Composite fermion picture for multi-component plasmas in 2D electron–hole systems in a strong magnetic field

A. Wójs[a,b,*], I. Szlufarska[a,b], K.-S. Yi[a,c], P. Hawrylak[d], J.J. Quinn[a]

[a] *Department of Physics, University of Tennessee, Knoxville, TN 37996-1501, USA*
[b] *Institute of Physics, Wroclaw University of Technology, Wroclaw 50-370, Poland*
[c] *Physics Department, Pusan National University, Pusan 609-735, South Korea*
[d] *Institute for Microstructural Sciences, National Research Council, Ottawa, Canada K1A 0R6*

Abstract

Low-lying states of a 2D electron–hole system contain electrons and one or more types of charged excitonic complexes. Binding energies and angular momenta of these excitonic ions, and the pseudopotentials describing their interactions with electrons and with one another are obtained from numerical studies of small systems. Incompressible fluid ground states of such multi-component plasmas are found in exact numerical diagonalizations. A generalized composite fermion (CF) picture involving Chern–Simons charges and fluxes of different types is proposed and shown to predict the low-lying states at any value of the magnetic field. © 2000 Published by Elsevier Science B.V. All rights reserved.

1. Introduction

In a 2D electron–hole system in a strong magnetic field, the only bound complexes are neutral excitons X^0 and spin-polarized charged excitonic ions X_k^- (k excitons bound to an electron) [1–6]. Other complexes found at lower fields [7,8] unbind due to a hidden symmetry [9]. The X_k^- ions are long lived fermions whose energy spectra contain Landau level structure [3–6]. By numerical diagonalization of small systems we can determine binding energies and angular momenta of the excitonic ions, and pseudopotentials which describe their interactions with electrons and with one another [5,6]. We show that a gas of X_k^-'s can form Laughlin [11] incompressible fluid states [5,6], but only for filling factors $v_k \leqslant (2k + 1)^{-1}$ (in the following, subscript k denotes X_k^-). Multi-component plasmas containing electrons and X_k^- ions of one or more different types can also form incompressible fluid states. A generalized composite fermion (CF) picture [12] is proposed to describe such a plasma [13]. It requires the introduction of Chern–Simons [14] charges and fluxes of different types (colors) in order to mimic generalized Laughlin-type correlations [15].

* Corresponding author. Tel.: +1-423-974-0771; fax: +1-423-974-6378.
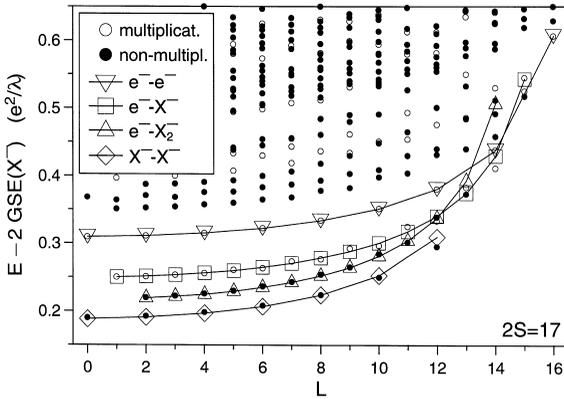*E-mail address:* awojs@utk.edu (A. Wójs)

Fig. 1. Energy spectrum of the four electron and two hole system at $2S = 17$.

Fig. 2. Energy spectra of the six electron and three hole system at $2S = 8$ and 11.

The predictions of this CF picture agree well with numerical results for systems containing up to 18 particles.

