Enigmatic 4/11 State: A Prototype for Unconventional Fractional Quantum Hall Effect

Sutirtha Mukherjee,[1] Sudhansu S. Mandal,[1] Ying-Hai Wu,[2] Arkadiusz Wójs,[3] and Jainendra K. Jain[2]
[1]*Department of Theoretical Physics, Indian Association for the Cultivation of Science, Jadavpur, Kolkata 700 032, India*
[2]*Department of Physics, 104 Davey Laboratory, Pennsylvania State University, University Park, Pennsylvania 16802, USA*
[3]*Institute of Physics, Wroclaw University of Technology, 50-370 Wroclaw, Poland*
(Received 15 July 2013; published 6 January 2014)

The origin of the fractional quantum Hall effect (FQHE) at 4/11 and 5/13 has remained controversial. We make a compelling case that the FQHE is possible here for fully spin polarized composite fermions, but with an unconventional underlying physics. Thanks to a rather unusual interaction between composite fermions, the FQHE here results from the suppression of pairs with a relative angular momentum of three rather than one, confirming the exotic mechanism proposed by Wójs, Yi, and Quinn [Phys. Rev. B **69**, 205322 (2004)]. We predict that the 4/11 state reported a decade ago by Pan *et al.* [Phys. Rev. Lett. **90**, 016801 (2003)] is a conventional partially spin polarized FQHE of composite fermions, and we estimate the Zeeman energy where a phase transition into the unconventional fully spin polarized state will occur.

DOI: 10.1103/PhysRevLett.112.016801 PACS numbers: 73.43.-f, 71.10.Pm

The fractional quantum Hall effect (FQHE) [1] is one of the cleanest and most nontrivial manifestations of interelectron interaction and has produced a string of surprising discoveries during the past three decades. A FQHE state is characterized by (f, γ, α), where f is the fraction appearing in the expression for the fractionally quantized Hall resistance $R_H = h/fe^2$ (indicating an incompressible state at filling factor $\nu = f$), γ is the spin or valley polarization, and α labels topologically distinct states with the same f and γ that may occur for different interactions. The richness of the physics is made evident by the remarkable fact that more than 75 fractions have been observed to date [2], and states with several different spin or valley polarizations occur at many of these fractions. Different physical mechanisms for the FQHE have been identified. Many FQHE states at filling factors of the form $\nu = j \pm n/(2pn \pm 1)$, where j, n, and p are integers, are explained as integer quantum Hall effects (IQHE) of composite fermions carrying $2p$ vortices [3], and the FQHE states at $\nu = 5/2$ and $7/2$ are modeled as chiral p-wave paired states of composite fermions [4]. Our focus here is on the FQHE at $\nu = 4/11$ and $5/13$ [5] which cannot be understood as either an IQHE or a paired state of composite fermions. We show below that their explanation requires yet another physical mechanism, thus adding to the richness of the FQHE and opening the exciting possibility of other FQHE states arising from this mechanism.

The 4/11 and 5/13 FQHE states are very delicate, appearing only in the highest quality samples [5]; in fact, a definitive observation, in the form of accurately quantized Hall plateaus with activated longitudinal resistance, is still lacking. These states were seen at fairly large magnetic fields ($\sim 11\,T$), where the Zeeman splitting (E_Z) is substantial, $\sim 3\,K$, and the resistance showed negligible variation upon an increase in E_Z; these facts were taken in Ref. [5] strongly to support a fully spin polarized FQHE. We will therefore look for a fully spin polarized state at these fractions, returning to the role of spin later. In this filling factor region, electrons capture two quantized vortices each to turn into composite fermions [3]. Composite fermions experience an effective magnetic field $B^* = B - 2\phi_0\rho$, where B is the external field, $\phi_0 = hc/e$ is the flux quantum, and ρ is the electron or composite fermion (CF) density. Composite fermions form Landau-like levels called Λ levels (ΛLs) in B^*, and their filling factor ν^* is related to the electron filling factor ν by the relation $\nu = \nu^*/(2\nu^* \pm 1)$. The IQHE of composite fermions at $\nu^* = n$ manifests as a FQHE at odd-denominator fractions of the form $\nu = n/(2n \pm 1)$. These will be referred to as the "conventional" FQHE states.

