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Physica E 34 (2006) 280–283
PHYSICA E
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Fractional quasiexcitons in incompressible electron liquids

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Available online 17 April 2006

Abstract

A new kind of many-body excitonic state composed of fractionally charged constituents is introduced. The constituents are a trion (X^-) embedded in an incompressible electron liquid and Laughlin quasiholes (QH's). Laughlin electron–trion correlations lead to an effective trion charge of $-e/3$. This many-body excitation is called "quasiexciton" and denoted by $\mathcal{X}^-$ to distinguish it from a normal trion. The $\mathcal{X}^-$ can bind one or two $(e/3)$-charged QH's, giving a neutral $\mathcal{X}$ or a positive $\mathcal{X}^+$. The energy spectra and photoluminescence from radiative quasiexciton decay are studied numerically and interpreted using a generalized composite Fermion model of the e–X^- fluid.

PACS: 71.35.Pq; 71.35.Ji; 73.43.−f

Keywords: Trion; Laughlin liquid; Photoluminescence

When a valence band hole (h) is introduced into a quasi-2D electron gas (2DEG), it can, under appropriate conditions, bind two electrons (e) to form a negatively charged exciton (X^-, also called trion). The binding energy Δ of the trion is sensitive to the width w and asymmetry of the quasi-2D layer, to the applied magnetic field B, to the electron filling factor ν, and to the spin state ($S = 0$ or 1) of the pair of bound electrons [1]. For very low values of ν, the photoluminescence (PL) resulting from the radiative decay of the trion is quite well understood [1,2]. Although PL has been studied at higher values of ν, where features coincident with the occurrence of Laughlin incompressible states [3] have been observed [4,5] there has been no explanation of the physics giving rise to the observed spectra. In this paper we report numerical studies for realistic experimental systems which show features that change as ν passes through the incompressible value of $\frac{1}{3}$. The results can be understood by using a generalized [6] composite Fermion (CF) [7] model describing correlations in the system containing a single X^- and N electrons. Our work was motivated by the wealth of unexplained PL data [4], particularly the most recent experiments [5], but the idea of quasiexcitons is both novel and applicable to a wide range of fractional quantum Hall systems.

We consider $w = 10$–$20\,\mathrm{nm}$ wide quantum wells with an electron concentration $n = 2\times10^{11}\,\mathrm{cm}^{-2}$ at filling factor ν approximately equal to $\frac{1}{3}$. The numerical diagonalization for the $Ne + h$ system ($N \leqslant 10$) is performed in the standard Haldane spherical geometry [8], with the lowest Landau level (LL) represented by an angular momentum shell with $l = Q$, and $2Q$ being the a variable integral parameter corresponding to the magnetic monopole strength in the units of hc/e. However, the cyclotron energies and interaction matrix elements used are appropriate to actual experimental systems. This is accomplished by first determining self-consistently [9] electron and hole subband wavefunctions and densities as a function of w and n, assuming asymmetric doping (i.e., donors on only one side of the quantum well). The results for three sample systems are shown in Fig. 1(a). The lowest subband envelope functions are then used in evaluating the Coulomb matrix elements. The energy spectra of a $2e + h$ system confined by this quantum well potential are then evaluated for both the singlet ($S = 0$) and triplet ($S = 1$) electron spin configurations. Despite an appreciable separation of the centroids of

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doi:10.1016/j.physe.2006.03.074

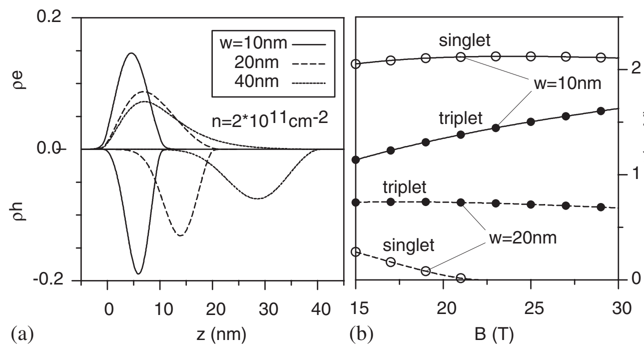

Fig. 1. (a) Density profiles in the normal direction $\varrho(z)$ for the lowest subband of electron and heavy hole, calculated for electron concentration $n = 2 \times 10^{11}\,\mathrm{cm}^{-2}$ and different well widths $w = 10$, 20, and 40 nm. (b) Binding energies of (bright) singlet and (dark) triplet trions for doped $w = 10$ and 20 nm wells as a function of magnetic field B.

the e and h, the X^- bound states are found for both $w = 10$ and 20 nm. This is illustrated in Fig. 1(b), showing the dependence of singlet and triplet trion binding energies Δ on the magnetic field B.

