



ELSEVIER

Physica E 11 (2001) 209–214

PHYSICA E

www.elsevier.com/locate/physce

Electron–hole systems in narrow quantum wells: excitonic complexes and photoluminescence

John J. Quinn^{a,*}, Arkadiusz Wójs^{a,b}, Kyung-Soo Yi^{a,c}, Izabela Szlufarska^{a,b}^a*Department of Physics, University of Tennessee, 200 South College, Knoxville, TN 37996, USA*^b*Institute of Physics, Wrocław University of Technology, Wrocław 50-370, Poland*^c*Physics Department, Pusan National University, Pusan 609-735, South Korea*

Abstract

The energy and photoluminescence (PL) spectra of a two-dimensional electron gas (2DEG) interacting with a valence-band hole are studied in the high-magnetic-field limit as a function of the filling factor ν and the separation d between the electron and hole layers. For d smaller than the magnetic length λ , the hole binds one or more electrons to form neutral (X^0) or charged (X^-) excitons, and PL probes the lifetime and binding energies of these complexes rather than the original correlations of the 2DEG. The low-lying states can be understood in terms of Laughlin-type correlations among the constituent negatively charged Fermions (electrons and X^- 's). For d , large compared to λ , the electron–hole interaction is not strong enough to bind a full electron, and fractionally charged excitons hQE_n (bound states of the hole and one or more Laughlin quasielectrons) are formed. The PL selection rule associated with rotational invariance (conservation of L) is only weakly violated in the interacting plasma, and the position and oscillator strengths of PL lines can be predicted and compared with numerical calculations. © 2001 Elsevier Science B.V. All rights reserved.

PACS: 71.35.Ji; 71.35.Ee; 73.20.Dx

Keywords: Charged magneto-exciton; Photoluminescence; Quantum well

1. Introduction

In order to obtain a better understanding of the photoluminescence (PL) process in fractional quantum Hall (FQH) systems, it is important to understand the nature of the low-lying eigenstates of the interacting

electron–hole system. In this note, we study the eigenstates of a system consisting of N electrons confined to a plane $z = 0$ and interacting with one another and with a valence-band hole (h) confined to a plane $z = d$, where d is measured in units of the magnetic length $\lambda = (\hbar c/eB)^{1/2}$. The cyclotron energy $\hbar\omega_c$ is assumed to be much larger than the Coulomb energy e^2/λ , so that only the lowest Landau level enters our calculations. Energy spectra obtained by exact numerical diagonalization for a nine-electron–one-hole system are

* Corresponding author. Tel.: +1-865-974-4089; fax: +1-865-974-6378.

E-mail address: jjquinn@utk.edu (J.J. Quinn).

presented for $d = 0, 1.5,$ and $4,$ and for $\nu = \frac{1}{3}$ plus $n = 0, 1, 2,$ or 3 Laughlin quasielectrons (QEs). The low-energy eigenstates can be interpreted in terms of excitonic complexes (or the hole) weakly interacting with the remaining electrons.

For $d \ll 1,$ the hole binds one or two electrons to form a neutral (X^0) or charged (X^-) exciton. The X^0 in its ground state is effectively decoupled from the remaining $N - 1$ electrons in a “multiplicative” state [1–3] whose energy is that of $N - 1$ electrons shifted by the X^0 binding energy. In contrast, the X^- is a charged Fermion, and it has Laughlin-like correlations with the remaining $N - 2$ electrons [4–6]. For $d \gg 1,$ the potential of the hole is a weak perturbation on the eigenstates of the N -electron system. The low-lying eigenstates can be understood in terms of the angular momenta of the Laughlin QEs and of the hole. For intermediate values of d ($1 \leq d \leq 2$), the potential of the hole is not strong enough to bind an electron, but it is not a weak perturbation on the eigenstates of the N -electron system, either. In this case the hole binds one or more Laughlin QEs to form fractionally charged excitons (FCXs). We denote a bound state of the hole and n QEs as hQE_n .