At 4/11 and 5/13 the CF filling is $\nu^* = 1 + 1/3$ and $\nu^* = 1 + 2/3$, and the question is what state composite fermions form at 1/3 and 2/3 filling in the second ΛL. Several proposals have been made, but all are subject to criticisms. A variational study [6] suggested that they form a crystal, while another [7] suggested a conventional Laughlin-type [8] FQHE state. The wave functions employed in these studies, however, have not been demonstrated to be sufficiently accurate to capture the subtle physics of this state. Reference [9] performed CF diagonalization [10,11] and also supported the conventional FQHE, primarily based on results for the 12 particle system; this system, however, was recently recognized [12] to "alias" with the anti-Pfaffian paired state at $\nu = 3/8$, thereby casting doubt on the conclusions of Ref. [9]. Wójs, Yi, and Quinn (WYQ) [13,14] modeled composite fermions in the second ΛL as fermions interacting via an effective two-body interaction, which is determined by placing two composite fermions in the second ΛL [6,15,16]. They studied the effective model by exact diagonalization and arrived at

 © 2014 American Physical Society

the surprising conclusion that composite fermions form "unconventional" 1/3 and 2/3 states. However, the two-body model is known sometimes to produce a wrong ground state [17], presumably because of the neglect of either three- and higher-body interaction between composite fermions or the filling factor dependence of the inter-CF interaction. The situation therefore remained unsettled.

Which state is energetically favored is determined by the very weak interaction between composite fermions. Fortunately, the method of CF diagonalization [10] (CFD) has been shown to capture the physics of inter-CF interaction extremely accurately in the region of interest, producing energies within ~0.05% of the exact energies. In this method, a correlated CF basis $\{\Psi_\nu^{\mathrm{CF},\alpha}\}$ is constructed starting from the known basis $\{\Phi_{\nu^*}^\alpha\}$ of degenerate ground states of noninteracting fermions at ν^*, and then the full Coulomb Hamiltonian is diagonalized within this basis. The basis functions $\Psi_\nu^{\mathrm{CF},\alpha}$ are much more complicated than the usual Slater determinants, but efficient methods have been developed to calculate with them [11,10]. The dimension of $\{\Psi_\nu^{\mathrm{CF},\alpha}\}$ is exponentially small compared to the dimension of the full lowest Landau level (LLL) Hilbert space, which allows CFD to treat much larger systems than possible for exact diagonalization. We stress that no assumption is made regarding the form of the interaction between composite fermions. More details can be found in Supplemental Material [18].

We use the spherical geometry [19], in which N electrons move on the surface of a sphere under the influence of a flux of $2Q(hc/e)$, where $2Q$ is an integer. The many particle eigenstates are labeled by the total orbital angular momentum L. Theoretical demonstration of incompressibility at a filling ν requires that the state at each N and $2Q$ satisfying $2Q = \nu^{-1}N - S$, where S is an N independent "shift," produces a uniform ($L = 0$) state separated from the excitations by a gap, and the gap extrapolates to a nonzero value in the limit $N \to \infty$. Candidate states with different values of S at a given ν are topologically distinct, and a determination of S by exact or CF diagonalization can identify which candidate state is viable. The shifts S^* for the conventional and WYQ states at 1/3 and 2/3 are given in Table I; these result in 4/11 and 5/13 states at shifts S shown in Table I (see Supplemental Material for details [18]).

