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Composite fermions and the fractional quantum Hall effect: essential role of the pseudopotential

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Abstract

The mean field (MF) composite fermion (CF) picture successfully predicts the band of low-lying angular momentum multiplets of fractional quantum Hall systems for any value of the magnetic field. This success cannot be attributed to a cancellation between Coulomb and Chern–Simons interactions between fluctuations beyond the mean field. It results instead from the short-range behavior of the Coulomb pseudopotential in the lowest Landau level (LL). The class of pseudopotentials for which the MFCF picture is successful can be defined, and used to explain the success or failure of the picture in different cases (e.g. excited LLs, charged magneto-excitons, and Laughlin uasiparticles in a CF hierarchy picture). © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Composite fermion; Pseudopotential

1. Introduction

The MFCF picture [1,2] does remarkably well in predicting the band of angular momentum (L) multiplets that form the low-energy sector of a 2D electron system in a strong magnetic field B. A Laughlin [3] incompressible $L=0$ ground state of an N electron system occurs when the magnetic monopole (which produces the radial magnetic field at the surface of the Haldane [4] sphere) has strength $2S_m = m(N-1)$, where m in an odd integer. For $2S$ different from $2S_m$ there will be $|2S-2S_m|$ quasiparticles (QPs). This is illustrated in Fig. 1, which displays the energy spectra of ten electrons on a Haldane sphere at monopole strength $25 \leqslant 2S \leqslant 29$. Frame (a) shows the Laughlin incompressible ground state at $L=0$. Frames (b) and (c) show states containing a single quasielectron QE (a) and quasihole QH (b) at $L=5$. In frames (d) and (e) the two QP states form the low-energy bands. In the MFCF picture, the effective monopole strength $2S^*$ is given by $2S^* = 2S - 2p(N-1)$, where p is an integer. S^* is the angular momentum l_0^* of a MF CF in the lowest CF Landau level. At $2S=27$ (with $p=1$), $l_0^* = \frac{9}{2}$ and the lowest shell accommodates $2l_0^*+1=10$ CF's, so that the shell is filled giving $L=0$. At $2S=27\pm1$ there will be one CF QHwith $l_{\mathrm{QH}}=5$ or one CF QE with $l_{\mathrm{QE}}=5$, giving $L=5$. At $2S=27\pm2$ there will be two CF QH each with $l_{\mathrm{QH}}=\frac{11}{2}$ giving $L=0$, 2, 4, 6, 8, 10, or two CF QE each with $l_{\mathrm{QE}}=9/2$ giving $L=0$, 2, 4, 6, 8.

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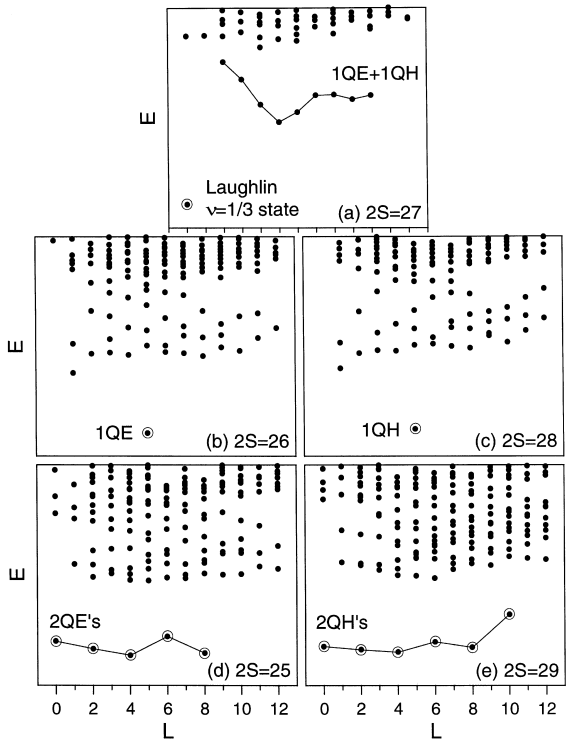

Fig. 1. Energy spectra of ten electrons in the lowest LL at $25 \leqslant 2S \leqslant 29$. Open circles mark lowest energy bands with fewest CF QPs.

