Composite fermion picture for the second generation of fractional quantum Hall liquids

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Abstract. A flux attachment transformation is derived for correlated composite fermions in their partially filled second Landau level. The transformation generates a hierarchy of "second generation" incompressible quantum liquids at electron filling factors $1/3 < \nu_e < 2/5$. The hierarchy stems from a zero-energy ground state of an appropriate model pseudopotential and it includes the entire family of quantum Hall states observed in experiment.

Keywords: quantum Hall effect, composite fermions, nonabelian statistics
PACS: 73.43.-f, 71.10.Pm, 05.30.-d

INTRODUCTION

In the "second generation" (G2) of fractional quantum Hall (FQH) states [1], composite fermions (CFs) [2] form incompressible liquids in their partially filled second Landau level (LL). Due to the specific form of CF–CF "residual" interaction [3], correlations in partially filled CF LLs are different [4] from correlations known in the LLs of electrons (e.g., from Laughlin or Moore–Read correlations). Therefore, flux attachment procedure converting a correlated CF liquid into filled shells of a new kind of hypothetical, less strongly interacting quasiparticles must differ from the original Jain's transformation [2] used to define the CFs from the electrons.

Such "G2 transformation" attaching flux to the CFs is derived here. It naturally generates a series of incompressible CF liquids at the correct (experimentally observed) electron LL filling factors $\nu_e = 3/8$ and $4/11$, and predicts a slightly weaker G2-FQH state at $\nu = 9/25$. Moreover, it correctly predicts the "shift" function $\gamma(\nu)$ relating LL degeneracy g to the particle number N in finite systems on a sphere, $g - 1 = \nu^{-1}N - \gamma(\nu)$, known to depend on the particular form of correlation responsible for the emergence of incompressibility at a given ν.

MODEL

In order to identify incompressible CF liquids we perform exact diagonalization for N interacting fermions of charge q confined to a Haldane sphere of unit radius [5]. The magnetic monopole of strength $2Q$ (i.e., flux $4\pi B = 2Q\phi_q$, where $\phi_q = hc/q$ is the flux quantum) produces radial field B yielding magnetic length scale $\lambda \equiv \sqrt{\hbar c/qB} = Q^{-1/2}$ at the surface. The s-th LL is a multiplet of single-particle angular momentum $l = Q + s$.

Interaction among CFs in their second LL is described by a pseudopotential (dependence of pair interaction energy V on relative angular momentum $\mathscr{R} = 1, 3, \dots$ [6]) that is dominated by a single largest coefficient at $\mathscr{R} = 3$ [3]. This is very different from repulsion in the lowest electron LL (strong maximum at $\mathscr{R} = 1$, yielding Laughlin correlations [7]), and also different from the behavior in higher electron LLs, in which always $V(1) > V(3)$.

RESULTS AND DISCUSSION

The G1-FQH states are predicted by Jain's CF theory [2], in which correlated electrons convert into nearly free CFs by binding some of the external magnetic field B. Flux $2\phi_e$ pointing opposite to B is attached to each electron, leaving a reduced effective field $B_{\rm CF} = B - 2\rho\phi_e$ (ρ being 2D concentration) seen by the CFs and corresponding to an increased effective filling factor $\nu_{\rm CF} = (\nu_e^{-1} - 2)^{-1}$. Electrons at ν_e become incompressible when CFs fill a number of shells, i.e., at $\nu_{\rm CF}$ equal to an integer.

The G2-FQH states [1] correspond to $1 < \nu_{\rm CF} < 2$. The strongest states $\nu_e = 4/11$ and $3/8$ have $\nu_{\rm CF} = 4/3$ and $3/2$, i.e., $\nu = 1/3$ and $1/2$ partial fillings of the second CF LL. For description of correlated CFs, an intuitive model analogous to Jain's flux attachment would be useful. Hence, we seek conversion of incompressible many-CF states to filled shells of (essentially) noninteracting hypothetical fermions to be called "G2-CFs" or CF*'s.

First we identify the maximum-density zero-energy ($E = 0$), zero-angular-momentum ($L = 0$) ground state of a model pseudopotential $V(\mathscr{R}) = \delta_3(\mathscr{R}) \equiv \delta_{\mathscr{R},3}$ which captures the essence of the actual CF–CF interaction. From exact diagonalization of $N \le 10$ fermions interacting through $V = \delta_3$ in LLs with different l we find such $E = L = 0$ series at $2l = 5N - 9$, extrapolating to

CP1199, *29th International Conference on the Physics of Semiconductors*, edited by M. J. Caldas and N. Studart

