

Excitonic complexes in quantum Hall systems

A Wójs and J J Quinn

University of Tennessee, Knoxville

Abstract. The formation and various possible decay processes of neutral and charged excitonic complexes in electronic integral and fractional quantum Hall systems are discussed. The excitonic complexes are bound states of a small number of the relevant negatively and positively charged quasiparticles (e.g., conduction electrons and valence holes, reversed-spin electrons and spin holes, Laughlin quasielectrons and quasiholes, composite fermions) that occur in an electron system under specific conditions (e.g., electron density, well width, electric and magnetic fields, or hydrostatic pressure). The examples of such bound states are interband neutral and charged excitons, fractionally charged “anyon excitons,” spin waves, skyrmions, or “skyrmion excitons.” Their possible decay processes include radiative recombination, experimentally observed in photoluminescence or far infrared emission, or spin transitions, important in the context of nuclear spin relaxation.

1. Introduction

The transport, optical, and spin properties of a two-dimensional electron gas (2DEG) in a high magnetic field have been intensively studied both experimentally [1, 2, 3, 4, 5, 6] and theoretically [7, 8, 9, 10, 11, 12, 13, 14, 15, 16] over more than a decade. Some of these studies have demonstrated that a common scenario of the formation of what can generally be called an excitonic complex occurs in various seemingly different physical situations. The excitonic complexes, consisting of a small number of appropriate elementary charged excitations (positively and negatively charged quasiparticles of various type depending on a particular form of the electron–electron correlations in the underlying 2DEG), can often be considered as nearly free particles with well defined single-particle properties. These properties, such as electric charge, characteristic size, longitudinal or angular momentum, spin, binding energy, or oscillator strength for a particular type of quasiparticle–antiquasiparticle recombination process, determine the response of the 2DEG to the experimental perturbation. In particular, being weakly coupled to one another or to the electrons, excitonic complexes recombine obeying simple selection rules that result from their geometric (2D translational) or dynamical (particle–hole) symmetries. These simple symmetries often persist under experimental conditions despite complicated electron–electron correlations or such typical symmetry-breaking mechanisms as disorder or collisions, and greatly simplify the measured response of the entire system. Sometimes, such simplification is even undesirable as it can make the experiment sensitive only to the simple properties of the excitonic complexes, and quite insensitive to the specific properties of the underlying 2DEG.

For example, it has long been predicted that the photoluminescence (PL) spectrum in an infinitely high magnetic field contains no information about the electron–electron correlations (e.g., the presence or charge of Laughlin quasiparticles in the fractional quantum Hall regime) regardless of possible disorder [15]. Instead, the spectrum is reduced to a single discrete transition corresponding to the recombination of a neutral exciton in the zero momentum ground state, and either decreasing the magnetic field in order to allow interactions to admix

higher Landau levels (LL's) or applying an electric field to spatially separate electrons and holes is needed for PL to become a useful tool for studying electron–electron interactions.

Another example is related to a prediction [17, 18] that the most strongly bound complex involving conduction electrons (e) and a valence hole (v) in very high magnetic fields is a triplet state of the charged exciton ($X^- = 2e + v$). This state is nonradiative because of both geometrical and dynamical symmetry, and has not been experimentally confirmed in earlier experiments in high magnetic fields [19], but only quite recently [20, 21], when special measures were taken to detect its weak PL signal. While breaking of the dynamical, particle–hole symmetry in a finite magnetic field is by no means surprising, the fact that collisions of an X^- with the surrounding electrons do not relax the geometrical selection rule associated with the angular momentum conservation is a nice demonstration of Laughlin correlations of the X^- with other negative charges [22, 23]. As a result of these correlations, at small values of the filling factor ν , the X^- 's remain spatially isolated and avoid high energy collisions with one other or with electrons to become true quasiparticles of a 2DEG containing additional valence holes [24].

In the following sections of this article we will review a few examples of excitonic complexes that form in electronic quantum Hall systems: interband excitonic complexes in Sec. 3, anyon excitons in Sec. 4, skyrmions in Sec. 5, and skyrmion excitons in Sec. 6. We will discuss the similarities and differences between all these complexes, and show the role they play in experimental studies of the 2DEG, particularly in PL.

2. Model

The numerical results presented here are obtained by exact numerical diagonalization of the interaction Hamiltonian of a finite number N of electrons (and, sometimes, one or more valence holes) confined on a spherical surface of radius R . In this model, the radial magnetic field B is due to a monopole placed in the center of the sphere [9]. The monopole strength $2Q$ is defined in the units of elementary flux $\phi_0 = hc/e$, so that $4\pi R^2 B = 2Q\phi_0$ and the magnetic length is $\lambda = R/\sqrt{Q}$. The single-particle states are the eigenstates of angular momentum l and its projection m and are called monopole harmonics. The energies ε fall into $(2l + 1)$ -fold degenerate angular momentum shells separated by the cyclotron energy $\hbar\omega_c$. The n -th ($n \geq 0$) shell (LL) has $l = Q + n$ and thus $2Q$ is a measure of the system size through the LL degeneracy. Due to the spin degeneracy, each l -shell is further split by the Zeeman gap, E_Z .

Using a composite index $i = [nm\sigma]$ (σ is the spin projection), the Hamiltonian of interacting particles can be written as $H = \sum c_{i\alpha}^\dagger c_{i\alpha} \varepsilon_{i\alpha} + \sum c_{i\alpha}^\dagger c_{j\beta}^\dagger c_{k\beta} c_{l\alpha} V_{ijkl\alpha\beta}$, where $c_{i\alpha}^\dagger$ and $c_{i\alpha}$ create and annihilate particle α (conduction electron e or valence hole v , reversed-spin electron e_R or spin hole h , etc.) in state i with energy $\varepsilon_{i\alpha}$, and $V_{ijkl\alpha\beta}$ are the interaction (Coulomb) matrix elements. Hamiltonian H is diagonalized in the basis of Slater determinants. The result of the diagonalization procedure is the set of many-body eigenenergies and eigenvectors. The energies E will be shown as a function of the conserved orbital (L and L_z) and spin (S and S_z) quantum numbers. To interpret the results obtained in the spherical geometry for the infinite planar system, L and L_z must be appropriately translated into the corresponding planar quantities [24, 25]. For example, for the (charge or spin) wave eigenstates that carry no net charge, angular momentum L must be replaced by wave vector $k = L/R$, while for the eigenstates corresponding to charged excitations L and L_z are connected with planar angular momentum projection \mathcal{M} and its center-of-mass component \mathcal{M}_{CM} . The eigenvectors $|\psi\rangle$ are needed to calculate spectral functions to describe PL or other decay processes, $\tau_{if}^{-1} = |\langle f | \mathcal{P} | i \rangle|^2$, where $\psi = i$ or f are the initial and final states, respectively, and \mathcal{P} is the appropriate transition operator.