It is quite remarkable that the MFCF picture works so well since its energy scale is $\hbar\omega_c^* = (2p+1)^{-1}\hbar\omega_c \propto B$, in contrast to the scale of the Coulomb interaction $e^2/\lambda \propto \sqrt{B}$, where λ is the magnetic length. The energy values obtained in the MFCF picture are totally incorrect, but the structure of the low-energy spectrum (which multiplets form the lowest lying band) is correct. As first suggested by Haldane [5], this is a result of the behavior of the pseudopotential $V(L')$ (interaction energy of a pair of electrons versus pair angular momentum) in the lowest LL.

2. Pseudopotential

In Fig. 2 we plot $V(L')$ versus $L'(L'+1)$ for the lowest ($n=0$) and first excited ($n=1$) LL for different values of $2l$ [6]. Note that for $n=0$ $V(L')$ rises

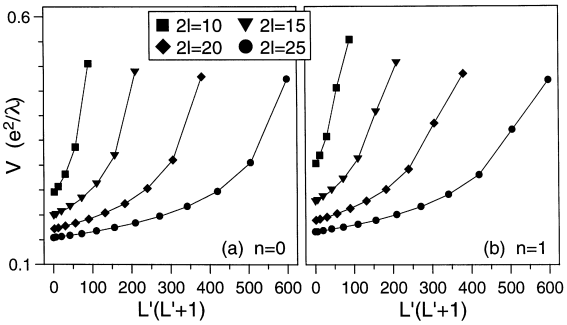

Fig. 2. Pseudopotentials V of the Coulomb interaction in the lowest (a), and first excited LL (b) as a function of squared pair angular momentum $L'(L'+1)$ for different values of $l = S + n$.

more steeply than linearly with increasing L' at all values of L', but for $n=1$ this is not true at the highest allowed values of L'.

A useful operator identity [7] relates the total angular momentum $\hat{L} = \sum_i \hat{l}_i$ to the sum over all pairs of the pair angular momentum $\hat{L}_{ij} = \hat{l}_i + \hat{l}_j$,

$$\sum_{i<j} \hat{L}_{ij}^2 = \hat{L}^2 + N(N-2)\,\hat{l}^2. \tag{1}$$

Here, each fermion has angular momentum l, so that $\hat{l}^2$ has the eigenvalue $l(l+1)$. From Eq. (1) it is not difficult to show that for a "harmonic" pseudopotential defined by $V_H(L') = A + B\,L'(L'+1)$, the energy $E_\alpha(L)$ of the αth multiplet with total angular momentum L would be independent of α, and that $E(L)$ would be of the form $a + b\,L(L+1)$ [7]. Because the actual pseudopotential is different from $V_H(L')$, the degeneracy of the multiplets α of the same L is lifted.

For a pseudopotential (which we will refer to as a short range, SR, potential) that rises more quickly with L' than $V_H(L')$, the lowest energy multiplets must, to the extent that it is possible, avoid having pair amplitude (or coefficient of fractional parentage [8]) from the largest values of L'. For $V_H(L')$ the lowest angular momentum states have the lowest energy. However, the difference $\Delta V(L') = V(L') - V_H(L')$ lifts the degeneracy of multiplets having the same value of L. If some low value of L has a very large number N_L of multiplets, $\Delta V(L')$ can push the lowest multiplet at that L value to a lower energy than any multiplet of a neighboring smaller L value for which N_L is much smaller.

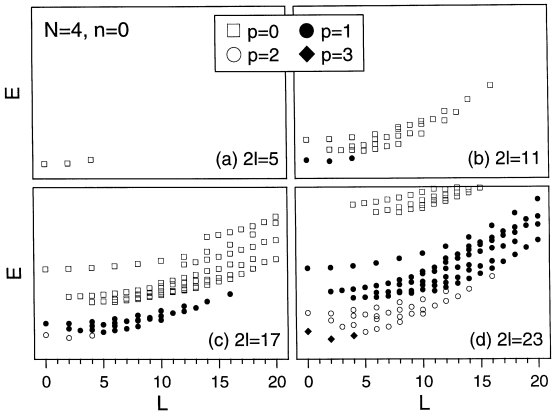

Fig. 3. Energy spectra of four electrons in the lowest LL. Different symbols mark subspaces $\mathscr{H}_p$ for $p = 0$, 1, 2, and 3.