The CFD spectra at the unconventional shifts are shown in Fig. 1. Several points are noteworthy. (i) The comparison with exact spectra, available for up to 16 particles (Fig. 1), demonstrates that the CFD spectra are to be treated as essentially exact for the ground states. (The CFD energies deviate from the exact ones by ~0.05%.) (ii) The ground state occurs at $L = 0$ for each value of N at the unconventional shifts. (iii) A reliable extrapolation of gaps to the thermodynamic limit is unfortunately not possible due to strong finite size effects, but the results are consistent with a nonzero value (see Fig. 2 for the energy of the lowest neutral excitation). The energy scale for the gaps (Fig. 2) is $\sim 0.002\, e^2/\epsilon\ell$, where $\ell = \sqrt{\hbar c/eB}$ is the magnetic length. This energy is approximately ~50 times smaller than the ideal theoretical gap of 1/3 ($0.1\, e^2/\epsilon\ell$), indicative of a much weaker interaction between composite fermions than that between electrons. (iv) The two-body interaction model of WYQ would produce identical spectra

TABLE I. Shifts for the conventional and unconventional states at 1/3, 2/3, 4/11, and 5/13.

ν	Conventional	Unconventional
1/3	$S^* = 3$	$S^* = 7$
2/3	$S^* = 0$	$S^* = -2$
4/11	$S = 4$	$S = 5$
5/13	$S = 17/5$	$S = 13/5$

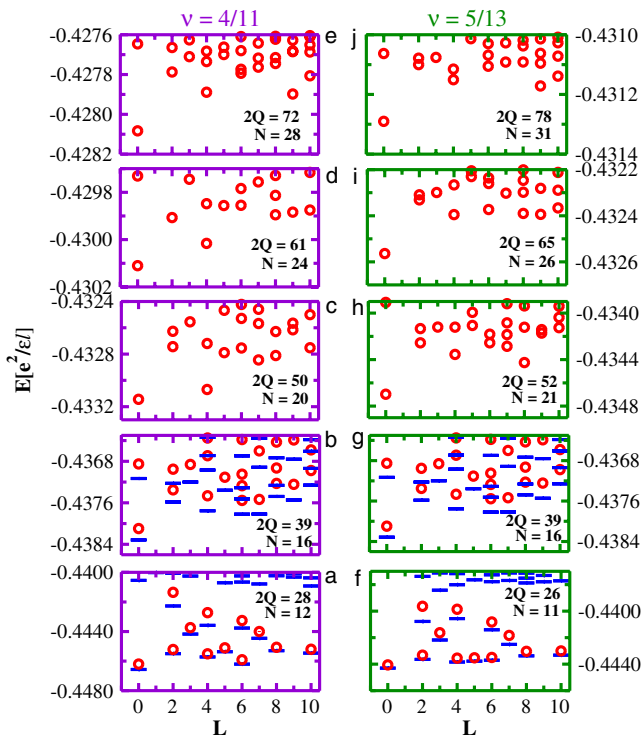

FIG. 1 (color online). Composite fermion spectra at 4/11 and 5/13. The circles show energies per particle obtained by CF diagonalization for the FQHE state at 4/11 (left) and 5/13 (right) at shifts $S = 5$ and 13/5, respectively. The dashes are obtained by exact diagonalization of the Coulomb interaction in the full LLL Hilbert space (only the very low energy states are shown; the dimensions of the full LLL basis and the CFD basis are given in Supplemental Material [18]). N is the number of particles, $2Q$ is the number of flux quanta passing through the sample, and L is the total angular momentum quantum number of the eigenstate. The energy per particle is quoted in units of $e^2/\epsilon\ell$, where $\ell = \sqrt{\hbar c/eB}$ is the magnetic length and ϵ is the dielectric constant of the host material. The energy includes the interaction with the positively charged neutralizing background. The $N = 16$ state at $2Q = 39$ occurs for both 4/11 and 5/13; we have included this panel twice for symmetry reasons.