There are two separate symmetries which dictate the rules for radiative recombination of an electron–hole pair. The most important one is translational invariance, which in the Haldane spherical geometry becomes rotational invariance. It requires the total angular momentum L and its z -component to be conserved in the radiative recombination process. Here L denotes the total angular momentum of the system, not just that of the excitonic complex involved in the recombination process. The second symmetry is called the “hidden symmetry” [7,8]. It results from the fact that the commutator of the interaction Hamiltonian with the photoluminescence operator $\hat{\mathcal{L}} = \int d^2r \hat{\Psi}_e(r) \hat{\Psi}_h(r)$ is proportional to $\hat{\mathcal{L}}$, whenever the magnitude of the electron–hole interaction $|V_{eh}|$ is equal to that of the electron–electron interaction $|V_{ee}|$. Due to this symmetry, when d is equal to zero, only “multiplicative” states can undergo radiative recombination. Therefore, when $d \ll 1,$ the PL spectrum contains information about the X^0 , while for $d > 1$ it contains information about the elementary excitations of the Laughlin fluid and their interactions with one another and with the hole.

2. Energy spectra

In Fig. 1 we present the examples of energy spectra of the nine-electron–one-hole system obtained by exact diagonalization in the spherical geometry. The separation d between the electron and hole planes is accounted for by taking $V_{eh}(r) = -e^2(r^2 + \lambda^2 d^2)^{-1/2}$. The radial magnetic field is given by $4\pi R^2 B = 2S\phi_0$, where R is the radius of the sphere, $\phi_0 = hc/e$ is the quantum of the flux, and the “monopole strength” $2S$ is equal to an integer. In different frames, the values of $2S$ are 21 (a–a’), 22 (b–b’), 23 (c–c’), and 24 (d–d’), and they correspond to the Laughlin $\nu = \frac{1}{3}$ state with 3, 2, 1, and 0 QE excitations, respectively. The interplanar separation equals $d = 0$ (a–d), 1.5 (a’–d’), and 4 (a’’–d’’).

2.1. Strong coupling

For $d = 0, X^0$ and X^- bound states occur. Due to the “hidden symmetry”, the multiplicative states containing an X^0 have the same spectrum as the eight-electron system shifted by the X^0 binding energy. The CF model [9,10] tells us that the effective monopole strength seen by one CF in a system of $N' = N - 1 = 8$ electrons near $\nu = \frac{1}{3}$ is $2S^* = 2S - 2(N' - 1)$. S^* plays the role of the angular momentum of the lowest CF shell (Landau level), therefore $S^* = 3.5, 4, 4.5,$ and 5 for the multiplicative states in frames (a), (b), (c), and (d) of Fig. 1, respectively. Since the lowest shell can accommodate $2S^* + 1$ CFs, it is exactly filled in Fig. 1(a), but there are 1, 2, and 3 excess CFs for Fig. 1(b), (c), and (d), respectively. The excess CFs go into the next shell as Laughlin QEs with $l_{QE} = S^* + 1$, giving one QE with $l_{QE} = 4$ (b), two QEs each with $l_{QE} = 4.5$ (c), and three QEs each with $l_{QE} = 5$ (d). The angular momenta of the lowest band of multiplicative states are obtained by the addition of the angular momenta of the QE excitations, remembering that they are identical Fermions. This gives $L = 0$ (a), $L = 4$ (b), $L = 0 \oplus 2 \oplus 4 \oplus 6 \oplus 8$ (c), and $L = 0 \oplus 2 \oplus 3 \oplus 4^2 \oplus 5 \oplus 6^2 \oplus 7 \oplus 8 \oplus 9 \oplus 10 \oplus 12$ (d). These states are shown as points surrounded by a small circle in all frames for $d = 0$. In the absence of QE–QE interactions (i.e. for mean-field CFs) all the states in the lowest CF band of each spectrum would degenerate, but QE–QE interactions remove this degeneracy. Higher-energy multiplicative states