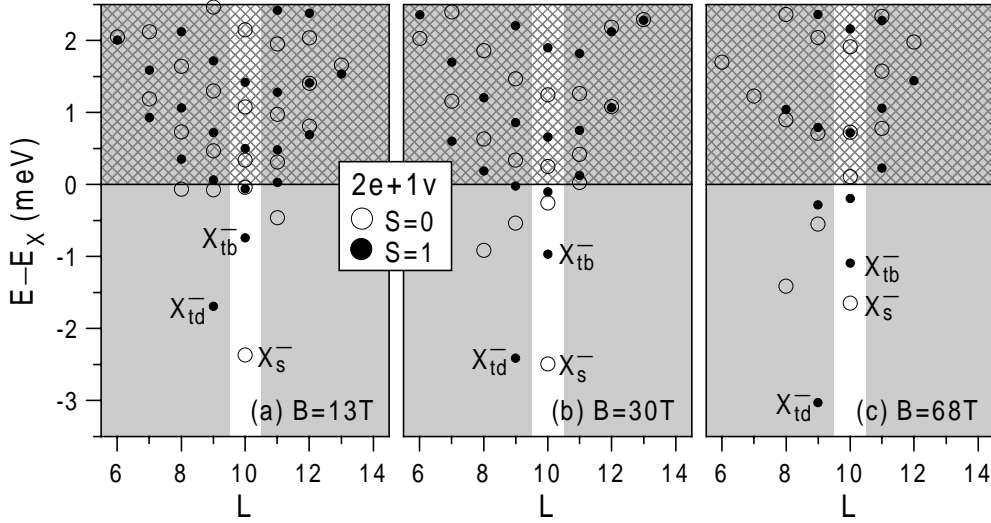


Figure 1. The energy spectra (energy E vs. angular momentum L) of the $2e + v$ system in a symmetric GaAs quantum well of width $w = 11.5$ nm at the magnetic field $B = 13$ T (a), 30 T (b), and 68 T (c), calculated on Haldane sphere with LL degeneracy $2Q + 1 = 21$.

3. Neutral and charged interband excitons

An $X^- = 2e + v$ consists of only three particles. The energy spectra of this simple system are shown in Fig. 1 for a GaAs symmetric quantum well of width $w = 11.5$ nm and for $B = 13$, 30, and 68 T. The effects of LL mixing, finite well width, anisotropy of the hole mass and its dependence on B , and the realistic Zeeman gap E_Z have all been included [24]. The energy E is measured from the exciton energy E_X , so that for the bound X^- states it gives the binding energy $\Delta = E_X - E$, and both singlet and triplet electron spin configurations are shown.

Because the emission of a photon does not change angular momentum of the (envelope) electron wave function, and because the electron left in the lowest LL after the radiative X^- recombination has $l = Q$, only those X^- states at $L = Q$ are optically active. Of all bound X^- states in Fig. 1, three are of particular importance. The X_s^- (singlet) and X_{tb}^- (triplet-bright) are the only strongly bound radiative states, while X_{td}^- (triplet-dark) has by far the lowest energy of all non-radiative states. The relative energy of different X^- states depends on experimentally variable parameters (e.g., B , w , or E_Z), and indeed, the transition between the X_s^- and X_{td}^- states can be seen in Fig. 1(b). The binding energies Δ and oscillator strengths τ^{-1} of the three X^- states, extrapolated to the $R/\lambda = \sqrt{Q} \rightarrow \infty$ limit, have been plotted in Fig. 2 as a function of B . The $X_s^- \leftrightarrow X_{td}^-$ transition is found at $B \approx 30$ T, and the X_{tb}^- state is about two times “brighter” than X_s^- (although both are considerably “darker” than the X).

Even in dilute systems, one might expect that collisions with surrounding electrons can affect the X^- recombination and in particular allow for weak emission from X_{td}^- . The surprising experimental fact that the effect of such collisions is minimal [19, 20, 21] results from Laughlin correlations between X^- and electrons in the fractional quantum Hall regime [22, 23]. In Fig. 3 we plot the energy spectra of $3e + v$ systems, in which the lowest bands of states describe repulsion of different $e-X^-$ pairs. The dependence of pair interaction energy V on pair angular momentum L is the interaction pseudopotential, which completely determines correlations in a degenerate LL. It is known that if $V(L)$ is “superharmonic” (V decreases more quickly than linearly as a function of separation $\langle r^2 \rangle$ when L is decreased), then Laughlin correlations occur [26]. It turns out that $e-X^-$ pseudopotential is superharmonic

