Excitonic ions and pseudopotentials in two-dimensional systems: Evidence for quantum Hall states of an X^- gas

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Systems of up to 12 electrons and six holes on the Haldane sphere are studied by exact numerical diagonalization. The low-lying states of the system involve bound excitonic complexes such as X_n^-. The angular momenta of these complexes and the pseudopotentials describing their interaction are determined. The similarity to the electron pseudopotential suggests the possibility of incompressible ground states of a gas of X^- ions for $\nu \leqslant 1/3$. The $\nu = 1/3$ states of three and four X^-'s are found. [S0163-1829(99)07639-0]

I. INTRODUCTION

In a quasi-two-dimensional system in a strong dc magnetic field, a pair of electrons (e^-) and a valence-band hole (h^+) can form a spin-triplet negatively charged exciton (X^-).[1–4] This state has lower energy than the multiplicative state predicted by hidden symmetry,[5] which consists of a neutral exciton (X) in its ground state and an unbound free electron. At sufficiently high fields, it also has lower energy than the extensively studied[6] spin-singlet X^-. Generally, in the system of electrons and holes confined to their lowest (spin-polarized) Landau levels, the only bound complexes are X and charged multiexciton complexes, or excitonic ions, X_n^- ($n = 1,2,\ldots$). Moreover, each X_n^- ion has only one bound state, and the binding energy, $\Delta_{X_n^-} = E_{X_{n-1}^-} + E_X - E_{X_n^-}$, quickly decreases with increasing n. The X^- and larger ions X_n^- are long-lived[4,7] composite particles with mass and charge; therefore, their lowest-energy states form a degenerate Landau level.[2] It seems plausible that such composite particles could form Laughlin[8] incompressible ground states at particular values of magnetic field.[9] An intriguing question is whether they would exhibit the fractional quantum Hall effect[10] in the thermodynamic limit.

In this paper we study by exact numerical diagonalization the energy spectra of small systems containing N_h holes and N_e electrons, confined to the surface of a Haldane sphere[11] of radius R and monopole strength $2S(hc/e)$. From our numerical results we are able to determine the angular momentum l_A of the composite particles ($A = e^-$, X^-, X_2^-, etc.) and the pseudopotentials $V_{AB}(L)$ describing the interaction of any pair AB as a function of the pair angular momentum L. At sufficiently large values of S and small values of L, i.e., at large average AB distance, the pseudopotentials of all pairs are very similar and can be well approximated by those of a pair of electrons (point particles) with individual angular momenta l_A and l_B. However, if A or B is a composite particle, the maximum allowed pair angular momentum (related to the minimum allowed average AB distance) is smaller than that of two point particles with angular momenta l_A and l_B. This is equivalent to a "hard-core" repulsion, effectively raising one or more highest pseudopotential parameters to infinity. Knowing binding energies and angular momenta of the composite particles and the $V_{X^-X^-}(L)$ pseudopotential, allows us to use the composite fermion (CF) picture[12] to predict the lowest-lying band of angular momentum multiplets of a system of X^- particles for various values of the magnetic field $B = (\hbar c/eR^2)S$. These predictions are compared to exact numerical results for the six-electron–three-hole and eight-electron–four-hole systems at $2S \leqslant 12$, and the agreement is found to be good. For larger systems it becomes difficult to carry out exact numerical diagonalization in terms of the individual electrons and holes, but the lowest-lying band of states will consist of X^-'s interacting through the pseudopotential $V_{X^-X^-}(L)$, which can be handled numerically.

II. MODEL

The single-particle states on the Haldane sphere are called monopole harmonics.[13] They are labeled by angular momentum $l \geqslant S$ and its projection m. The $(2S+1)$-fold degenerate lowest Landau level consists of the $l = S$ multiplet. The characteristic wave-function length scale is the magnetic length λ ($R^2 = S\lambda^2$).