3. Energy spectra of SR pseudopotential

Fig. 3 displays some very informative results [7] for a simple four particle system at different values of the single particle angular momentum l (which differs by $\Delta l = p(N-1)$, $p = 1, 2, \ldots$). Note that the set of multiplets at $l - p(N-1)$ is always the subset of the multiplets at l. The SR pseudopotential appears to have the property that its Hilbert space $\mathscr{H}$ splits into subspaces $\mathscr{H}_p$ containing states with no parentage from pair angular momentum $L' = 2(l-p) - 1$. $\mathscr{H}_0$ is the entire space, $\mathscr{H}_1$ is the subspace that avoids $L'_{\mathrm{MAX}} = 2l - 1$, $\mathscr{H}_2$ avoids $L = 2l - 1$ and $2l - 3$, etc. Since the interaction energy in each subspace $\mathscr{H}_p$ is measured on the scale $V(L' = 2(l-p) - 1)$, the spectrum splits into bands with gaps between bands associated with the differences in appropriate pseudopotential coefficients. The largest gap is always between the zeroth and first band, the next largest between the first and second, etc. Note that the subset of multiplets at $l' = l - p(N-1)$ is exactly the subset chosen by the MFCF picture. In addition, at the Jain values $\nu = n(1 + 2pn)^{-1}$, where $n = 1, 2, \ldots$, there is only a single multiplet at $L = 0$ in the "lowest subset" for an appropriate value of p.

These ideas can be made more formal by using the algebra of angular momentum addition and the "coefficients of fractional parentage" familiar to atomic and nuclear physicists. The conclusions are quite clear. There is really only one energy scale, that of the Coulomb interaction e^2/λ. Laughlin states occur when the fractional parentage for electrons (or holes) allows avoidance of the pseudopotential $V(2(l-p) - 1)$ for $p = 0, 1, \ldots$. Jain states occur when the fractional parentage of the appropriate $V(2(l-p) - 1)$ is much smaller (but not zero) for $L = 0$ than for other allowed multiplets. The MFCF picture works only if $V(L')$ is a SR potential that rises like $[L'(L'+1)]^{\beta}$ with $\beta > 1$ [7].

4. Other pseudopotentials

For the $n = 1$ and higher LL's, $V(L')$ is not SR for all values of L'. For $n = 1$, $V(L')$ is essentially harmonic at $L' = L'_{\mathrm{MAX}}$, and for $n > 1$ it is subharmonic at the largest values of L'. Therefore, even if ground states at filling factors like $\nu = 2 + \frac{1}{3}$ have $L = 0$, they are not Laughlin-type incompressible states which avoid pair angular momentum $L'_{\mathrm{MAX}} = 2l - 1$.

A CF hierarchy scheme was proposed by Sitko et al. [9] in which the CF transformation was reapplied to QPs in partially filled shells. The application of the MFCF approximation was found to work in some cases but not in others. Some idea of when the MFCF approximation is valid can be obtained from looking at the 2QE and 2QH states in Fig. 1. The QH pseudopotential is SR at $L = 10$, but not at $L = 8$. The QE pseudopotential is certainly not SR at $L = 8$, but at $L = 6$ it might be. This suggests that Laughlin states will be formed by QHs of the $\nu = \frac{1}{3}$ state at $\nu_{\mathrm{QH}} = \frac{1}{3}$ and by QEs of the $\nu = \frac{1}{3}$ state at $\nu_{\mathrm{QE}} = 1$, explaining the strong FQHE of the underlying electron system at the Jain $\nu = \frac{2}{7}$ and $\frac{2}{5}$ filling factors. In contrast, no FQHE at $\nu_{\mathrm{QH}} = \frac{1}{5}$ ($\nu = \frac{4}{13}$ electron filling) or $\nu_{\mathrm{QE}} = \frac{1}{3}$ ($\nu = \frac{4}{11}$) would be expected because the QP pseudopotentials are not SR at these values.