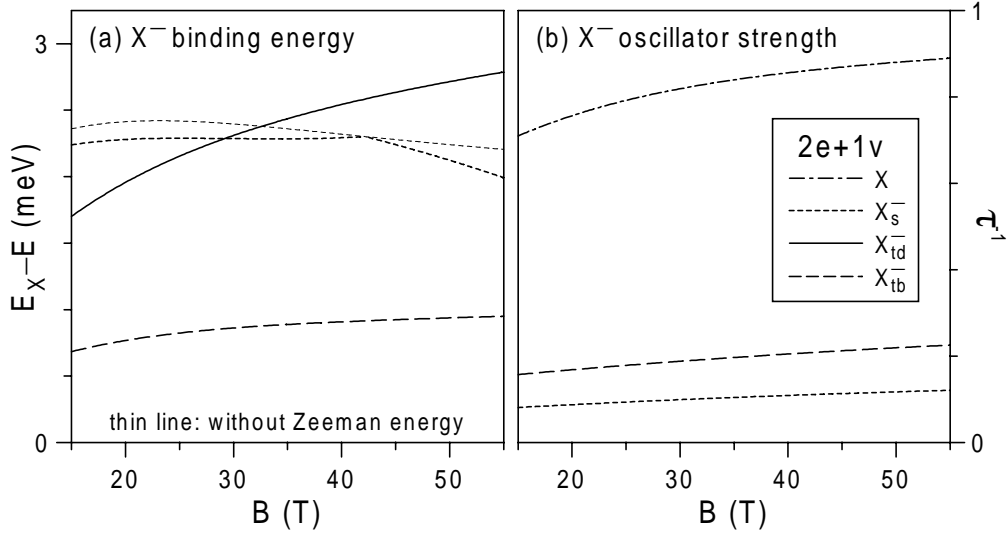


Figure 2. The binding energies Δ (a) and oscillator strengths τ^{-1} (b) of different X^- states in a symmetric GaAs quantum well of width $w = 11.5$ nm, as a function of magnetic field B .

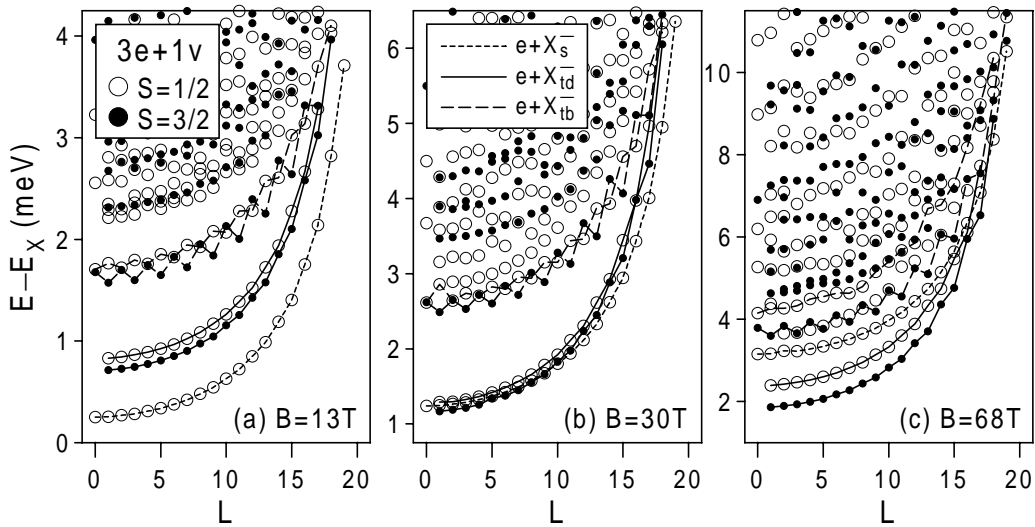


Figure 3. The same as in Fig. 1 but for the $3e + v$ system.

(similar to the e - e pseudopotential in the lowest LL). The resulting Laughlin correlations between an X^- and the electrons mean that one or more e - X^- pair states of highest repulsion are maximally avoided, or in other words, that the high energy e - X^- collisions do not occur.

In Figs. 4 and 5 we plot the oscillator strengths τ^{-1} and emission energies $\hbar\omega$ for the $3e + v$ eigenstates corresponding to an X^- interacting with an electron. In both figures, the horizontal axes give pair angular momentum L which in a Laughlin correlated system is simply related to the LL filling factor ν (only the $L \leq l_{X^-} + l_e - \mu$ pair states occur at $\nu \leq \mu^{-1}$). As expected, for small L (i.e., very dilute 2DEG) both $\hbar\omega$ and τ^{-1} converge to the values appropriate for single X^- 's plotted in Fig. 2, meaning that there is no significant effect of the e - X^- interactions on the X^- recombination at small ν . Somewhat surprisingly, the Laughlin correlations prevent considerable increase of the τ_{td}^{-1} through interaction with

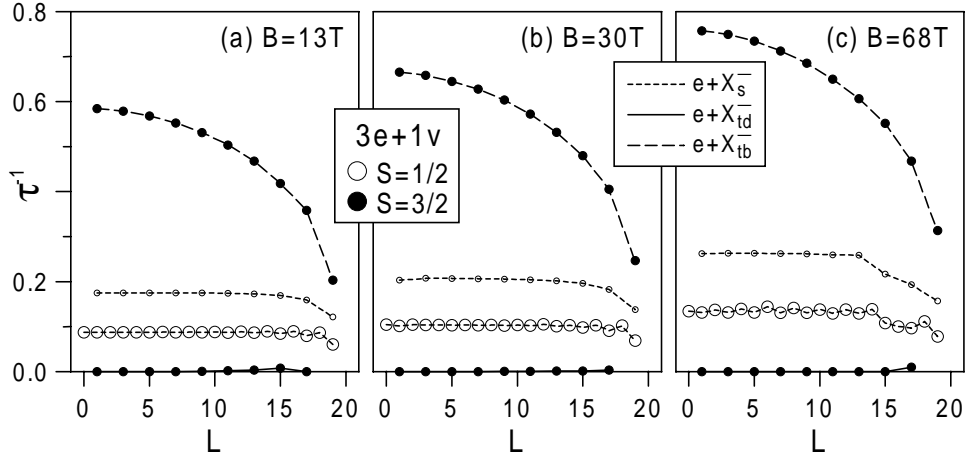


Figure 4. The oscillator strengths τ^{-1} of different X^- states interacting with an electron in a symmetric GaAs quantum well of width $w = 11.5$ nm at the magnetic field $B = 13$ T (a), 30 T (b), and 68 T (c), calculated on Haldane sphere with LL degeneracy $2Q + 1 = 21$, and plotted as a function of the $e-X^-$ pair angular momentum L .