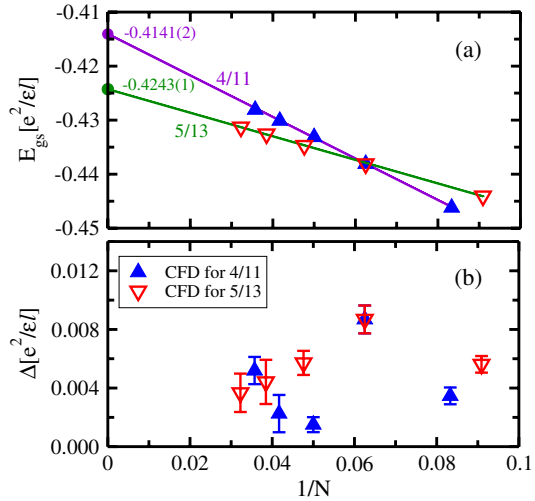

FIG. 2 (color online). (a) The ground state energy per particle for 4/11 and 5/13 as a function of $1/N$, extrapolated to the thermodynamic limit. (b) The energy gap of the lowest neutral excitation.

for the horizontally neighboring panels in Fig. 1. Substantial differences seen in the CFD spectra indicate the importance of the three- and higher-body interactions between composite fermions. (v) Finally, it is interesting to note that for the 4/11 state with 12, 16, and 28 particles, the full dimensions of the $L = 0$ sector (in the LLL) are 902, 250 256, and $\sim 2 \times 10^{13}$, respectively, whereas the dimensions of the corresponding CF bases are 1, 3, and 28.

We next show, by constructing explicit trial wave functions, that the origin of this FQHE is captured by the mechanism proposed by WYQ. A pairwise interaction for fermions confined to any Landau level (LL) can, in general, be parametrized as [19]

$$V = \sum_m V_m |m\rangle\langle m|, \tag{1}$$

where $|m\rangle$ denotes the two particle state with relative angular momentum m, and the pseudopotential V_m is the energy of this state. For the Coulomb interaction V_1 dominates, and the conventional states $n/(2n \pm 1)$ are produced in a model $V_m = \delta_{m,1}$. WYQ consider instead the model interaction $V_m = \delta_{m,3}$ and find, by numerical diagonalization, that it produces incompressible $L = 0$ ground states at 1/3 and 2/3, but at shift 7 and -2, respectively, as opposed to the conventional shifts of 3 and 0 produced by the Coulomb (or $V_m = \delta_{m,1}$) interaction. The WYQ states are thus topologically distinct from Laughlin's 1/3 and 2/3 states. Explicit wave functions for the WYQ states are not known, but can be generated exactly for up to 15 particles by a brute force numerical diagonalization. The WYQ states do not have zero energy, implying that they minimize, but do not eliminate, occupation of pairs with relative angular momentum 3. Given that there is no repulsion in the angular momentum $m = 1$ channel, one might expect pairing correlations, but the actual FQHE state does not involve pairing, as evidenced by the fact that an incompressible state is produced for both even and odd N. We have performed an extensive investigation of the 1/3 WYQ state through exact diagonalization on sphere, torus, and disk, as well as through its entanglement spectrum. These studies, reported in the Supplemental Material [18], clarify that its excitations carry local charge 1/3, its excitations obey Abelian braid statistics, it is topologically distinct from the usual 1/3 state, it has a complex edge with multiple channels, and its edge does not support, in the absence of reconstruction, backward moving neutral modes.