2. Four electron–two hole system

Understanding of the energy spectrum of this simple system is essential for our considerations. Result of the numerical diagonalization in Haldane spherical geometry [16,17], for the magnetic monopole strength $2S = 17$, is shown in Fig. 1. Open and solid circles mark multiplicative and non-multiplicative states [9,10], respectively. For $L < 12$ there are four low-lying bands, which we have identified, in order of increasing energy, as two X^-s, an electron and an X_2^-, an electron and an X^- and a decoupled X^0, and finally two electrons and two decoupled X^0s. We find that the X_k^- has an angular momentum $l_k = S - k$ in contrast to an electron which has $l_0 = S$. All relevant binding energies and pseudopotentials are also determined. An important observation is that the pseudopotential of composite particles ($k > 0$) is effectively infinite (hard core) if L exceeds a particular value. This is due to unbinding of ions at too small separation. Once the maximum allowed Ls for all pairings are established, the four bands in Fig. 1 can be approximated by the pseudopotentials of electrons (point charges) with angular momenta l_A and l_B, shifted by the appropriate binding energies (large symbols).
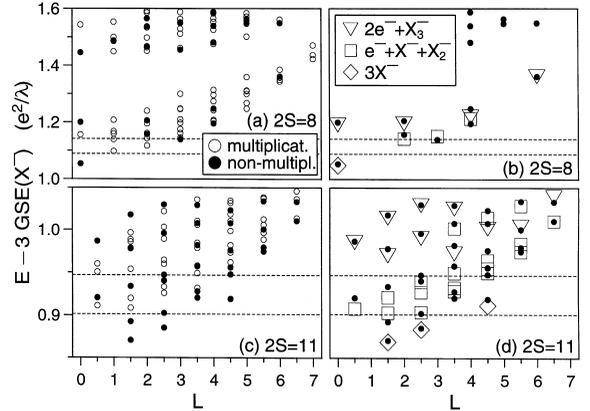
3. Larger systems

We know from exact calculations for up to 11 electrons [18–20] that the CF picture correctly predicts the low-lying states of the fractional quantum Hall systems. The reason for this success is [18–20] the ability of the electrons in states of low L to avoid large fractional parentage (FP) [18–20] from pair states associated with large values of the Coulomb pseudopotential. In particular, for the Laughlin $v_0 = \frac{1}{3}$ state, the FP from pair states with maximum pair angular momentum $L = 2l_0 - 1$ vanishes. We hypothesize that the same effect should occur for an X^- system when $l_0 = S$ is replaced by $l_1 = S - 1$. We define an effective X^- filling factor as $v_1(N, S) = v_0(N, S - 1)$ and expect the incompressible X^- states at all Laughlin and Jain fractions for $v_1 \leqslant \frac{1}{3}$. States with $v_1 > \frac{1}{3}$ cannot be constructed because they would have some FP from pair states forbidden by the hard core repulsion [5,6].

Fig. 2 shows energy spectra of the 6e + 3h system at $2S = 8$ and 11. Both multiplicative (open circles) and non-multiplicative (solid circles) states are shown in frames (a) and (c). In frames (b) and (d) only the non-multiplicative states are plotted, together with the approximate spectra (large symbols) obtained by diagonalizing the system of three ions with the actual pseudopotentials appropriate to the three possible charge configurations: $3\,X^-$ (diamonds), $e^- + X^- + X_2^-$ (squares), and $2e^- + X_3^-$ (triangles).

Good agreement between the exact and approximate spectra in Figs. 2b and d allows identification of the three ion states and confirms our conjecture about incompressible states of a X^- gas. States corresponding to different charge configurations form bands At low L, the bands are separated by gaps, predominantly due to different total binding energies of different configurations. The lowest state in each band corresponds to the three ions moving as far from each other as possible. If the ion–ion repulsion energies were equal for all configurations (a good approximation for dilute systems), the two higher bands would lie above dashed lines, marking the ground state energy plus the appropriate difference in binding energies. The low-lying multiplicative states can also be identified as $3e^- + 3X^0$, $2e^- + X^- + 2X^0$, $2e^- + X_2^- + X^0$, and $e^- + 2X^- + X^0$. The bands of three ion states are separated by a rather large gap from all other states, which involve excitation and breakup of composite particles.