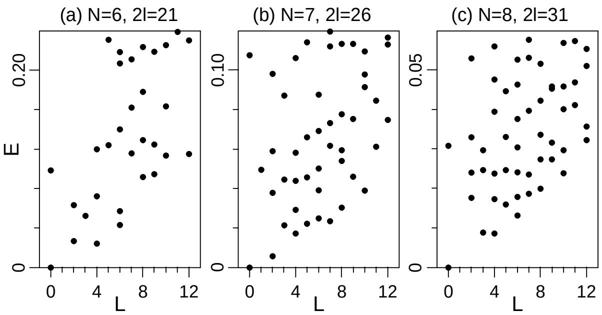

FIGURE 1. Energy spectra (energy E vs. angular momentum L) of $N \leq$ particles with model interaction $V = \delta_3$, calculated on a sphere with shell angular momentum $l = (5N-9)/2$.

$N/2l \to \nu = 1/5$ in large systems. The energy spectra for $N \leq 8$ are shown in Fig. 1. CF pairing in these ground states is evident from Haldane amplitudes (not shown).

Conversion from $2l = 5N-9$ to $2l^* = N-1$ of a filled CF* shell is achieved by the following transformation

$$2l^* = 2l - 4(N-2). \tag{1}$$

Attributing degeneracy of the CF-LL to fictitious magnetic flux, Eq. (1) can be interpreted as attachment of $p = 4$ flux quanta to each CF. The factor $(N-2)$ suggests that each CF* sees an average flux from all but one other CFs, which simply reflects the CF pairing.

Transformation (1) can be naturally extended to

$$2l^* = |2l - p(N-2)|. \tag{2}$$

with an arbitrary number p of flux quanta attached to each CF (odd values of p must also be admitted due to pairing: path of a given particle can only encircle a whole other pair). Let us consider an arbitrary number $|n|$ of completely filled CF* shells, with the effective magnetic field pointing either in the same or in opposite direction to the fictitious external field giving rise to the degeneracy of the CF LL. The latter case, corresponding to $2l < p(N-2)$, will be conveniently distinguished by a negative sign of n.

The filling of CF* shells yields a family of CF states at

$$2l = (p + n^{-1})N - (2p + n), \tag{3}$$

extrapolating to $\nu \equiv \lim(N/2l) = (p + n^{-1})^{-1}$ on a plane (some fractions ν result for two combinations of p, n). By construction, Eq. (3) includes the $(p,n) = (4,1)$ zero-energy state at $2l = 5N-9$. Remarkably, the $\nu = 1/3$ state at $2l = 3N-7$ also emerges as $(4,-1)$, while $(1,1)$ reproduces the familiar $\nu = 1/2$ series at $2l = 2N-3$.

To check which of the (p,n) states of Eq. (3) actually occur for the interacting CFs, we computed ground state energies as a function of N and $2l$. Results are shown in Fig. 2. The largest excitation gaps Δ occur for $(p,n) =$ $(4,-1)$ and $(1,1)$, corresponding to the known [1] FQH states at $\nu_e = 4/11$ and $3/8$. Sizeable gap is also found for $(4,-2)$, suggesting a new FQH state at $\nu_e = 9/25$. Other states show only marginal incompressibility.

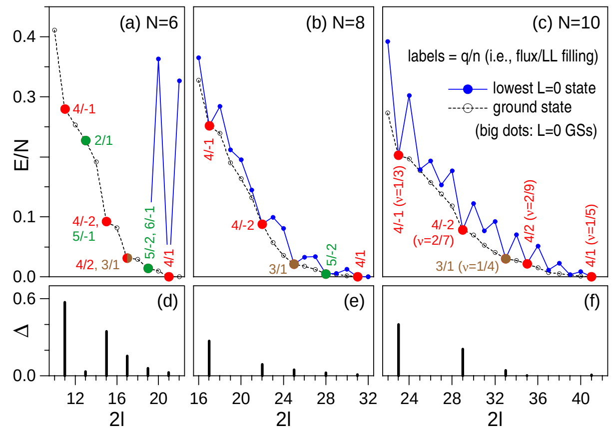

FIGURE 2. Top: ground state energy per particle E/N (also, lowest energy at $L = 0$) of $N \leq 10$ particles with model interaction $V = \delta_3$, on a sphere, as a function of shell angular momentum l. Bottom: excitation gaps of $L = 0$ ground states.

CONCLUSION

We have used numerical calculations on a sphere to identify the nondegenerate zero-energy ground state of a model pseudopotential describing CF–CF interaction in their second LL. This ground state was converted to a filled shell of next-generation CFs by a flux attachment procedure which is different from the one applicable to electrons in their lowest LL. This procedure was then used to generate the entire hierarchy of second-generation FQH states at $1/3 < \nu_e < 2/5$, in good agreement with experiments and calculations.

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