For the 10 nm well the ground state is the "bright singlet" X_s^- with $S = 0$ and angular momentum $L = Q$ (on a sphere), the value corresponding to a relative angular momentum $M = 0$ on a plane. This singlet is the trion ground state at $B = 0$, analog of the Hydrogen ion H^-. For the 20 nm well the trion ground state is the "dark triplet" X_t^- with $S = 1$ and $L = Q - 1$ [1], corresponding to $M = -1$. This triplet is the only bound state in the lowest LL (in symmetric wells). Other known trion states, "bright triplet" with $S = 1$ and $M = 0$, and "dark singlet" with $S = 0$ and $M = -2$ (neither shown in Fig. 1(b)) are at most marginally bound in those wells. Somewhat surprisingly, the trion spectrum for the wider well is more like the ideal system (with the *hidden symmetry*) than that of the narrow well. This behavior results from partial cancellation of the symmetry-breaking effect of hole LL mixing (which favors the singlet trion) by the e–h layer separation in the wider well.

When trions are embedded into a 2DEG, they interact with electrons through an effective Haldane electron–trion pseudopotential [8], in a similar way as electrons interact with each other. In the lowest LL this causes Laughlin e–X^- and e–e correlations, which can be described in the generalized CF picture [6,7] by attaching to each electron or trion a $2p$ magnetic field flux quanta. The electrons converted to CF_e's fill the lowest LL in effective magnetic field $B^* = B/(2p+1)$ at the Laughlin filling factor $\nu_L = (2p+1)^{-1}$. While the following discussion can readily be generalized to any Laughlin or Jain incompressible liquids (corresponding to one or more completely filled CF LL's), let us concentrate on the Laughlin $\nu = \frac{1}{3}$ state with $2p = 2$. At ν larger or smaller than $\nu_L = \frac{1}{3}$, Laughlin quasiparticles will occur in the form of quasielectrons (QE's) or quasiholes (QH's), each carrying an effective charge $\varepsilon = \pm e/(2p+1) = e/3$. In the same way an X^- in a Laughlin liquid is converted into a CF_{X^-} with charge $q = -\varepsilon = -e/3$.

An X^- coupled to electron liquid in Laughlin state and carrying fractional charge $-e/3$ is a many-body excitation, a *quasiexciton* $\mathcal{X}^-$. Because the $\mathcal{X}^-$ is negatively charged, it interacts with quasiparticles. At $\nu < \frac{1}{3}$ the $\mathcal{X}^-$ binds one quasihole to become neutral $\mathcal{X}$ with the binding energy Δ^0. This $\mathcal{X}$ may then attract another QH and become positively charged $\mathcal{X}^+$, with binding energy denoted as Δ^+. At $\nu > \frac{1}{3}$ only QE's are present at very low temperature, and the $\mathcal{X}^-$ is stable unless Δ^0 is larger than the Laughlin energy gap $\Delta_L = \varepsilon_{QE} + \varepsilon_{QH}$.

In general, different quasiexcitons have different energies, and PL peaks due to their radiative decay should appear at different energies. However, within the quantum Hall plateau, some QP's and some quasiexcitons can be in localized traps and unable to form the most stable excitonic bound states before undergoing radiative decay. Thus, we expect that some peaks due to each of the quasiexcitons should appear near the center of the Hall plateau. As ν moves across the plateau to smaller (larger) values, the $\mathcal{X}^-$ ($\mathcal{X}^+$) peaks should weaken and disappear. This kind of behavior was seen [4] over a decade ago but never explained.