The Hamiltonian of an interacting system of N_e electrons and N_h holes in the lowest Landau level contains the constant cyclotron energy and the e-e, e-h, and h-h Coulomb interactions (for the matrix elements see Ref. 14). The Hilbert space of e-h states is spanned by single-particle configurations classified by the total angular momentum projection M. The eigenstates, obtained through numerical diagonalization of the many-body Hamiltonian in the M

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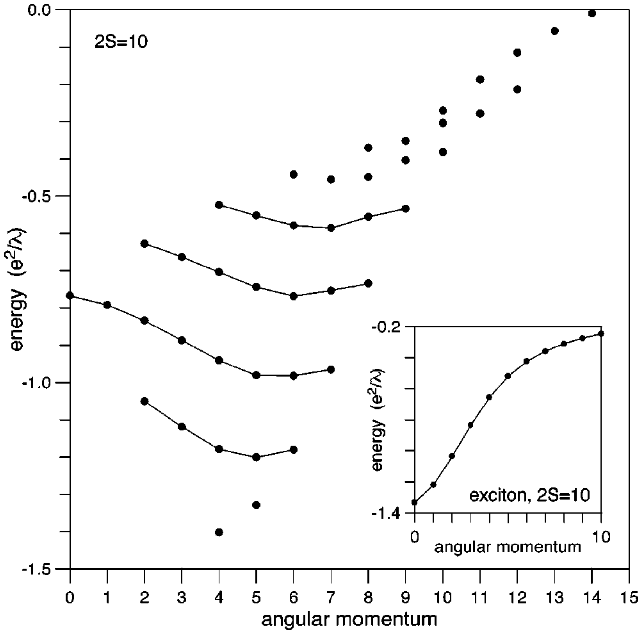

FIG. 1. Energy spectrum of two electrons and one hole at $2S=10$. Inset: energy spectrum of an electron-hole pair.

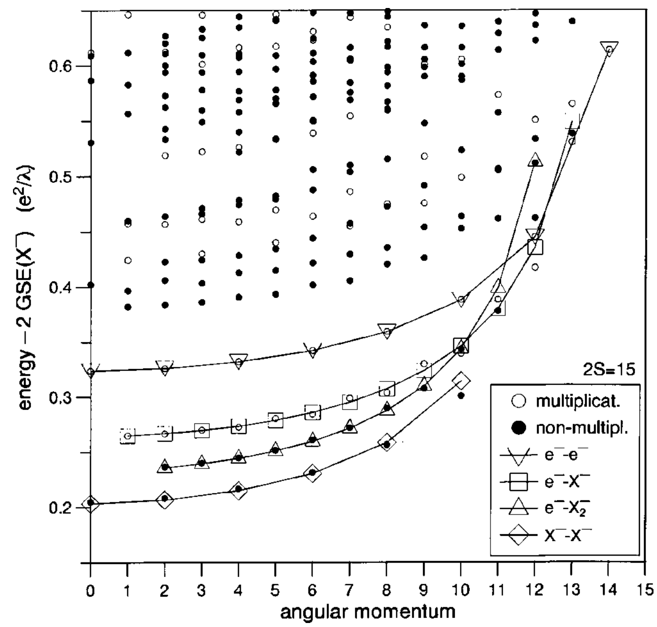

FIG. 2. Energy spectrum of four electrons and two holes at $2S=15$. Open circles, multiplicative states; solid circles, nonmultiplicative states; triangles, squares, and diamonds, approximate pseudopotentials.

eigensubspaces, fall into degenerate total angular momentum (L) multiplets.

Because of the hidden symmetry[5] due to symmetric e-e, e-h, and h-h interactions, the number N_X of decoupled excitons is another good quantum number labeling many-body e-h states. The states with $N_X=0$ are called nonmultiplicative, and the states with $N_X>0$ are multiplicative. The multiplicative states are obtained by addition of a zero angular momentum exciton into the eigenstate of an e-h system with one less e-h pair.

III. TWO-ELECTRON–ONE-HOLE SYSTEM

In Fig. 1 we show the energy spectrum (in magnetic units) of a system of two electrons and one hole at $2S=10$ as a function of the total angular momentum L. The lowest-energy state at $L=S$ is the multiplicative state with one neutral exciton in its $l_X=0$ ground state and one electron of angular momentum $l_e=S$. Only one state of lower energy occurs in the spectrum. It appears at $L=S-1$ and corresponds to the only bound state of the negatively charged exciton X^-.[9] The value of the X^- angular momentum, $l_{X^-}=S-1$, can be understood by noticing that the lowest-energy single-particle configuration of two electrons and one hole is the "compact droplet," in which the two electrons have z component of angular momentum $m=S$ and $m=S-1$, and the hole has $m=-S$, giving $M=S-1$.

As marked with lines in Fig. 1, unbound states above the multiplicative state form bands, which arise from the e-h interaction and are separated by gaps associated with the characteristic excitation energies of an e-h pair (the e-h pseudopotential, i.e., the energy spectrum of an exciton, is shown in the inset). These bands are rather well approximated by the expectation values of the total (e-e and e-h) interaction energy, calculated in the eigenstates of the e-h interaction alone (without e-e interaction).