The largest systems for which we performed exact calculations are the 6e + 3h and 8e + 4h systems at $2S$ up to 12 (Laughlin $v_1 = \frac{1}{5}$ state of three X^-s and one quasi-X^--hole in the $v_1 = \frac{1}{3}$ state of four X^-s). In each case the CF picture applied to the X^- particles works well. For larger systems the exact diagonalization becomes difficult. For example, for the 12e + 6h system we expect the $v_1 = \frac{1}{3}, \frac{2}{7}, \frac{2}{9}$, and $\frac{1}{5}$ incompressible states to occur at $2S = 17, 21, 23$, and 27, respectively. We managed to extrapolate the involved pseudopotentials making use of their regular dependence on $2S$, and use them, together with the binding energies, to determine approximate low-lying bands in the energy spectra, as shown in Fig. 3. At $2S = 17$, the only state of the $6X^-$ configuration is the $L = 0$ ground state (filled circle); other $6X^-$ states are forbidden by the hard core. The low-lying states of other low-energy configurations, $e^- + 5X^- + X^0$ (open circles) and $e^- + 4X^- + X_2^-$ (open squares) are separated from the $6X^-$ ground state by a gap. At $2S = 21$, 23, and 27, all features predicted by the CF picture occur for the $6X^-$ states.

4. Generalized composite fermion picture

In order to understand all of the numerical results presented in Fig. 3a, we introduce a generalized CF picture by attaching to each particle fictitious flux
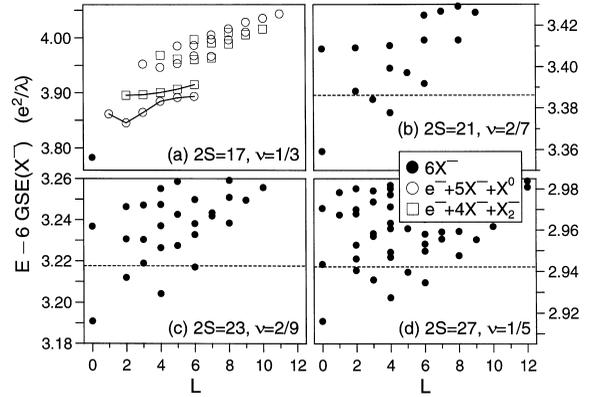

Fig. 3. Low-energy spectra of different charge configurations of the 12 electron and six hole system at $2S = 17$, 21, 23, and 27.

tubes carrying an integral number of flux quanta ϕ_0. In the multi-component system, each a-particle carries flux $(m_{aa} - 1)\phi_0$ that couples only to charges on all other a-particles and fluxes $m_{ab}\phi_0$ that couple only to charges on all b-particles, where a and b are any of the types of fermions. The effective monopole strength [12,18–20] seen by a CF of type a (CF-a) is $2S_a^* = 2S - \sum_b (m_{ab} - \delta_{ab})(N_b - \delta_{ab})$. For different multi-component systems we expect generalized Laughlin incompressible states when all the hard cores are avoided and CFs of each type fill completely an integral number of their CF shells. In other cases, the low-lying multiplets will contain different types of quasiparticles (QP-a, QP-b, ...) or quasiholes (QH-a, QH-b, ...) in the neighboring incompressible state.

Our multi-component CF picture can be applied to the system of excitonic ions, where the CF angular momenta are given by $l_k^* = |S_k^*| - k$. As an example, let us consider Fig. 3a and make the following CF predictions. For six X^-s we obtain the Laughlin $v_1 = \frac{1}{3}$ state at $L = 0$. Because of the X^--X^- hard core, it is the only state of this configuration. For the $e^- + 5X^- + X^0$ configuration we set $m_{11} = 3$ and $m_{01} = 1$, 2, and 3. For $m_{01} = 1$ we obtain $L = 1, 2, 3^2, 4^2, 5^3, 6^3, 7^3, 8^2, 9^2, 10$, and 11; for $m_{01} = 2$ we obtain $L = 1, 2, 3, 4, 5$, and 6; and for $m_{01} = 3$ we obtain $L = 1$. For the $e^- + 4X^- + X_2^-$ configuration we set $m_{11} = 3$, $m_{02} = 1$, $m_{12} = 3$, and $m_{01} = 1, 2$, or 3. For $m_{01} = 1$ we obtain $L = 2, 3, 4^2, 5^2, 6^3, 7^2, 8^2, 9$, and 10; for $m_{01} = 2$ we obtain $L = 2, 3, 4, 5$, and 6; and for $m_{01} = 3$ we obtain $L = 2$. Note that the sets of multiplets obtained for

higher values of m_{01} are subsets of the sets obtained for lower values; we would expect them to form lower energy bands since they avoid additional large values of $e^- - X^-$ pseudopotential. As marked with lines in Fig. 3a, this is indeed true for the states predicted for $m_{01} = 2$. However, the states predicted for $m_{01} = 3$ do not form separate bands. This is because $e^- - X^-$ pseudopotential increases more slowly than linearly as a function of $L(L + 1)$ in the vicinity of $L = l_0 + l_1 - m_{01}$; in such case the CF picture fails [18–20].