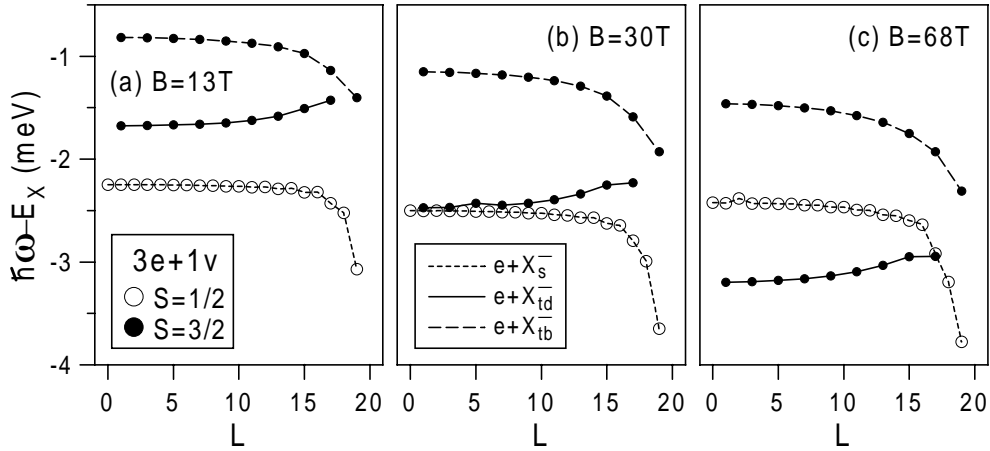


Figure 5. The same as in Fig. 4 but with recombination energy $\hbar\omega$ shown on vertical axes.

electrons even at $\nu \approx \frac{1}{3}$. This justifies a simple picture of PL in a dilute 2DEG, according to which emission occurs from isolated, well-defined bound complexes (X and X^- 's), and hence it is virtually insensitive to ν . In particular, this explains the absence of an X_{td}^- peak even in the PL spectra [19] showing strong recombination of a higher-energy triplet state X_{tb}^- (although the X_{td}^- emission has been eventually detected at very low temperatures [20, 21]). An interesting feature in Fig. 5 is also merging of $\hbar\omega_{tb}$ and $\hbar\omega_{td}$ which has actually also been observed experimentally at $\nu \approx \frac{1}{3}$ [20].

4. Anyon excitons

The fractionally charged ‘‘anyon excitons’’ have been predicted to form in strongly asymmetric quantum wells or heterostructures, in which the perpendicular electric field produced by the doping layer spatially separates conduction electron (e) and valence hole (v) layers by a

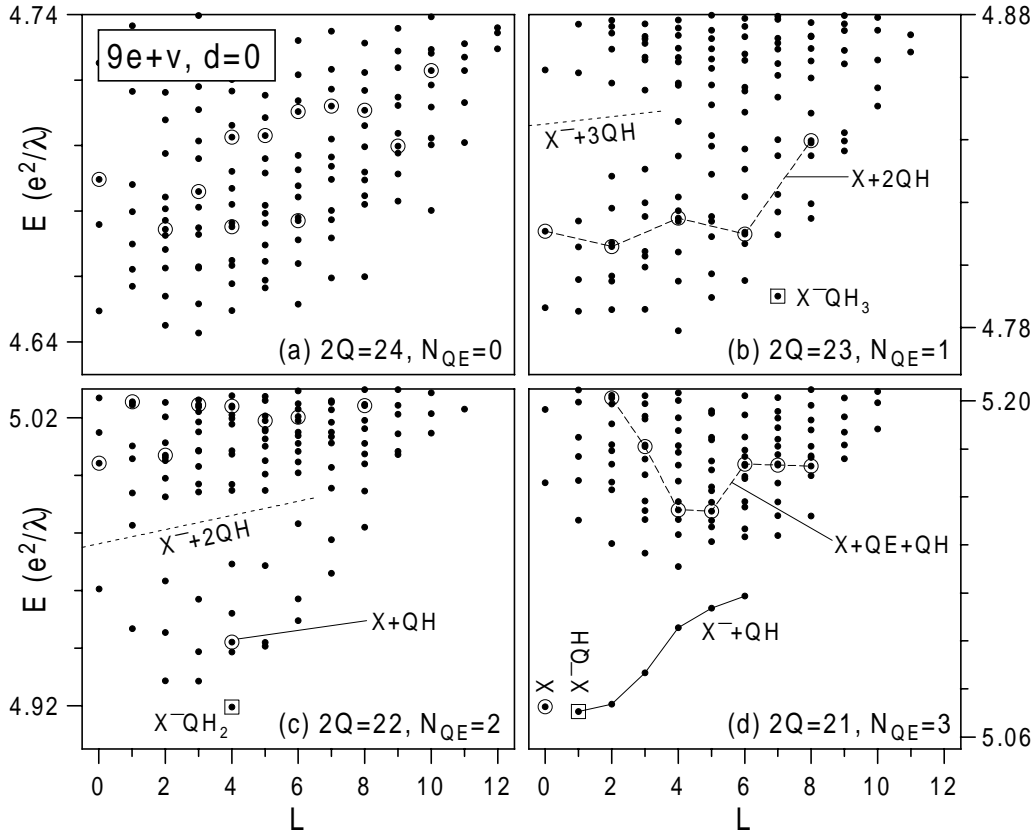


Figure 6. The energy spectra (energy E vs. angular momentum L) of an ideal $9e+v$ system (no LL mixing and zero quantum well width) calculated on Haldane sphere with LL degeneracy $2Q + 1 = 24$ (a), 23 (b), 22 (c), and 21 (d). The $e-v$ layer separation is $d = 0$.

distance $d \sim \lambda$ [27, 28, 29]. In such situation, the $v-e$ attraction becomes too weak on the characteristic 2DEG correlation energy scale and the resolution of the attractive Coulomb potential of the hole becomes too low on the characteristic 2DEG length scale, and the 2DEG retains its original Laughlin correlations even in the presence of the hole injected optically in a PL experiment. Unlike in symmetric structures, because of the reversed ordering of the $e-e$ and $v-e$ energy scales, the charge of the hole v injected into the 2DEG is no longer screened with “real” electrons e , but with the fractionally charged Laughlin quasielectrons (QE’s) [29] or reversed-spin quasielectrons (QE_R) [11, 30].