IV. FOUR-ELECTRON–TWO-HOLE SYSTEM

In Fig. 2 we display the energy spectrum of a system of four electrons and two holes at $2S=15$. The states marked by open and solid circles are multiplicative (containing one or more decoupled X's) and nonmultiplicative states, respectively. For $L<10$ there are four rather well defined low-lying bands. Two of them begin at $L=0$. The lower of these consists of two X^- ions interacting through a pseudopotential $V_{X^-X^-}(L)$. The upper band consists of states containing two decoupled X's plus two electrons interacting through $V_{e^-e^-}(L)$. The band that begins at $L=1$ consists of one X plus an X^- and an electron interacting through $V_{e^-X^-}(L)$, while the band that starts at $L=2$ consists of an X_2^- interacting with a free electron.

Remember that $l_e=S$, $l_{X^-}=S-1$, and $l_{X_2^-}=S-2$, and that decoupled excitons do not carry angular momentum ($l_X=0$). For a pair of identical fermions of angular momentum l, the allowed values of the pair angular momentum are $L=2l-j$, where j is an odd integer. For a pair of distinguishable particles with angular momenta l_A and l_B, the total angular momentum satisfies $|l_A-l_B|\leq L\leq l_A+l_B$. The states containing two free electrons and two decoupled neutral excitons fit exactly the pseudopotential for a pair of electrons at $2S=15$; the maximum pair angular momentum is $L^{\max}=14$ as expected. The states containing two X^-'s terminate at $L=10$. Since the X^-'s are fermions, one would have expected a state at $L^{\max}=2l_{X^-}-1=12$. This state is missing in Fig. 2. By studying $2X^-$ states for low values of S, we surmise that the state with $L=L^{\max}$ does not occur because of the finite size of the X^-. Large pair angular momentum corresponds to the small average distance, and two X^-'s in the state with $L^{\max}$ would be too close to one another for the bound X^-'s to remain stable. We can think of this as a "hard-core" repulsion for $L=L^{\max}$. Effectively, the corresponding pseudopotential parameter $V_{X^-X^-}(L^{\max})$ is infinite. In a similar way, $V_{e^-X^-}(L)$ is effectively infinite for $L=L^{\max}=14$, and $V_{e^-X_2^-}(L)$ is infinite for $L=L^{\max}=13$.[15]

Once the maximum allowed angular momenta for all four pairings AB are established, all four bands in Fig. 2 can be roughly approximated by the pseudopotentials of a pair of electrons (point charges) with angular momenta l_A and l_B, shifted by the binding energies of appropriate composite particles. For example, the X^--X^- band is approximated by the e^--e^- pseudopotential for $l=l_{X^-}=S-1$ plus twice the X^- energy. The agreement is demonstrated in Fig. 2, where the squares, diamonds, and two kinds of triangles approximate the four bands in the four-electron–two-hole spectrum. The fit of the diamonds to the actual X^--X^- spectrum is quite good for $L<10$. The fit of the e^--X^- squares to the open circle multiplicative states is reasonably good for $L<12$, and the e^--X_2^- triangles fit their solid circle nonmultiplicative states rather well for $L<11$. At sufficiently large separation (low L), the repulsion between ions is weaker than their binding, and the bands for distinct charge configurations do not overlap.

V. MANY X^- SYSTEM

We know from exact diagonalization calculations for up to 11 electrons[14,16] that the CF picture correctly predicts the low-lying states of the fractional quantum Hall systems. The reason for this success has been shown[16] to be the ability of the electrons in states of low total angular momentum to avoid large fractional parentage[17] from pair states associated with large repulsive values of the Coulomb pseudopotential $V_{e^-e^-}(L)$. In particular, for the Laughlin $\nu_e=1/3$ state, the fractional parentage from pair states with the pair angular momentum of $L^{\max}$ vanishes. We hypothesize that the same effect should occur for a system of X^-'s when $l_e=S$ is replaced by $l_{X^-}=S-1$. Defining the effective X^- filling factor of the NX^- system at the monopole strength of $2S$ as $\nu_{X^-}(N,S)=\nu_e(N,S-1)$, we expect the occurrence of incompressible $L=0$ ground states of charged excitons at all Laughlin and Jain fractions for $\nu_{X^-}\leqslant 1/3$. States with $\nu_{X^-}>1/3$ cannot be constructed because they have some fractional parentage from pair angular momentum $L^{\max}$, which is essentially infinite due to the hard-core repulsion.

