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Collective excitations in a 2D electron system: Canted field geometry

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Abstract

We investigate the charge and spin collective modes induced in a 2D electron gas by a weak electromagnetic perturbation in the presence of a dc magnetic field which makes an angle θ with the electron layer. The excitation frequencies are determined within the framework of the Landau–Silin theory of Fermi liquids by solving a semi-classical equation for transport in the self-consistent electromagnetic field associated with particle density fluctuations. The quasiparticle interaction is spin dependent and varies parametrically with the degree of spin polarization. In the long wavelength limit, we obtain analytic results for the frequencies of the collective modes as functions of θ.

Keywords: Collective excitations; Spin polarization

In a 2D electron gas – n electrons per unit area in the $\hat{x}-\hat{z}$ plane imbedded in a positive background – coherent propagation of spin and charge density waves occurs at certain values of the frequency ω and wave-vector $\boldsymbol{k}$, for which the resonance of the response functions to an electromagnetic perturbation is realized. We investigate the existence of these collective modes in the case when a dc magnetic field is applied at a small angle, θ, to the electron layer. In equilibrium, a strong magnetic field, $\boldsymbol{B}$, creates a difference in the number, n_σ, of electronic spins parallel to $\boldsymbol{B}$ and $n_{\bar{\sigma}}$, the number of antiparallel spins. The degree of spin polarization, $\zeta=(n_\sigma-n_{\bar{\sigma}})/(n_\sigma-n_{\bar{\sigma}})$ is a continuous function of $\boldsymbol{B}$ and takes on any value between -1 and 1. The orthogonal component, $B_y=B\sin\theta\sim B\theta$, drives a weak cyclotron motion of the electrons, with a frequency, $\omega_c^*=eB\theta/m^*c$, dependent on the electron effective mass, m^* (m^* includes the band structure effects).

The electron gas behaves essentially like a Fermi liquid, and in many instances a phenomenological description based on the Landau–Silin theory of Fermi liquids predicted correctly its properties [1]. A semi-classical approach, which doesn't consider the Landau quantization of the electron orbits in the magnetic field, is possible when the Zeeman spin-splitting energy, γ^*B, is much larger than ω_c^*.

The elementary excitations of an interacting electron system are quasiparticles of momentum $\boldsymbol{k}$ and spin $\boldsymbol{\sigma}$. The quasiparticles are described by the deviation $\delta n_{k\sigma}$ from thermal equilibrium. The thermal equilibrium distribution function arises from the non-interacting ground state (consisting of two Fermi discs of radii, $k_{F\sigma}=\sqrt{4\pi n_\sigma}$) by

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adiabatically turning on the electron-electron interaction. The interaction between quasiparticles is described, in a most general way, by $\Phi_{k\sigma;k'\sigma'} = \phi_{k;k'} + (\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')\psi_{k,k'}$. Both $\phi_{k;k'}$ and $\psi_{k,k'}$ depend parametricaly on ζ. In the presence of a dc magnetic field which polarizes the electron gas at an angle θ, δn_k^0 is written, in a notation employing the usual Pauli spin matrices, as

$$\delta n_k^0 = \begin{pmatrix} \delta n_{k\sigma}^0 & -i\frac{\theta}{2}(\delta n_{k\sigma}^0 - \delta n_{k\bar{\sigma}}^0) \\ i\frac{\theta}{2}(\delta n_{k\sigma}^0 - \delta n_{k\bar{\sigma}}^0) & \delta n_{k\bar{\sigma}}^0 \end{pmatrix}. \quad (1)$$

The interaction of the electron gas with a weak electromagnetic perturbation creates new quasiparticles and consequently, induces charge and spin fluctuations, described by a new distribution, δn_k. In a matrix notation, the diagonal terms of δn_k correspond to the charge and longitudinal (parallel to $\hat{z}$) spin response, while the off-diagonal elements represent spin-flip processes which generate the magnetization along the $\hat{x}$ and $\hat{y}$ axes. The dynamics of these fluctuations are determined, in a semi-classical approximation, by the solution of a transport equation [2]. In the vicinity of a Fermi surface, where quasiparticles are well defined [3], the derivative of the equilibrium distribution function in respect with the quasiparticle energy, ϵ_k, behaves like a delta function. We introduce a new function, v_{kl}, to write a solution to the transport equation as $\delta n_k = v_k(-\mathrm{d}\delta n_k^0/\mathrm{d}\epsilon_k)$. Because of the interaction, the equations for different $\boldsymbol{k}$ are coupled. It is then preferable to solve for the Fourier components, indexed by an integer, l, of v_k, which is considered a periodic function of φ, the angle made by the quasiparticle momentum with the $\hat{z}$ axis. For a sinusoidal variation of the electromagnetic perturbation, $\sim \mathrm{e}^{\mathrm{i}(\omega t - qx)}$, v_l satisfies

$$(-i\omega + il\omega^*\alpha_l)v_l + il\omega^*\beta_l v_l - \frac{qv_\mathrm{F}}{2} \times [\alpha_{(l-1)}v_{(l-1)} + \beta_{(l-1)}v_{(l-1)} - \alpha_{(l+1)}v_{(l+1)} - \beta_{(l+1)}v_{(l+1)}] = \frac{ev_\mathrm{F}}{2}(E_-\delta_{l,1} + E_+\delta_{l,-1}). \quad (2)$$