The energy spectra of $9e + v$ systems at different values of the monopole strength $2Q$ corresponding to $N_{\text{QE}} = 1, 2$, and 3 QE’s in the Laughlin $\nu = \frac{1}{3}$ state of 9 electrons interacting with the hole have been shown in Figs. 6, 7, and 8 for different values of the $v-e$ layer separation, $d = 0, \lambda$, and 2λ . These spectra have been calculated for a very ideal situation, without taking into account the LL mixing or finite well width, so d/λ must be regarded as an effective parameter controlling the strength and resolution of the perturbation potential introduced in the 2DEG by the presence of the hole, rather than as an actual displacement of e and v wave functions in an experimental system [29].

In Fig. 6 ($d = 0$; the “strong coupling” regime), the X^-_{id} , which is the only bound X^- state in the lowest LL, is the most stable quasiparticle, and the anyon excitons do not form. The open circles mark the so-called “multiplicative” states in which the $L = 0$ exciton decouples from the remaining 8 electrons due to the “hidden” symmetry (the exact

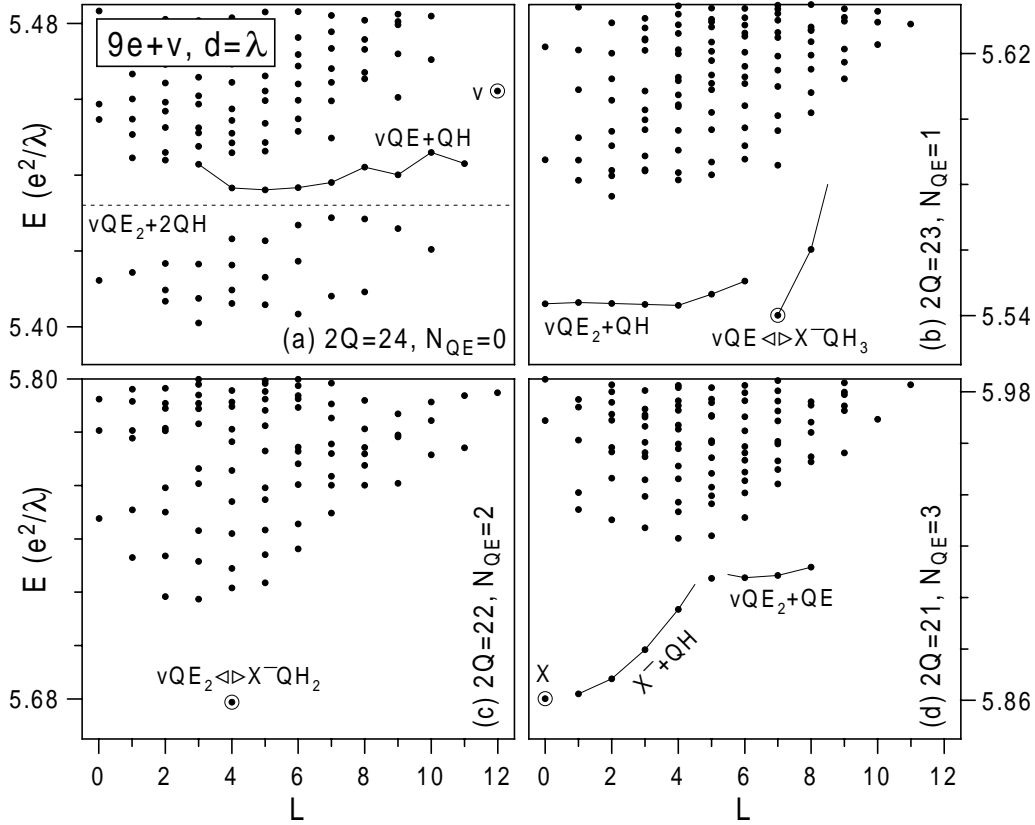


Figure 7. The same as in Fig. 6 but for $d = \lambda$.

$e-v$ particle-hole symmetry in the lowest LL) [14]. All other, non-multiplicative low-energy $9e + v$ states contain an X^- interacting with the remaining 7 electrons. These states can be well described within the generalized composite fermion model [23] for the two-component $(7e + X^-)$ Laughlin liquid. Depending on the value of $2Q$ that varies between 24 and 21, the lowest-energy $7e + X^-$ states contain between zero and three quasiholes (QH's) analogous to Laughlin quasiholes of a one-component electron liquid. The residual QH- X^- attraction whose pseudopotential can be extracted from the $X^- + \text{QH}$ band marked in frame (d), leads to the formation of $X^- \text{QH}$ and $X^- \text{QH}_2$ very weakly bound states and of an excited (unstable) $X^- \text{QH}_3$ states, identified in frames (b), (c), and (d), respectively.

In Fig. 7 ($d = \lambda$; intermediate-coupling regime), new low-energy bands of states emerge in addition to those containing the X or X^- 's. We interpret these new states as the anyon exciton states. In some cases the two type of states occur in the same spectrum. For example, the $v\text{QE}_2$ -QE band in Fig. 7(c) coexists with the X state and the X^- -QH band. In other cases, the low-lying X or X^- states occur at the same L as the low-lying anyon exciton states, and the transition between the two is continuous. For example, $v\text{QE}_2$ is mixed with $X^- \text{QH}_2$ in Fig. 7(b), and $v\text{QE}$ is mixed with $X^- \text{QH}_3$ in Fig. 7(a).

In Fig. 8 ($d = 2\lambda$; weak-coupling regime), well developed anyon exciton bands occur. The isolated $v\text{QE}$, $v\text{QE}_2$, and $v\text{QE}_3$ states are the ground states in the spectra corresponding to $N_{\text{QE}} = 1, 2$, and 3, respectively. Their angular momenta l_{AX} are obtained by adding $l_h = Q$ and $l_{\text{QE}} = Q^* + 1$, where $2Q^* = 2Q - 2(N - 1)$ is the effective monopole strength in the composite fermion picture [13, 26] and $2Q = 3(N - 1) - N_{\text{QE}}$. Similarly, the angular momenta of states containing an anyon exciton and the excess QP's result from adding l_{AX} and l_{QP} .