In Fig. 3 we present numerical results for a system of six electrons and three holes, for values of $2S=8$ and $2S=11$. Both multiplicative (open circles) and nonmultiplicative (solid circles) states are shown in frames (a) and (c). In frames (b) and (d) only the non-multiplicative states are plotted, together with the approximate spectra marked with large open symbols. The approximate spectra are obtained by diagonalizing the system of three negatively charged particles with the actual pseudopotentials (see Fig. 2), appropriate to the three possible groupings of six electrons and three holes into three ions: X^--X^--X^- (diamonds), e^--X^--X_2^- (squares), and e^--e^--X_3^- (triangles).

Good agreement between the exact and approximate spectra in Figs. 3(b) and 3(d) allows identification of the three ion states in the spectrum and confirms our conjecture about incompressible states of a gas of X^-'s for $\nu_{X^-}\leqslant 1/3$. States corresponding to different groupings form bands. The bands

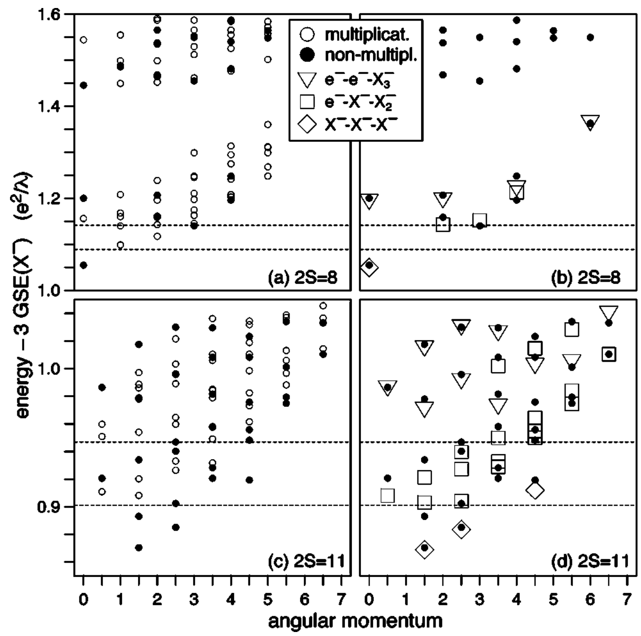

FIG. 3. Energy spectra of six electrons and three holes at $2S=8$ and 11. Open circles, multiplicative states; solid circles, nonmultiplicative states; triangles, squares, and diamonds, approximate spectra; dashed lines, estimated lower bounds of spectra given by triangles and squares.

overlap slightly at higher L, but at low L they are separated by gaps reflecting different energies of ions in different groupings. The lowest-energy state within each band corresponds to the three ions moving as far from each other as possible (maximally avoiding high pair angular momenta). If the ion-ion repulsion energies were equal for all groupings (a good approximation for dilute systems), the two higher bands would lie above dashed lines, marking the ground-state energy plus the appropriate difference in binding energies. The low-lying multiplicative states in Figs. 3(a) and 3(c) can also be identified as three ion states and fall into following bands: $3X$-e^--e^--e^-, $2X$-e^--e^--X^-, X-e^--e^--X_2^-, and X-e^--X^--X^-, whose bottom edges can be estimated based on binding energies. The bands of three ion states are separated by a rather large gap from all other states, which involve excitation and breakup of composite particles.

It follows from above discussion that the energy spectrum of N_e electrons and N_h holes contains well-developed low-energy bands of states containing particular combinations of bound charged composite particles (ions) and decoupled excitons. The relative position of bands is governed by the binding energies of ions. The X_n^- binding energy decreases sufficiently quickly with increasing n, that if $N_e=2N_h$, the lowest band consists only of states of N_h identical X^- ions.

Knowing that the lowest-lying states in Fig. 3 contain three X^-'s (states approximated by the diamonds) and using the arguments on the correspondence between the Ne^- system at the monopole strength $2S$ and the NX^- system at $2(S-1)$, we can make the following identification: (i) The lowest-energy state at $2S=8$ is an $L=0$ incompressible ground state of three X^-'s corresponding to $\nu_{X^-}=1/3$. In fact, this is the only state of three X^-'s for this value of $2S$ (other states of three X^-'s are forbidden by the hard-core repulsion). Note that it occurs at a magnetic field that is larger by 1/3 than that for Laughlin $\nu=1/3$ state of three electrons. (ii) The lowest-energy state at $2S=11$ corresponds to one quasielectron with $l_{\mathrm{QE}}=3/2$ in the $\nu=1/5$ Laughlin state and thus can be thought of as one quasi-X^- in the $\nu_{X^-}=1/5$ state.