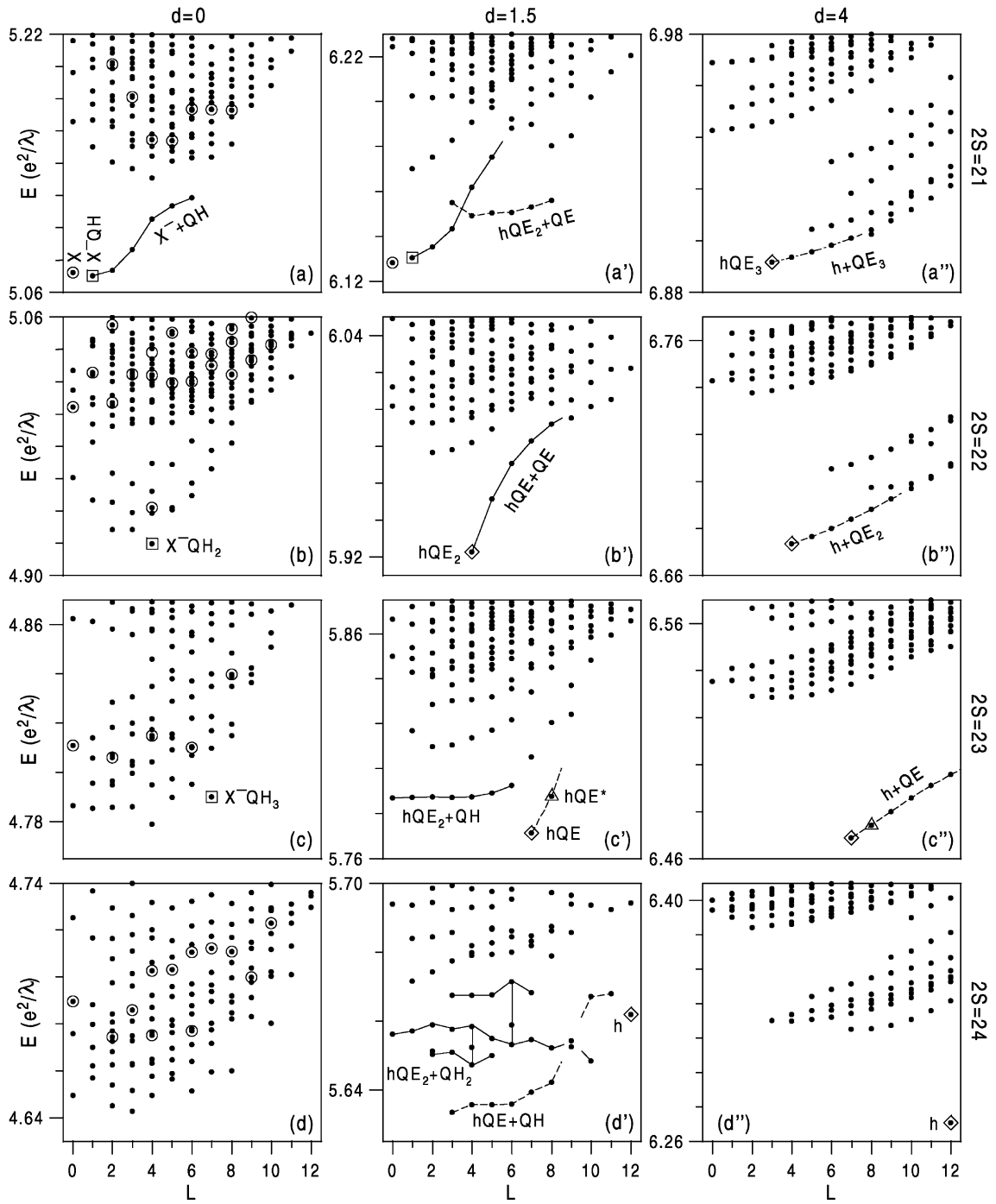


Fig. 1. Energy spectra of the nine-electron-one-hole system for the monopole strength $2S=21, 22, 23, 24$ (from top to bottom), and for the interplane separation $d=0, 1.5, 4$ (from left to right). Lines and open symbols mark the low-energy states containing different bound excitonic complexes.

that appear in the figure contain additional QE–QH pairs.