While the WYQ states clearly represent a new kind of order, one may ask if they are at all realizable. The interaction $V_m = \delta_{m,3}$ appears unphysical, because it penalizes pairs with relative angular momentum $m = 3$ but has no repulsion in the angular momentum $m = 1$ channel. However, this interaction is realized precisely for composite fermions in the second ΛL. References [6,15,16] have shown that for composite fermions in this filling factor region, the V_3 pseudopotential dominates (which was the motivation for WYQ considering this interaction). To test if this physics actually underlies the 4/11 and 5/13 FQHE, we obtain the unconventional WYQ ground states $\Phi_{4/3}^{\rm uncon}$ and $\Phi_{5/3}^{\rm uncon}$ at 4/3 and 5/3 by an exact numerical diagonalization of the WYQ interaction $V_m = \delta_{m,3}$, and then composite fermionize them to obtain explicit trial wave functions for the CF states at 4/11 and 5/13, denoted $\Psi_{4/11}^{\rm uncon}$ and $\Psi_{5/13}^{\rm uncon}$. (See Supplemental Material for details [18].) A direct comparison with the CFD ground states, shown in Table II, provides strong support that these wave functions capture the physics of the actual 4/11 and 5/13 FQHE states. In other words, incompressibility at these fractions results because the occupation of CF pairs with relative angular momentum $m = 3$ is minimized, distinct from the usual mechanism for FQHE at $n/(2n \pm 1)$, which minimizes occupation of electron pairs with $m = 1$.

The high overlaps of the 4/11 and 5/13 ground states with the composite-fermionized WYQ wave functions demonstrates that the three-body terms in the inter-CF interaction do not significantly affect the nature of the 4/11 and 5/13 ground states. This is somewhat surprising because the three-body terms are responsible for substantial differences between the excitation spectra of the corresponding systems (paired horizontally in Fig. 1) at 4/11 and 5/13.

Our work has a number of experimental implications. First, it implies that fully spin polarized FQHE is possible at 4/11 and 5/13 under appropriate conditions. The analogy to the WYQ states implies that the 4/11 (5/13) state does not involve pairing, supports charge 1/11 (1/13) excitations with Abelian braid statistics, has multiple edge channels, and does not have (has) backward moving neutral modes. The absence of a well-defined magnetoroton branch

TABLE II. Testing trial wave functions for the unconventional states at $4/11$ and $5/13$. This table gives the overlap between $\Psi^{\rm uncon}$ and $\Psi^{\rm CFD}$ for $4/11$ and $5/13$. The trial wave functions $\Psi^{\rm uncon}_{4/11}$ and $\Psi^{\rm uncon}_{5/13}$ are derived by composite fermionization of $\Psi^{\rm uncon}_{4/3}$ and $\Psi^{\rm uncon}_{5/3}$, which, in turn, are exact ground states of the WYQ interaction that select states that minimize the occupation of pairs with relative angular momentum 3. $\Psi^{\rm CFD}_{4/11}$ and $\Psi^{\rm CFD}_{5/13}$, obtained by CF diagonalization, are essentially exact. The spherical geometry is used. N is the total number of electrons or composite fermions, N^* is the number of composite fermions in the second ΛL, and $2Q$ is the number of flux quanta passing through the surface of the sphere.

N	$2Q$	N^*	$\langle\Psi^{\rm CFD}_{4/11}\vert\Psi^{\rm uncon}_{4/11}\rangle$
12	28	5	1.000
16	39	6	0.9985(1)
20	50	7	0.9834(1)
24	61	8	0.9351(2)
28	72	9	0.9627(2)
N	$2Q$	N^*	$\langle\Psi^{\rm CFD}_{5/13}\vert\Psi^{\rm uncon}_{5/13}\rangle$
11	26	4	1.000
16	39	6	0.9985(1)
21	52	8	0.9962(1)
26	65	10	0.9875(2)
31	78	12	0.9428(3)

in the finite-system spectra indicates that their quasiparticles and quasiholes are large and complex, as has been found even for the $7/3$ state [20].