As an example, we study the $\mathcal{X}^-$ in a Laughlin $\nu = \frac{1}{3}$ liquid formed in 10 and 20 nm wells, doped on one side to concentration $n = 2 \times 10^{11}\,\mathrm{cm}^{-2}$. For the wide well the only bound trion is the X_t^-. As it has high ($>90\%$) squared projection onto the lowest LL, we may restrict our consideration to the lowest spin-polarized LL. The results for $N = 9$ are shown in Fig. 2.

These spectra can be understood using the generalized CF picture [6] and angular momenta addition rules. On a sphere, the CF transformation introduces an effective monopole strength $2Q^* = 2Q - 2(K-1)$ where $K = N - 1$ is the total number of free electrons and trions. The angular momenta of constituent QP's are $\ell_{QH} = Q^*$, $\ell_{QE} = Q^* + 1$ and $\ell_{\mathcal{X}^-} = Q^* - 1$ (for $\mathcal{X}_t^-$). Hence $2Q^* = 6$, 7, and 8 for frames (a)–(c) in Fig. 2.

The $\mathcal{X}^-$ is the dark ground state in frame (a) at $L = \ell_{\mathcal{X}^-} = Q^* - 1 = 2$. The $\mathcal{X}^+$ is found in frame (c) at $L = \ell_{\mathcal{X}^+} = 4$. The allowed values of the angular momentum of the pair of QH's are given by $2Q^* - j$, where j is an odd integer. Since $2Q^* = 22 - 2(8-1) = 8$ and $\ell_{\mathcal{X}^-} = Q^* - 1 = 3$, the most strongly bound $\mathcal{X}^+$ has $\ell_{2QH} = 7$ and $L = \ell_{2QH} - \ell_{\mathcal{X}^-} = 4$. In frame (b) there is one $\mathcal{X}^-$ with $\ell_{\mathcal{X}^-} = 5/2$ and one QH with $\ell_{QH} = 7/2$. The band of $\mathcal{X} = \mathcal{X}^- + \mathrm{QH}$ states extends from $L = 1$ to 6. The radiative ground state at $L = 0$ is (approximately) a multiplicative state, containing a $k = 0$ exciton only very weakly coupled to the remaining $K = 8$ electrons. It opens the band of $\mathcal{X} = \mathcal{X}^- + \mathrm{QH}$ pair states [10], interpreted earlier [11] as "dressed neutral exciton" resulting from the interaction of the neutral exciton X with the magnetoron.

This can be understood in terms of the in-plane dipole moment and its proportionality to the wave vector $k = L/R$. The continuous dispersion of the $\mathcal{X}$

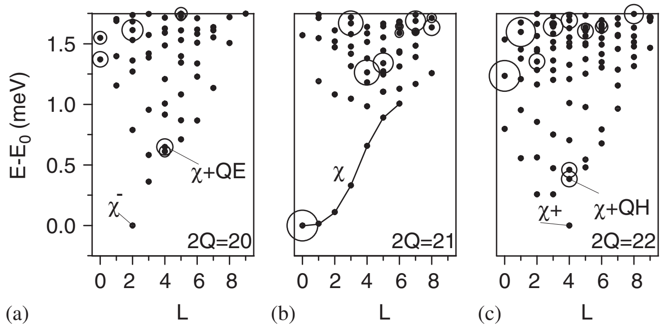

Fig. 2. Energy E as a function of angular momentum L for a $9e+h$ system in a LL shell with $20\leqslant 2Q\leqslant 22$, and corresponding to a $w=20\,\text{nm}$ quantum well doped to electron concentration $n=2\times 10^{11}\,\text{cm}^{-2}$, at magnetic field $B=25\,\text{T}$. Circle diameters give oscillator strengths τ^{-1}.