A final interesting example is that of a multicomponent plasma of electrons and one or more negatively charged excitonic ions X_k^- (a bound state of k neutral excitons and an electron) formed in an electron–hole system. These excitonic ions are long lived fermions with LL structure [10,11]. The angular momentum of an X_k^- on a Haldane sphere is $l_k = S - k$. The pseudopotentials describing the interactions of X_k^- ions with electrons and with one another can be shown to be SR. In fact, $V_{AB}(L')$, where A or B or both are composite particles has

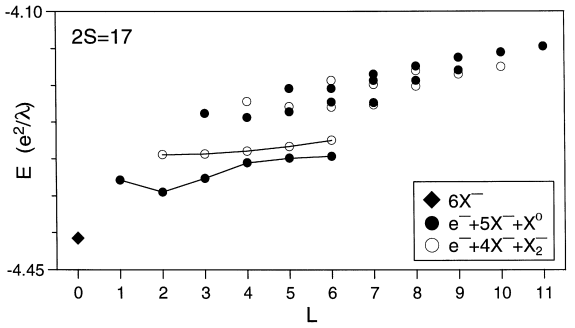

Fig. 4. Low energy spectra of different charge configurations of the 12e + 6h system on a Haldane sphere at $2S = 17$.

a "hard core" for which one or more of the largest values of $V_{AB}(L')$ are effectively infinite.

The following configurations of ions have low energy in the twelve electron–six hole system at $2S = 17$. The $6X^-$ configuration (i) has the maximum total binding energy ε_i. Other expected low lying bands are: (ii) $e^- + 5X^- + X^0$ with ε_{ii} and (iii) $e^- + 4X^- + X_2^-$ with ε_{iii}. Here, $\varepsilon_i > \varepsilon_{ii} > \varepsilon_{iii}$ are all known. Although we are unable to perform an exact diagonalization for this system in terms individual electrons and holes, we can use appropriate pseudopotentials and binding energies to obtain the low-lying states in the spectrum. The results are presented in Fig. 4. There is only one $6X^-$ state (the $L = 0$ Laughlin $\nu_{X^-} = \frac{1}{3}$ state [3]) and two bands of states in each of groupings (ii) and (iii). A gap of $0.0626e^2/\lambda$ separates the $L = 0$ ground state from the lowest excited state.

5. Generalized composite fermion picture

In order to understand the numerical results obtained in Fig. 4, it is useful to introduce a generalized CF picture by attaching to each particle fictitious flux tubes carrying an integral number of flux quanta ϕ_0. In the multi-component system [12], each a-particle carries flux $(m_{aa} - 1)\phi_0$ that couples only to charges on all other a-particles and fluxes $m_{ab}\phi_0$ that couple only to charges on all b-particles, where a and b are any of the types of fermions. The effective monopole strength seen by a CF of type a (CF-a) is $2S_a^* = 2S - \sum_b (m_{ab} - \delta_{ab})(N_b - \delta_{ab})$. For different multi-component systems we expect generalized Laughlin incompressible states when all the hard core pseudopotentials are avoided and CF's of each kind fill completely an integral number of their CF shells (e.g. $N_a = 2l_a^* + 1$ for the lowest shell). In other cases, the low-lying multiplets are expected to contain different kinds of quasiparticles (QP-A, QP-B, ...) or quasiholes (QH-A, QH-B, ...) in neighboring filled shells. Our multi-component CF picture can be applied to the 12e + 6h spectrum given in Fig. 4. The agreement is really quite remarkable and strongly indicates that our multi-component CF picture is correct.

In this work we have emphasized that the success of the MFCF picture is critically dependent on the nature of the pseudopotential. We have presented several examples of SR pseudopotentials for which the CF picture works well, and several subharmonic pseudopotentials for which it does not.

Acknowledgements

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