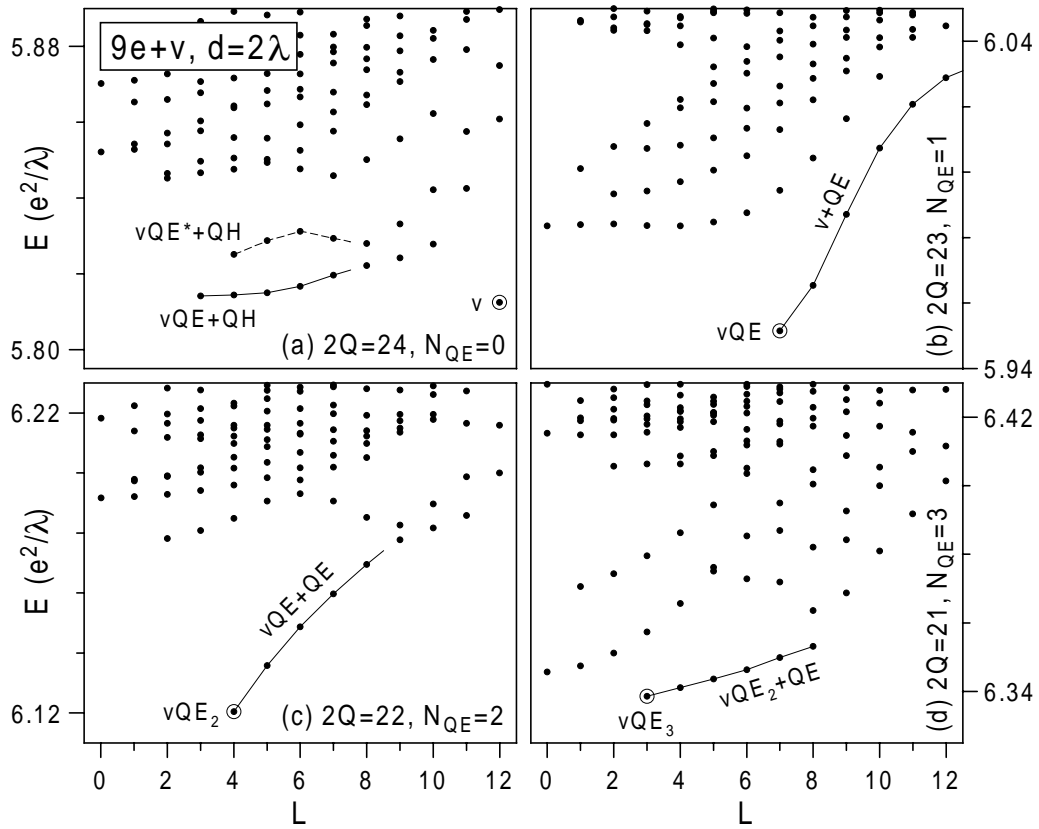


Figure 8. The same as in Fig. 6 but for $d = 2\lambda$.

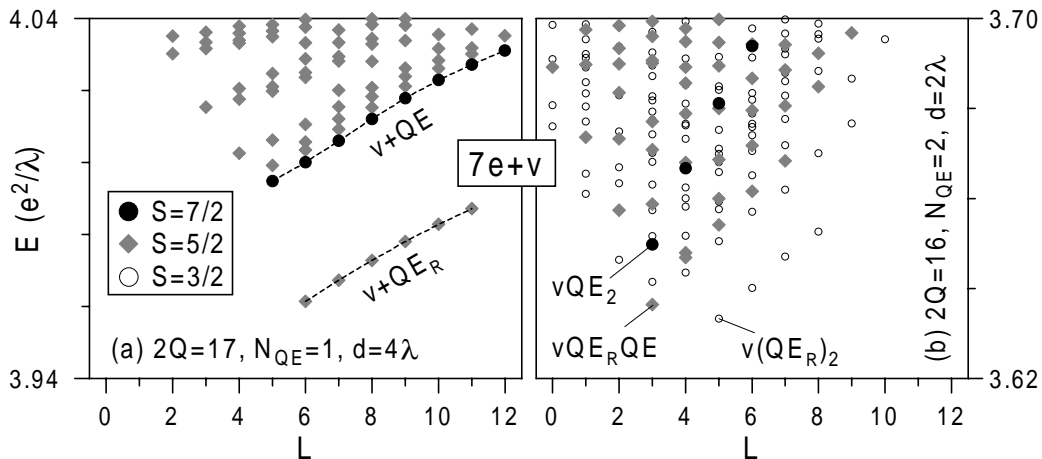


Figure 9. Similar to Fig. 6 but for $7e + v$ system at LL degeneracy $2Q + 1 = 17$ (a) and 16 (b), and including electron spin excitations. The $e-v$ layer separation is $d = 4\lambda$ (a) and 2λ (b).

In Fig. 9 we show similar spectra for the $7e + v$ system, but now including the possible electron spin excitations [30]. In addition to the spinless anyon excitons vQE and vQE_2 , the “reversed-spin anyon excitons” vQE_R , and vQE_RQE , and $v(QE_R)_2$ can be identified, in which one or more QE’s are replaced by the reversed-spin quasielectrons, QE_R ’s.