The largest systems for which we performed exact calculations are the six-electron–three-hole and eight-electron–four-hole systems at the values of $2S$ up to 12 (Laughlin $\nu_{X^-}=1/5$ state of three X^-'s and one quasi-X^--hole in the $\nu_{X^-}=1/3$ state of four X^-'s, respectively). In each case the CF picture applied to the X^- particles works well. For larger systems the exact diagonalization becomes quite difficult. For example, for the twelve-electron–six-hole system, we expect the $\nu_{X^-}=1/3$, 2/7, 2/9, and 1/5 incompressible states to occur at $2S=17$, 21, 23, and 27, respectively. Unfortunately, the exact diagonalization of this 18-particle system at such values of $2S$ is beyond our current computer capability. However, we managed to extrapolate the $V_{X^-X^-}(L)$ pseudopotential making use of its very regular dependence on $2S$ and use it to determine approximate bands of $6X^-$ states. The spectra obtained in this way are shown as the solid circles in Fig. 4. At $2S=17$ [Fig. 4(a)], the only state of the $6X^-$ charge configuration is the $L=0$ ground state; higher configurations are forbidden by the hard core of $V_{X^-X^-}(L)$ at $L=14$. The low-lying states of the neighboring charge configurations, X-$5X^-$-e^- and X_2^--$4X^-$-e^-, are shown as open circles and open squares, respectively. These are obtained using the appropriate pseudopotentials $V_{AB}(L)$ and binding energies, and the results demonstrate that the energies of charge configurations other than six X^- ions are separated from its $L=0$ ground state by a gap. At $2S=23$ and 27 [Figs. 4(b) and 4(c)], all features predicted by the CF picture occur.

Note also that the number of NX^- states lying below higher bands, corresponding to other combinations of ions (below dashed lines in Fig. 4), increases when ν_{X^-} decreases. This is because the ion binding energies increase and the ion-ion repulsion energy decreases when the magnetic field increases. At sufficiently low ν_{X^-}, a significant low-energy part of the spectrum remains unaffected by the possibility of appearance of other combination of ions.

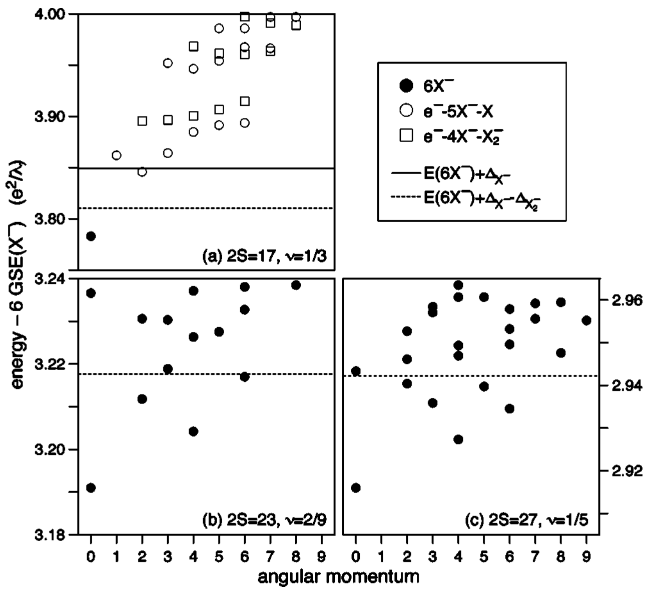

FIG. 4. Approximate $6X^-$ bands (solid circles) in the spectra of twelve electrons and six holes at $2S=17$, 23, and 27. Open symbols at $2S=17$-X-$5X^-$-e^- and X_2^--$4X^-$-e^- higher bands. Horizontal lines, estimated lower bounds of higher bands.

VI. CONCLUSION

The low-lying states of a spin-polarized system of electrons and holes involve bound composite particles (X, X^-, and X_n^-), and a gas of X^- ions can form Laughlin incompressible states for $\nu_{X^-}\leqslant 1/3$. We find that these charged complexes remain stable for a separation d between electron and hole layers up to at least 0.7λ, and that the $V_{X^-X^-}$ pseudopotential increases with d for $d<0.7\lambda$, which further stabilizes the X^- Laughlin ground states for $\nu_{X^-}\leqslant 1/5$. A clear signature of the X^- will be visible in the photoluminescence spectrum; however, systems containing a gas of both electrons and X^- ions must be analyzed. These results will be reported in a subsequent publication.

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