We now show that the electron spin also plays an interesting role. A "conventional" partially spin polarized $4/11$ state has been proposed in the past [21–23], wherein composite fermions fill lowest spin-up ΛL completely and form a conventional $1/3$ state in the spin-reversed lowest ΛL, giving a polarization $\gamma = (\nu^*_\uparrow - \nu^*_\downarrow)/(\nu^*_\uparrow + \nu^*_\downarrow) = 1/2$, where ν^*_σ represents the filling factor of composite fermions with spin σ. The conventional mechanism for the partially spin polarized state has been confirmed by CFD [22]. The interaction energy of the partially polarized ground state [21,22], $-0.4205(2)e^2/\epsilon\ell$, is less than that of the fully spin polarized state, $-0.4141(2)e^2/\epsilon\ell$ (Fig. 2), indicating that the partially polarized state is stabilized at sufficiently low Zeeman splitting E_Z, defined as the energy required to flip a single spin. Equating the per-particle Coulomb energy difference to $E_Z/4$ (as $1/4$ of the composite fermions flip their spin in going from fully to partially polarized state), a phase transition from the partially spin polarized conventional state to a fully spin polarized unconventional state is predicted to occur at $\kappa \equiv E_Z/(e^2/\epsilon\ell) = 0.025$. For GaAs parameters (band mass $m_b = 0.067m_{\rm e}$, Landé g factor $g = -0.44$, background dielectric function $\epsilon = 13.6$), this translates into a transition at a magnetic field $B_{\rm crit} \sim 19$ T. (Finite width corrections are not considered explicitly here, but are expected to reduce $B_{\rm crit}$ by 10%–20% [24].) Our detailed calculations thus lead to the surprising prediction, at variance with the earlier conclusion [5], that the $4/11$ state observed in Ref. [5] is partially spin polarized with $\gamma = 1/2$ even though it occurs at a magnetic field as high as ∼11 T. (The insensitivity of resistance to variations in E_Z [5] can be explained by noting that the lowest gap in the partially polarized state corresponds to an excitation within the spin-reversed sector [21,22], and therefore does not involve a spin reversal.) An experimental verification of this predictions, as well as of a magnetic transition at $\kappa \approx 0.025$ (for $\nu = 4/11$), will serve as nontrivial confirmation of the physics described above. The spin polarizations and spin phase transitions at $\nu = n/(2pn \pm 1)$ have been measured by transport [25–30], optical [31–34], NMR [35–39], and compressibility [40] measurements; analogous valley polarization transitions have been observed in AlAs quantum wells [41–43], and the experimental observations are generally consistent with the CF theory [44,24].

We have also evaluated the pair correlation functions as well as the density profiles of the quasiparticle and quasihole for the conventional and the unconventional states. The differences between them for conventional and unconventional states are substantial for $1/3$ but less so for $4/11$, as shown in the Supplemental Material [18].

It is interesting to ask what other analogous unconventional CF liquids are possible. We have investigated this question by diagonalizing both the second ΛL interaction given in Ref. [6] and the model $V_m = \delta_{m,3}$ interaction for a wide range of particle numbers and flux values, and found that, in the range $2 > \nu^* > 1$, it produces incompressibility only at $\nu^* = 4/3$, $5/3$, $6/5$, and $9/5$. To the extent this model is applicable, our study implies that unconventional CF states occur at $4/11$, $5/13$, $6/17$, and $9/23$, which, along with $3/8$ [5,12], exhaust all possible FQHEs in the range $2/5 > \nu > 1/3$ for a fully spin polarized system.

We have thus shown that the fully spin polarized FQHE at $4/11$ and $5/13$ originates from a novel mechanism, due to a peculiar interaction between composite fermions. We have predicted that the previously observed state at $4/11$ [5] is partially spin polarized, and that a transition into a fully polarized state will occur at $\kappa \equiv E_Z/(e^2/\epsilon\ell) \approx 0.025$. We close with a further remarkable implication of our study. It is well appreciated that the nature of the FQHE depends sensitively on the interaction pseudopotentials. That is the reason why FQHE in the second LL of GaAs is different from that in the lowest LL, and no FQHE is seen in yet higher LLs. That is also partially responsible for differences between FQHE in GaAs and graphene, and between FQHE of electrons and hard core bosons. Composite fermions in various ΛLs provide yet another system of particles with rather unusual interactions [6,15,16], which can possibly spawn new unconventional quantum liquids. The higher ΛLs of composite fermions are likely to serve as a playground for the possible discovery of new topological states as the sample quality improves in coming years.