The agreement of our CF predictions with the exact spectra of different systems, as in Figs. 2 and 3, is really quite remarkable and strongly indicates that our multi-component CF picture is correct. We are actually able to confirm predicted Laughlin-type correlations [15] in the low-lying states by calculating their FP coefficients [18–20]. In view of the results obtained for many different systems that we were able to treat numerically, we conclude that if exponents m_{ab} are chosen correctly, the CF picture works well in all cases.

5. Summary

Low-lying states of electron–hole systems in a strong magnetic field contain charged excitonic ions X_k^- interacting with one another and with electrons. For different combinations of ions occurring at low-energy, we introduced general Laughlin-type correlations into the wave functions and demonstrated formation of incompressible fluid states of such multi-component plasmas at particular values of the magnetic field. We also proposed a generalized multi-component CF picture and successfully predicted lowest bands of multiplets for various charge configurations at any value of the magnetic field. It is noteworthy that the fictitious Chern–Simons fluxes and charges of different types or colors are needed in the generalized CF model. This strongly suggests that the effective magnetic field seen by the CFs does not physically exist and that the CF picture should be regarded as a mathematical convenience rather than physical reality. Our model also suggests an explanation of some perplexing observations found in photoluminescence, but this topic will be addressed in a separate publication.

References

[1] A.J. Shields, M. Pepper, M.Y. Simmons, D.A. Ritchie, Phys. Rev. B 52 (1995) 7841.
[2] G. Finkelstein, H. Shtrikman, I. Bar-Joseph, Phys. Rev. B 53 (1996) 1709.
[3] A. Wójs, P. Hawrylak, Phys. Rev. B 51 (10) (1995) 880.
[4] J.J. Palacios, D. Yoshioka, A.H. MacDonald, Phys. Rev. B 54 (1996) 2296.
[5] A. Wójs, P. Hawrylak, J.J. Quinn, Physica B 256–258 (1998) 490.
[6] A. Wójs, P. Hawrylak, J.J. Quinn, Phys. Rev. Lett. (cond-mat/9810082), submitted for publication.
[7] K. Kheng, R.T. Cox, Y. Merle d'Aubigne, F. Bassani, K. Saminadayar, S. Tatarenko, Phys. Rev. Lett. 71 (1993) 1752.
[8] H. Buhmann, L. Mansouri, J. Wang, P.H. Beton, N. Mori, M. Heini, M. Potemski, Phys. Rev. B 51 (1995) 7969.
[9] I.V. Lerner, Yu.E. Lozovik, Sov. Phys. JETP 53 (1981) 763.
[10] A.H. MacDonald, E.H. Rezayi, Phys. Rev. B 42 (1990) 3224.
[11] R.B. Laughlin, Phys. Rev. Lett. 50 (1983) 1395.
[12] J.K. Jain, Phys. Rev. Lett. 63 (1989) 199.
[13] A. Wójs, I. Szlufarska, K.-S. Yi, J.J. Quinn, Phys. Rev. Lett. (cond-mat/9904395), submitted for publication.
[14] A. Lopez, E. Fradkin, Phys. Rev. B 44 (1991) 5246.
[15] B.I. Halperin, Helv. Phys. Acta 56 (1983) 75.
[16] F.D.M. Haldane, Phys. Rev. Lett. 51 (1983) 605.
[17] T.T. Wu, C.N. Yang, Nucl. Phys. B 107 (1976) 365.
[18] A. Wójs, J.J. Quinn, Solid State Commun. 108 (1998) 493.
[19] A. Wójs, J.J. Quinn, Solid State Commun. 110 (1999) 45.
[20] A. Wójs, J.J. Quinn, Phys. Rev. B (cond-mat/9903145), submitted for publication.