628.
- [40] V.M. Apalkov, E.I. Rashba, Phys. Rev. B 48 (1993) 18–312.