We are grateful to A. C. Balram, T. H. Hansson, J. J. Quinn, and S. H. Simon for fruitful discussions. We acknowledge financial support from CSIR, Government of India (S. M.), DOE Grant No. DE-SC0005042 (Y.-H. W., J. K. J.), and the Polish NCN Grant No. 2011/01/B/ST3/04504 and EU Marie Curie Grant No. PCIG09-GA-2011-294186 (A. W.). The computations were performed using computing facilities of the Department of Theoretical Physics at IACS, Cyfronet, and WCSS (both parts of PL-Grid Infrastructure), and the Research Computing and Cyberinfrastructure, a unit of Information Technology Services at Pennsylvania State University. Y.-H. W. thanks the authors of the DiagHam package for sharing their programs.

[1] D. C. Tsui, H. L. Störmer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[2] W. Pan (private communication).
[3] J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989).
[4] G. Moore and N. Read, Nucl. Phys. **B360**, 362 (1991).
[5] W. Pan, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **90**, 016801 (2003).
[6] S.-Y. Lee, V. W. Scarola, and J. K. Jain, Phys. Rev. B **66**, 085336 (2002).
[7] M. O. Goerbig, P. Lederer, and C. M. Smith, Phys. Rev. Lett. **93**, 216802 (2004).
[8] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).
[9] C.-C. Chang and J. K. Jain, Phys. Rev. Lett. **92**, 196806 (2004).
[10] S. S. Mandal and J. K. Jain, Phys. Rev. B **66**, 155302 (2002).
[11] J. K. Jain and R. K. Kamilla, Phys. Rev. B **55**, R4895 (1997); Int. J. Mod. Phys. B **11**, 2621 (1997).
[12] S. Mukherjee, S. S. Mandal, A. Wójs, and J. K. Jain, Phys. Rev. Lett. **109**, 256801 (2012).
[13] A. Wójs, K.-S. Yi, and J. J. Quinn, Phys. Rev. B **69**, 205322 (2004).
[14] A. Wójs, D. Wodziński, and J. J. Quinn, Phys. Rev. B **71**, 245331 (2005).
[15] P. Sitko, S. N. Yi, K. S. Yi, and J. J. Quinn, Phys. Rev. Lett. **76**, 3396 (1996).
[16] S.-Y. Lee, V. W. Scarola, and J. K. Jain, Phys. Rev. Lett. **87**, 256803 (2001).
[17] For example, for $(N, 2Q) = (20, 49)$ and $(24, 61)$, the WYQ model predicts ground states at $L = 0$ and 6, whereas the CFD finds $L = 6$ and 0, respectively, [12].
[18] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.112.016801 for details on the calculation of shifts, CFD, composite fermionization, Hilbert space dimensions, and also for a comparison of pair correlation function and quasiparticle and quasihole charge densities of conventional and WYQ states at $1/3$. It also includes a detailed study of the properties of the WYQ $1/3$ state.
[19] F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983).
[20] A. C. Balram, Y.-H. Wu, G. J. Sreejith, A. Wójs, and J. K. Jain, Phys. Rev. Lett. **110**, 186801 (2013).
[21] K. Park and J. K. Jain, Phys. Rev. B **62**, R13274 (2000).