For the non-multiplicative states we have one X^- and $N_e = N - 2$ remaining electrons. The generalized CF picture [4] allows us to predict the lowest energy band in the spectrum in the following way. The effective monopole strength seen by the electrons is $2S_e^* = 2S - 2(N_e - 1) - 2N_{X^-}$, while that seen by the X^- is $2S_{X^-}^* = 2S - 2N_e$. Here, we have attached to each Fermion (electron and X^-) two fictitious flux quanta and used the mean-field approximation to describe the effective monopole strength seen by each particle (note that a CF does not see its own flux). The angular momentum of the lowest CF electron shell is $l_0^* = S_e^*$, while that of the CF X^- shell is $l_{X^-}^* = S_{X^-}^* - 1$ [11,12]. For the system with $N_e = 7$ and $N_{X^-} = 1$ at $2S = 21, 22, 23$, and 24 , the generalized CF picture leads to: one QH with $l_{QH} = 3.5$ and one X^- with $l_{X^-}^* = 2.5$, giving a band at $1 \leq L \leq 6$ for Fig. 1(a); two QHs with $l_{QH} = 4$ and one X^- with $l_{X^-}^* = 3$ giving $L = 0 \oplus 1 \oplus 2^3 \oplus 3^3 \oplus 4^4 \oplus 5^3 \oplus 6^3 \oplus 7^2 \oplus 8^2 \oplus 9 \oplus 10$ for Fig. 1(b); three QHs with $l_{QH} = 4.5$ and one X^- with $l_{X^-}^* = 3.5$ giving $L = 0 \oplus 1^4 \oplus 2^6 \oplus 3^7 \oplus \dots \oplus 11^3 \oplus 12^2 \oplus 13 \oplus 14$ for Fig. 1(c); and four QHs with $l_{QH} = 5$ plus one X^- with $l_{X^-}^* = 4$ giving $L = 0^3 \oplus 1^6 \oplus \dots \oplus 16^2 \oplus 17 \oplus 18$ for Fig. 1(d). In the figure, we have restricted the values of L and of E , so that not all the states are shown.

2.2. Weak coupling

For $d \gg 1$, the electron–hole interaction is a weak perturbation on the energies obtained for the N -electron system [13–15]. The numerical results can be understood by adding the angular momentum of the hole, $l_h = S$, to the electron angular momenta obtained from the simple CF model. The predictions are: for $2S = 21$ there are three QEs each with $l_{QE} = 3.5$ and the hole has $l_h = 10.5$; for $2S = 22$ two QEs each with $l_{QE} = 4$ and $l_h = 11$; for $2S = 23$ one QE with $l_{QE} = 4.5$ and $l_h = 11.5$; and for $2S = 24$ no QEs and $l_h = 12$. Adding the angular momenta of the identical Fermion QEs gives L_e , the electron angular momenta of the lowest band; adding to L_e the angular momentum l_h gives the set of allowed multiplets appearing in the low-energy sector. For example, in Fig. 1(b'') the allowed values of L_e are $1 \oplus 3 \oplus 5 \oplus 7$, and the multiplets at 7 and 3 have lower energy than those at 1

and 5. Four low-energy bands appear at $4 \leq L \leq 18$, $8 \leq L \leq 14$, $6 \leq L \leq 16$, and $10 \leq L \leq 12$, resulting from $L_e = 7, 3, 5$, and 1, respectively.

2.3. Intermediate coupling

For $d \approx 1$, the electron–hole interaction results in formation of bound states of a hole and one or more QEs. In the two-electron–one-hole system, the X^0 and X^- unbind for $d \approx 1$, but interaction with the surrounding unbound electrons in a larger system can lead to persistence of these excitonic states beyond $d = 1$. For example, the band of states at $d = 0$ in Fig. 1(a) that we associated with an X^- interacting with a QH persists at $d = 1.5$ in Fig. 1(a'). However, it appears to cross another low-energy band that extends from $L = 3$ to 8. This latter band can be interpreted in terms of three QEs interacting with the hole as was done in the weak-coupling limit shown in Fig. 1(a''). The other bands of the weak-coupling regime (those beginning at $L = 5, 6, 7, 8$, and 9) have disappeared into the continuum of higher states as a result of the increase of V_{eh} .