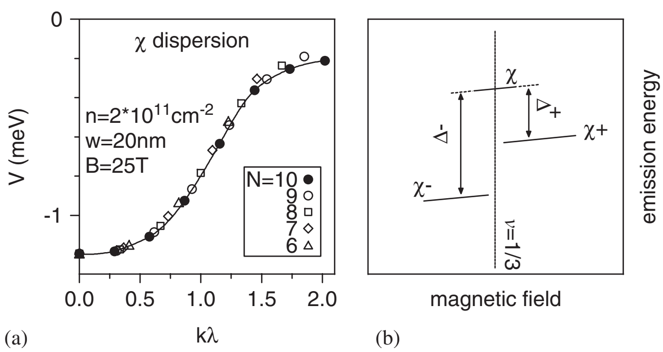

Fig. 3. (a) Dispersion of the neutral quasiexciton ($\mathscr{X}^- - \text{QH}$ interaction energy V as a function of wave vector k; $\lambda=(hc/eB)^{-1/2}$ is the magnetic length) in a $w=20\,\text{nm}$ quantum well doped to electron concentration $n=2\times 10^{11}\,\text{cm}^{-2}$, at magnetic field $B=25\,\text{T}$. (b) Schematic behavior of the PL spectrum, discontinuous around $\nu=\frac{1}{3}$ due to emission from different quasiexcitons.

(the $\mathscr{X}^- - \text{QH}$ pseudopotential) at $\nu=\frac{1}{3}$, obtained by extrapolation of data for $N\leqslant 10$, is shown in Fig. 3(a). Its energy and length scales are comparable to the dispersion of X rescaled to account for the $e\to\varepsilon$ reduction in effective charge.

The $\mathscr{X}^-$ binding energies $\Delta^{\pm}$ may be estimated by first identifying in spectra the so-called multiplicative states (by comparison with spectra of the ideal systems) and then knowing that $\mathscr{X}=\text{X}$ at $k=0$. These values were then extrapolated to $N\to\infty$. The $\mathscr{X}$ and $\mathscr{X}^+$ binding energies Δ^0 and Δ^+ were obtained in the same way and the $V(0)\equiv\Delta^0$ equivalence was used to vertically shift dispersion curves in Fig. 3(a).

Our estimates of the energies involved (GaAs, $w=20\,\text{nm}$, $n=2\times 10^{11}\,\text{cm}^{-2}$, $B=25\,\text{T}$) are: $\varepsilon_{\text{QH}}=0.73$, $\varepsilon_{\text{QE}}=1.05$, $\Delta^0=1.20$, $\Delta^-=0.52$ and $\Delta^+=0.27\,(\text{meV})$. They may be verified in a high-resolution PL experiment and the $\Delta^-\neq\Delta^+$ asymmetry is expected to make emission energy discontinuous at $\nu=\frac{1}{3}$. PL spectra qualitatively similar to our prediction in Fig. 3(b) have already been observed [4,5].

The quasiexciton formation may be described by a sequence of three processes: (i) trion binding: $2e+h\to\text{X}^-$, (ii) Laughlin correlation with surrounding electrons $\text{X}^-\to\mathscr{X}^-$, (iii) QH capture $\mathscr{X}^-\to\mathscr{X}$ or $\mathscr{X}^+$. Hence the $\mathscr{X}^-$, $\mathscr{X}$ and $\mathscr{X}^+$ are in fact the same X^-, the only difference being their separation from the surrounding 2DEG. This was confirmed by evaluating the e–h pair-distribution function $g_{eh}(r)$ for each of the three quasiexcitons and comparing them with the pair-correlation function accurate for the triplet state X^-. Moreover, integration of $[g_{eh}(r)-\frac{1}{3}]$ for each of the quasiexcitons confirms the assigned total charge of $-e/3$, 0, and $+e/3$ for the $\mathscr{X}^-$, $\mathscr{X}$, and $\mathscr{X}^+$ respectively.

The second system we studied had the same density n but a smaller $w=10\,\text{nm}$. The quasiexcitons now contain an X_s^- whose binding energy depends on LL mixing. To simplify our numerical diagonalization we reduced the highest Haldane pseudopotential V_0 by 10%. This affects interactions only within the trion and induces an accurate X_s^- ground state in the lowest LL. It also yields the correct pair-distribution functions g_{eh} and g_{ee}, which determine the coupling to the electrons.

The numerical results for the $8e+h$ system calculated with one reversed spin electron and the reduced V_0 are presented in Fig. 4. In contrast to the wider well, the low lying states involve the bright X_s^- and are radiative. Surprisingly, the (approximately) multiplicative states with an $\mathscr{X}$ (nearly) uncoupled from the additional QP form the lowest bands at $2Q=17$ and 19. The charged QX's are found to be excited states in these spectra (in the calculation of L note that now $\ell_{\mathscr{X}^-}=Q^*$).