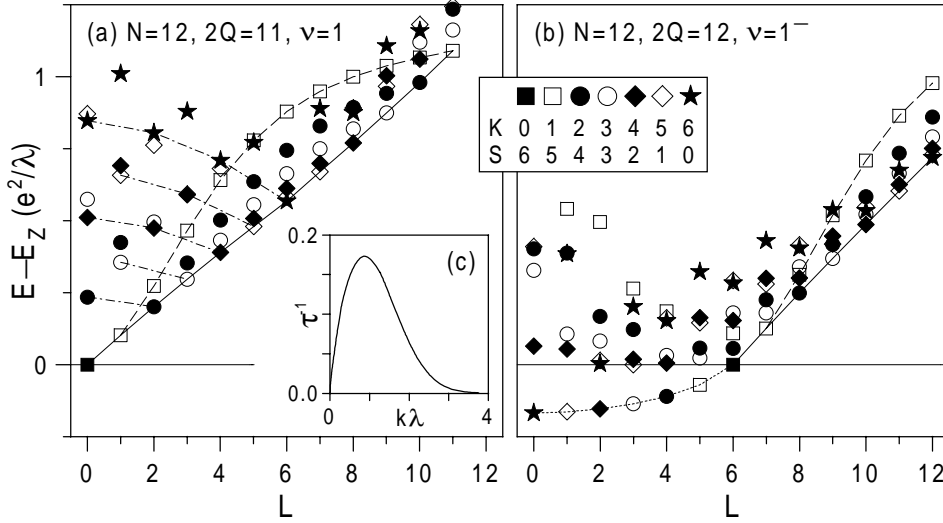


Figure 10. The energy spectra (energy E vs. angular momentum L) of an ideal $12e$ system (no LL mixing and zero quantum well width) with spin excitations in the integral quantum Hall regime, calculated on Haldane sphere with LL degeneracy $2Q + 1 = 12$ (a) and 13 (b).

Similarly as it was for X^- 's, the translational symmetry of an isolated anyon exciton leads to the conservation of its L and L_z in the emission process. This leads to the strict optical selection rules that can only be broken by collisions or disorder. The recombination of an anyon exciton state formed in a Laughlin $\nu = (2p + 1)^{-1}$ electron liquid occurs through annihilation of a well defined number of QE's and creation of an appropriate number of QH's [27, 28]. It turns out that the processes involving more than the minimum number of QP's all have negligible intensity, which for $p = 1$ ($\nu = \frac{1}{3}$) leaves only the following four possible recombination events: $\nu + n\text{QE} \rightarrow (3 - n)\text{QH} + \gamma$, where $n = 0, 1, 2$, or 3 , and γ denotes the photon. When the angular momentum conservation law is applied to the above recombination events, we obtain [31] that the only radiative anyon excitons are νQE^* (the first excited state of a $\nu\text{-QE}$ pair), νQE_R , and νQE_2 , while all others (including νQE) are “dark.”

Because the formation of radiative anyon excitons depends on the presence of QE's or QE_R 's in the 2DEG, the magneto-PL spectrum is expected to change discontinuously at $\nu = \frac{1}{3}$. Such anomalous behavior has actually been observed experimentally [3].

5. Spin waves and skyrmions

The integral quantum Hall system near $\nu = 1$ with spin excitations contains a small number of reversed-spin electrons e_R and spin holes h , and it is very similar to the dilute system of conduction electrons e and valence holes v in the lowest LL. The important difference is that the energy of a $k = 0$ spin wave (which plays the role of an interband exciton) is equal to the electron Zeeman splitting, E_Z , which can be made small compared to the characteristic interaction energy, e^2/λ . Therefore, it is possible to achieve experimentally the situation in which the skyrmions (the e_R-h analogues of interband X^- 's) are truly stable ground states of the system [16, 18], with infinite lifetimes which are not limited by radiative recombination.

In Fig. 10 we present the low energy spectra of the $\nu = 1$ and $\nu = 1^-$ (a single spin hole in $\nu = 1$) states. In this and all other spectra, only the lowest state at each L and S is shown and $K = \frac{1}{2}N - S$ counts the number of spin flips away from the fully polarized ground state.

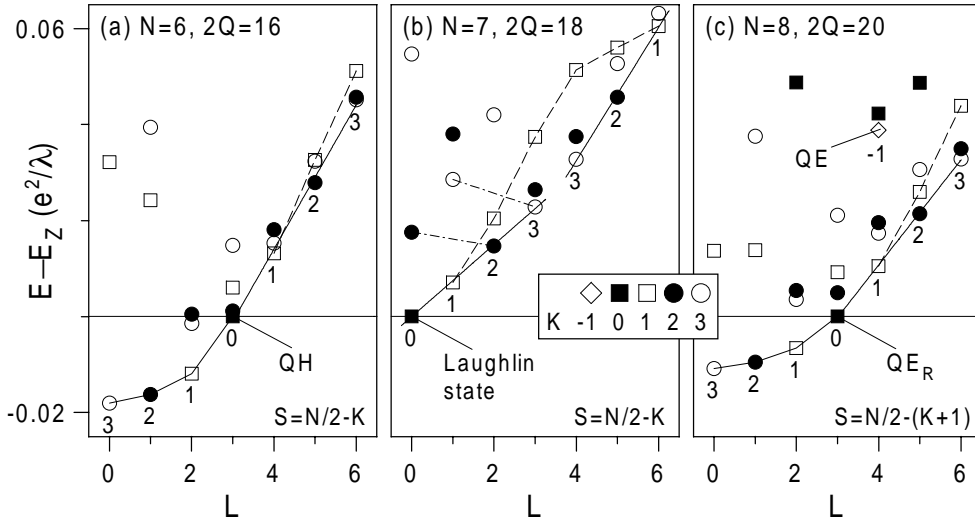


Figure 11. Similar to Fig. 10 but for the fractional quantum Hall regime ($\nu \approx \frac{1}{3}$).

In Fig. 10(a), the ground state is the ferromagnetic integral quantum Hall $\nu = 1$ state at $L = K = 0$. Because the Zeeman energy E_Z is omitted, this state is degenerate with many other states with the same $L = K = 0$ but with different values of S_z , and corresponding to a number $S - S_z$ of $k = 0$ spin waves, each having energy $E_Z = 0$ and decoupled from one another and from the underlying $\nu = 1$ state (the analogues of the $e-v$ “multiplicative” states). Remarkably, the low-energy excited states in Fig. 10(a) form a linear band with $L = K = 1, 2, \dots$. These states contain a number K of spin waves each with $L = 1$ and moving in the same direction so as to build up the maximum total $L = K$. The linear dependence of E on K within this band can be also interpreted as decoupling of so correlated $L = 1$ spin waves from one another, although different from decoupling of $k = 0$ spin waves [25]. In particular, note that a pair of $L = 1$ spin waves can be in two states of total angular momentum $L = 0$ or 2 , and only the latter is noninteracting.