For $2S = 22$, the lowest band can be interpreted in terms of one X^- interacting with two QHs of the generalized CF picture. The X^- has $l_{X^-}^* = 3$ and the QHs each have $l_{QH} = 4$. The allowed values of L_{2QH} are 7, 5, 3, and 1, and the ‘‘molecular’’ state QH_2 which has the smallest average QH–QH distance would have $l_{QH_2} = 7$. This gives a band of $X^- + QH_2$ states going from $L = l_{QH_2} - l_{X^-}^* = 4$ to $l_{QH_2} + l_{X^-}^* = 10$. A higher band might result from the 2QH state at $L_{2QH} = 5$ interacting with the X^- to give $2 \leq L \leq 8$. The lower band beginning at $L = 4$ could also be interpreted as a hole interacting with two QEs of the nine-electron system (each QE having $l_{QE} = 4$). This would produce a band of states with $4 \leq L \leq 18$ (arising from $l_{QE_2} = 7$ and $l_h = 11$). Since the states with $L \geq 8$ merge with the continuum, we cannot determine which of these descriptions are more appropriate for $d \approx 1.5$ based on the energy spectra alone (to do so we must analyze the eigenstates).

For $2S = 23$, there are two low-lying bands. The first contains a hole with $l_h = 11.5$ and a QE with $l_{QE} = 4.5$. This gives rise to a band extending from $L = 7$ to 16. The second band contains an additional QE–QH pair. The cost of energy in creating this additional pair is comparable to the energy gained through the interac-

tion of the additional QE with the hole. The lowest hQE₂ state occurs at $l_{\text{hQE}_2} = l_{\text{h}} - l_{\text{QE}_2} = 3.5$ (this results from choosing $l_{2\text{QE}} = 8$, the largest value from the set of allowed $l_{2\text{QE}} = 8, 6, 4, 2$, and 0) and adding l_{hQE_2} to $l_{\text{QH}} = 3.5$ to obtain a band with $0 \leq L \leq 7$. The state with $L = 7$ is missing, undoubtedly due to the large QE–QH repulsion at $l_{\text{QE–QH}} = 1$ [16].

For $2S = 24$, the ground state at $d = 1.5$ contains one hole with $l_{\text{h}} = 12$ and QE–QH pair with $l_{\text{QE}} = 5$ and $l_{\text{QH}} = 4$. The hole and QE bind giving a set of states with l_{hQE} satisfying $7 = l_{\text{h}} - l_{\text{QE}} \leq l_{\text{hQE}} \leq l_{\text{h}} + l_{\text{QE}} = 17$. The most strongly bound state has $l_{\text{hQE}} = 7$. Adding $l_{\text{hQE}} = 7$ to $l_{\text{QH}} = 4$ gives band $3 \leq L \leq 11$ marked in Fig. 1(d'). This band has lower energy than the Laughlin state of nine electrons and the hole which occurs at $L = l_{\text{h}} = 12$.

3. Photoluminescence

Exact numerical diagonalization gives both the eigenvalues and the eigenfunctions. The low-energy states $|i\rangle$ of the initial N -electron–one-hole system have just been discussed. The final states $|f\rangle$ contain $N' = N - 1$ electrons and no holes. The recombination of an electron–hole pair is proportional to the square of the matrix element of the photoluminescence operator $\hat{\mathcal{L}}$. We have evaluated $|\langle f | \hat{\mathcal{L}} | i \rangle|^2$ for all of the low-lying initial states and have found the following results [17,18]. (i) Conservation of the total angular momentum L is at most weakly violated through the scattering of “spectator” particles (electrons or quasiparticles) which do not participate directly in the recombination process if the filling factor ν is less than or equal to approximately $\frac{1}{3}$. (ii) In the strong-coupling region, the neutral X^0 line is the dominant feature of the PL spectrum. The X^- –QH₂ state has very small oscillator strength for radiative recombination. (iii) For intermediate coupling, the hQE₂ and an excited state of the hQE (which we denote by hQE*) are the only states with large oscillator strength for photoluminescence.