[22] C. C. Chang, S. S. Mandal, and J. K. Jain, Phys. Rev. B **67**, 121305(R) (2003).
[23] A. Wójs, G. Simion, and J. J. Quinn, Phys. Rev. B **75**, 155318 (2007).
[24] A. C. Archer and J. K. Jain, Phys. Rev. Lett. **110**, 246801 (2013).
[25] J. P. Eisenstein, H. L. Stormer, L. Pfeiffer, and K. W. West, Phys. Rev. Lett. **62**, 1540 (1989).
[26] R. R. Du, A. S. Yeh, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **75**, 3926 (1995).
[27] H. Cho, J. B. Young, W. Kang, K. L. Campman, A. C. Gossard, M. Bichler, and W. Wegscheider, Phys. Rev. Lett. **81**, 2522 (1998).
[28] A. S. Yeh, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **82**, 592 (1999).
[29] J. Eom, H. Cho, W. Kang, K. L. Campman, A. C. Gossard, M. Bichler, and W. Wegscheider, Science **289**, 2320 (2000).
[30] B. A. Piot, J. Kunc, M. Potemski, D. K. Maude, C. Betthausen, A. Vogl, D. Weiss, G. Karczewski, and T. Wojtowicz, Phys. Rev. B **82**, 081307(R) (2010); C. Betthausen *et al.*, report.
[31] I. V. Kukushkin, K. v. Klitzing, and K. Eberl, Phys. Rev. Lett. **82**, 3665 (1999); I. V. Kukushkin, J. H. Smet, K. von Klitzing, and K. Eberl, *ibid.* **85**, 3688 (2000).
[32] G. Yusa, H. Shtrikman, and I. Bar-Joseph, Phys. Rev. Lett. **87**, 216402 (2001).
[33] J. G. Groshaus, P. Plochocka-Polack, M. Rappaport, V. Umansky, I. Bar-Joseph, B. S. Dennis, L. N. Pfeiffer, K. W. West, Y. Gallais, and A. Pinczuk, Phys. Rev. Lett. **98**, 156803 (2007).
[34] J. Hayakawa, K. Muraki, and G. Yusa, Nat. Nanotechnol. **8**, 31 (2012).
[35] S. Melinte, N. Freytag, M. Horvatic, C. Berthier, L. P. Levy, V. Bayot, and M. Shayegan, Phys. Rev. Lett. **84**, 354 (2000); N. Freytag, M. Horvatic, C. Berthier, M. Shayegan, and L. P. Levy, *ibid.* **89**, 246804 (2002).
[36] J. H. Smet, R. A. Deutschmann, W. Wegscheider, G. Abstreiter, and K. von Klitzing, Phys. Rev. Lett. **86**, 2412 (2001).
[37] S. Kraus, O. Stern, J. G. S. Lok, W. Dietsche, K. von Klitzing, M. Bichler, D. Schuh, and W. Wegscheider, Phys. Rev. Lett. **89**, 266801 (2002).
[38] J. H. Smet, R. A. Deutschmann, F. Ertl, W. Wegscheider, G. Abstreiter, and K. von Klitzing, Phys. Rev. Lett. **92**, 086802 (2004).
[39] L. Tiemann, G. Gamez, N. Kumuda, and K. Muraki, Science **335**, 828 (2012).
[40] B. E. Feldman, A. J. Levin, B. Krauss, D. Abanin, B. I. Halperin, J. H. Smet, and A. Yacoby, Phys. Rev. Lett. **111**, 076802 (2013).
[41] N. C. Bishop, M. Padmanabhan, K. Vakili, Y. P. Shkolnikov, E. P. De Poortere, and M. Shayegan, Phys. Rev. Lett. **98**, 266404 (2007).
[42] M. Padmanabhan, T. Gokmen, and M. Shayegan, Phys. Rev. B **80**, 035423 (2009); Phys. Rev. B **81**, 113301 (2010).
[43] T. Gokmen, M. Padmanabhan, and M. Shayegan, Nat. Phys. **6**, 621 (2010).
[44] K. Park and J. K. Jain, Phys. Rev. Lett. **80**, 4237 (1998).