The reason for such different behavior in the narrow well seems to be the smaller size of the $\mathscr{X}_s^-$ compared to the $\mathscr{X}_t^-$. This increases the $\mathscr{X}_s^- - \text{QH}$ attraction compared to the $\mathscr{X}_t^- - \text{QH}$ one. The energies obtained for the singlet case ($\Delta^0\simeq\Delta_{\text{L}}\simeq 2\,\text{meV}$) suggest that the $\mathscr{X}_s^-$ binds QH so strongly that even in the absence of QH's it spontaneously creates a QE–QH pair and binds the QH. As a result, the neutral $\mathscr{X}$ is the most stable quasiexciton independent of the presence of QE's or QH's. This precludes discontinuity in PL at $\nu=\frac{1}{3}$ in the 10 nm quantum well.

In summary, we have introduced novel many-body excitonic states composed of fractionally charged

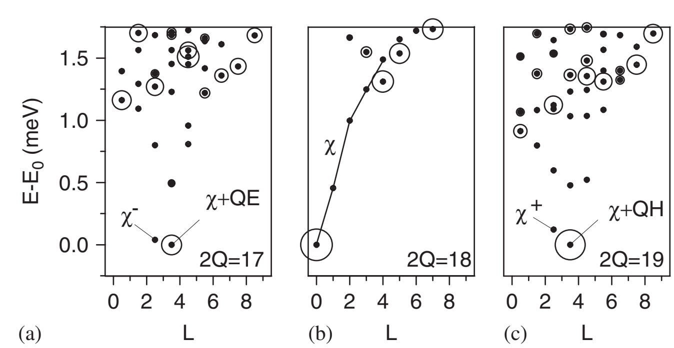

Fig. 4. Same as Fig. 2, but for the $8e+h$ system with one spin-flip ($7e\uparrow+1e\downarrow$) in a LL shell with $17\leqslant 2Q\leqslant 19$, and for $w=10\,\text{nm}$ (and same $n=2\times 10^{11}\,\text{cm}^{-2}$ and $B=25\,\text{T}$).

constituents, and have used them to study the behavior of PL near a Laughlin incompressible filling factor. Our model is supported by numerical diagonalization studies of $Ne + h$ systems ($N \leqslant 10$), and it gives reasonable qualitative agreement with published experimental results.

We thank M. Byszewski, M. Potemski, P. Hawrylak, W. Bardyszewski, and L. Bryja for helpful discussions. Work supported by Grant DE-FG 02-97ER45657 of the U.S. Department of Energy and Grant 2P03B02424 of the Polish KBN.

References

[1] A. Wójs, J.J. Quinn, P. Hawrylak, Phys. Rev. B 62 (2000) 4630; A. Wójs, J.J. Quinn, P. Hawrylak, Physica E 8 (2000) 254.
[2] G. Yusa, H. Shtrikman, I. Bar-Joseph, Phys. Rev. Lett. 87 (2001) 216402; I. Bar-Joseph, Semicond. Sci. Technol. 20 (2005) R29.
[3] R.B. Laughlin, Phys. Rev. Lett. 50 (1983) 1395.
[4] D. Heiman, et al., Phys. Rev. Lett. 61 (1988) 605; B.B. Goldberg, et al., Phys. Rev. Lett. 65 (1990) 641.
[5] M. Byszewski, et al., SemiMag-16, Tallahassee, USA, August 2–6 (2004), private communication.
[6] A. Wójs, I. Szlufarska, K.-S. Yi, J.J. Quinn, Phys. Rev. B 60 (1999) 11273; A. Wójs, P. Hawrylak, J.J. Quinn, Phys. Rev. B 60 (1999) 11661.
[7] J.K. Jain, Phys. Rev. Lett. 63 (1989) 199.
[8] F.D.M. Haldane, Phys. Rev. Lett. 51 (1983) 605.
[9] I.H. Tan, et al., J. Appl. Phys. 68 (1990) 4071.
[10] A. Wójs, J.J. Quinn, Phys. Rev. B 63 (2001) 045303; A. Wójs, J.J. Quinn, Phys. Rev. B 63 (2001) 045304.
[11] V.M. Apalkov, F.G. Pikus, E.I. Rashba, Phys. Rev. B 52 (1995) 6111.