At zero temperature ($T = 0$), all initial states must be ground states of the N -electron–one-hole system. At finite but low temperatures, excited initial states contribute to the PL spectrum. The photoluminescence intensity is proportional to

$$w_{i \rightarrow f} = \frac{2\pi}{\hbar} \mathcal{Z}^{-1} \sum_{i,f} e^{-i\beta E_i} |\langle f | \hat{\mathcal{L}} | i \rangle|^2 \times \delta(E_i - E_f - \hbar\omega), \quad (1)$$

where $\beta^{-1} = k_{\text{B}}T$ and $\mathcal{Z} = \sum_i e^{-\beta E_i}$. It is worth illustrating how the hQE₂ → QH + $\hbar\omega$ process satisfies the $\Delta L = 0$ selection rule. An initial state containing one hole and two QEs of an N -electron system must have $2S = 3(N - 1) - 2 = 3N - 5$. Each QE will have $l_{\text{QE}} = \frac{1}{2}(N - 1)$ and the hole has $l_{\text{h}} = S = \frac{3}{2}N - \frac{5}{2}$. The most strongly bound hQE₂ state has $l_{\text{QE}_2} = 2l_{\text{QE}} - 1 = N - 2$ and $l_{\text{hQE}_2} = l_{\text{h}} - l_{\text{QE}_2} = \frac{1}{2}(N - 1)$. The final state contains $N' = N - 1$ electrons and a single QH with $l_{\text{QH}} = S - (N' - 1) = \frac{1}{2}(N - 1)$. Thus, the initial and final states each have $L = \frac{1}{2}(N - 1)$, so the $\Delta L = 0$ selection rule is satisfied. The same thing can be done for the excited hQE* complex (with L larger by one unit than the ground state of the hQE). It satisfies the $\Delta L = 0$ selection rule, but the ground state hQE does not.

Acknowledgements

The authors would like to acknowledge the support of the Materials Research Program of Basic Energy Sciences, US Department of Energy. A.W. and I.S. acknowledge the partial support from grant 2P03B05518 of the Polish Sci. Comm., and K.-S.Y. acknowledges the support from the Korean Research Foundation.

References

- [1] V.M. Apalkov, E.I. Rashba, Phys. Rev. B 46 (1992) 1628.
- [2] V.M. Apalkov, E.I. Rashba, Phys. Rev. B 48 (1993) 18312.
- [3] A.B. Dzyubenko, Yu.E. Lozovik, Fiz. Tverd. Tela 25 (1983) 1519 [Sov. Phys. Solid State 25 (1983) 874].
- [4] A. Wójs, I. Szlufarska, K.S. Yi, J.J. Quinn, Phys. Rev. B 60 (1999) R11273.
- [5] A. Wójs, J.J. Quinn, P. Hawrylak, Phys. Rev. B 62 (2000) 4630.
- [6] A. Wójs, J.J. Quinn, P. Hawrylak, Physica E 8 (2000) 254.
- [7] I.V. Lerner, Yu.E. Lozovik, Zh. Eksp. Teor. Fiz. 80 (1981) 1488 [Sov. Phys. JETP 53 (1981) 763].
- [8] A.H. MacDonald, E.H. Rezayi, Phys. Rev. B 42 (1990) 3224.
- [9] J.K. Jain, Phys. Rev. Lett. 63 (1989) 199.
- [10] X.M. Chen, J.J. Quinn, Solid State Commun. 92 (1994) 865.
- [11] A. Wójs, P. Hawrylak, J.J. Quinn, Physica B 256–258 (1998) 490.

- [12] A. Wójs, P. Hawrylak, J.J. Quinn, Phys. Rev. B 60 (1999) 11661.
- [13] X.M. Chen, J.J. Quinn, Phys. Rev. Lett. 70 (1993) 2130.
- [14] X.M. Chen, J.J. Quinn, Phys. Rev. B 50 (1994) 2354.
- [15] X.M. Chen, J.J. Quinn, Phys. Rev. B 51 (1995) 5578.
- [16] P. Sitko, S.N. Yi, K.S. Yi, J.J. Quinn, Phys. Rev. Lett. 76 (1996) 3396.
- [17] A. Wójs, J.J. Quinn, Phys. Rev. B 63 (2001) 45303.
- [18] A. Wójs, J.J. Quinn, Phys. Rev. B 63 (2001) 45304.