The e_R-h annihilation process analogous to the $e-v$ radiative emission can be achieved by hyperfine coupling of a 2DEG to localized nuclear spins. However, the selection rule for such process is completely different from that governing PL. The appropriate spectral function $\tau^{-1}(k)$ for the spin wave creation/destruction is shown in Fig. 10(c). It has a maximum at $k\lambda \sim 1$, corresponding to the characteristic size of the electron cyclotron orbit [32].

In Fig. 10(b) for $\nu = 1^-$ the band of states with $L = S = Q - K$ and $E < 0$ appears. These are the (anti)skyrmion states, $S_K^+ = Ke_R + (K + 1)h$, analogous to the interband charged excitons X_K^- in the lowest LL [16, 18, 25]. These states are not only truly long-lived (provided that E_Z can be made sufficiently small, e.g., by application of pressure or appropriate doping), but unlike the X_K^- states they are connected with one another through a sequence of spin-flip transitions induced by the hyperfine interaction with a nuclear spin [32]. The $S_K^- \leftrightarrow S_{K+1}^-$ spin-flip transitions are the analogues of the photon emission for the X_K^- states. However, the different selection rule described by the spectral function $\tau^{-1}(k)$ the inset (instead of a strict $k = 0$ rule for the interband emission) allows these transitions in contrast to the forbidden PL of the X_K^- states in the lowest LL. Actually, the $S_K^- \leftrightarrow S_{K+1}^-$ process is believed to be largely responsible for the nuclear spin relaxation in quantum Hall systems.

In Fig. 11 we show similar spectra to Fig. 10, but for the fractional quantum Hall regime, near $\nu = \frac{1}{3}$. Again, despite different character of the constituents – elementary charge

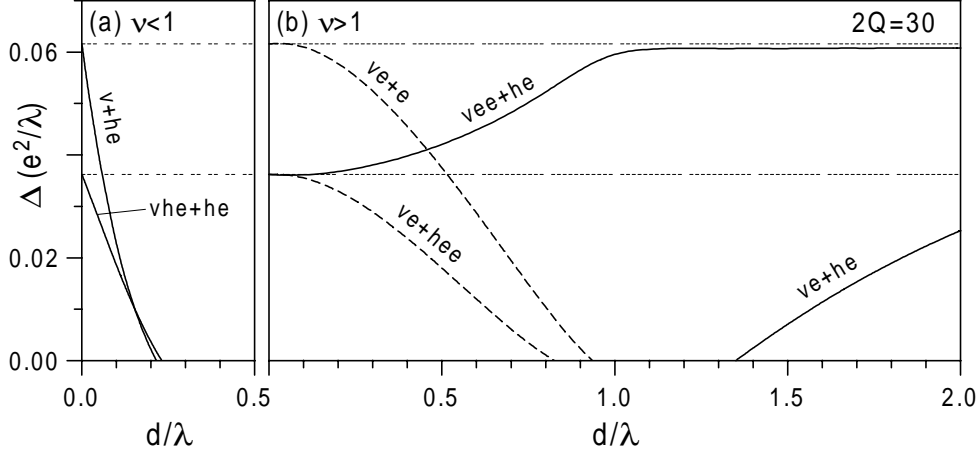


Figure 12. The binding energies Δ of various skyrmion exciton states calculated in an ideal system (no LL mixing and zero quantum well width), as a function of $eh-v$ layer separation d .

excitations (QE_R and QH replacing e_R and h), the same type of bound excitonic complexes are identified. These are spin waves $QE_R + QH$, skyrmions $S_K^- = (K + 1)QE_R + KQH$, and antiskyrmions $S_K^+ = KQE_R + (K + 1)QH$.

6. Skyrmion excitons

When a valence hole v is introduced into a quantum Hall system with a small value of E_Z , it seems possible that it might substitute for one of the spin holes h in a skyrmion or antiskyrmion bound state to form yet another type of excitonic complexes, a skyrmion exciton [33, 34]. Such a complex shares the properties of both pure interband and pure spin excitonic complexes, and for example it might both recombine radiatively via photon emission and couple to nuclear spins via hyperfine interaction. It also has a richer energy spectrum as the two kinds of holes, h and v become distinguishable under actual experimental conditions. Unlike in a dilute $e-v$ system with spin excitations where also three kinds of particles (e could have two different spins) were involved in a X_s^- state, different orbitals of h and v holes (e.g., due to different effective masses or different response to the electric field) make the $e-h$ and $e-v$ interactions different. This prevents the mapping of a $h-v-e_R$ system on a simple two-(iso)spin $e\uparrow-e\downarrow-v$ system with (iso)spin-symmetric interactions.

One possible scenario for the skyrmion exciton creation might be the following. When a v is added to a quantum Hall state at $\nu \leq 1$, there are no negatively charged excitations it could bind. But if E_Z is sufficiently small, v may induce and bind one or more spin waves to form a skyrmion exciton, $v \rightarrow vhe \rightarrow v(he)_2 \rightarrow \dots$. The binding energies of these mixed complexes are shown in Fig. 12(a) as a function of the $eh-v$ layer separation d (note that we skip subscript ‘‘R’’ in symbol e_R in this figure). The situation is different and quite more complicated at $\nu > 1$, in the presence of free reversed-spin electrons or skyrmions. Being negatively charged, they are attracted to the added hole v , and, depending on E_Z , d , and other parameters, they can bind to it to form a rich variety of neutral or negatively charged $h-v-e_R$ states, some of which have been indicated in Fig. 12(b). The fact that the binding energy for the $ve_R + he_R \rightarrow vh(e_R)_2$ process remains negative for $d \leq 1.35\lambda$ suggests that in symmetric structures the attraction between v and $S_1^- = h(e_R)_2$ (or a larger skyrmion) causes breakup of the latter and emission of free spin waves: $v + e_R(he_R)_K \rightarrow ve_R + K \times he_R$. This would make

the equilibrium PL signal come from the same excitonic complex, ve_R , regardless of the size of the skyrmions present in the 2DEG before illumination. On the other hand, the ve_R exciton might attract a second e_R or S^- to acquire charge and become able to induce and bind one or more spin waves. So far these ideas have only been tested in an ideal system (only lowest LL included, no disorder, and zero well width), and more realistic calculation will be needed to verify their significance in actual PL experiments.

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