$\alpha_l = 1 + (m^*/2\pi\hbar^2)\int_0^{2\pi} \mathrm{d}\varphi \mathrm{e}^{-\mathrm{i}l\varphi}(\phi+\psi)_{k_{\mathrm{F}\sigma};k_{\mathrm{F}\sigma}}$ is generated by the spin-symmetric part of the interaction, whereas $\beta_l = (m_\sigma^*/2\pi\hbar^2)\int_0^{2\pi} \mathrm{d}\varphi \mathrm{e}^{-\mathrm{i}l\varphi}(\phi - \psi)_{k_{\mathrm{F}\sigma};k_{\mathrm{F}\bar{\sigma}}}$ is the Fourier coefficient of the spin-antisymmetric interaction. $E_\pm = E_x \pm iE_y$ is the local electric field associated with the charge and spin density fluctuations. It is related to the electric current, $\boldsymbol{j}$, through Maxwell's equations, which for the chosen geometry lead to $i\omega(E_x qc^2/2\pi\omega^2, -E_y\epsilon_0/2\pi q) = \boldsymbol{j}$ [4]. The electric current is just the sum of all momenta of the bare electrons weighted by the deviation from equilibrium of the quasiparticle distribution function, $\boldsymbol{j} = \sum_k(\hbar\,\boldsymbol{k}/m^*)\delta n_k$.

The condition of self-consistent oscillations is equivalent to finding those values of the frequency ω for which the determinant of the homogeneous system satisfied by v_l, developed from Eq. (2), is zero. An important simplification occurs in the long wavelength limit, when qv_F is much smaller than ω_c^*. Then, up to terms quadratic in $qv_\mathrm{F}/\omega_\mathrm{c}^*$, the various excitations are linearly independent, and analytic results can be obtained for $\omega(q)$. Also, the coupling induced by the external dc magnetic field is considered up to terms proportional to θ^2.

The modes that propagate in the system under the effect of the self-consistent electric field are magnetoplasma oscillations which correspond to $|l| \leqslant 1$. If $\omega_{p\sigma} = 2\pi n_\sigma e^2 q/\epsilon_s m^*$ is the plasma frequency of a 2D electron system of spin σ, in a dielectric medium of permitivity ϵ_s, the low frequency collective excitation is

$$\omega_-^2(q) = \frac{\theta^2}{2}\left[\omega_{p\sigma}^2(\alpha_{1\sigma} - \beta_{1\sigma})(1 - \sqrt{n_{\bar{\sigma}}/n_\sigma}) + \omega_{p\bar{\sigma}}^2(\alpha_{1\bar{\sigma}} - \beta_{1\bar{\sigma}})(1 - \sqrt{n_\sigma/n_{\bar{\sigma}}})\right] + q^2 v_{f\sigma}v_{f\bar{\sigma}}(\alpha_{1\sigma}\alpha_{1\bar{\sigma}} - \beta_{1\sigma}\beta_{1\bar{\sigma}}) \times \frac{\left[(\alpha_{0\sigma} - \sqrt{\frac{n_\sigma}{n_{\bar{\sigma}}}}\beta_{0\sigma}) + \frac{m_\sigma^*}{m_{\bar{\sigma}}^*}(\alpha_{0\bar{\sigma}} - \sqrt{\frac{n_{\bar{\sigma}}}{n_\sigma}}\beta_{0\bar{\sigma}})\right]}{\left[(\sqrt{\frac{n_\sigma}{n_{\bar{\sigma}}}}\alpha_{1\sigma} - \beta_{1\sigma}) + \frac{m_\sigma^*}{m_{\bar{\sigma}}^*}(\sqrt{\frac{n_{\bar{\sigma}}}{n_\sigma}}\alpha_{1\bar{\sigma}} - \beta_{1\bar{\sigma}})\right]}. \quad (3)$$

The first term of Eq. (3) originates in the spin-flip processes along the direction of the dc magnetic field $\boldsymbol{B}$, which generate contributions to the magnetization along the z axis, proportional to θ^2. The second term is a spin wave, $\omega_- \sim q^2$, driven by the $l=0$ and $l=1$ Fourier components of the spin-antisymmetric part of the quasiparticle interaction, weighted by the ratio of the spin population.

The high frequency solution is a superposition of two magnetoplasmons, $\omega_+^2(q) = \bar{\omega}_\sigma^2 + \bar{\omega}_{\bar{\sigma}}^2$, with $\bar{\omega}_\sigma$, the plasma frequency for an electron gas of spin σ modified by the quasiparticle interaction

$$\bar{\omega}_\sigma^2 = \omega_p^2 \Big\{ \alpha_{1\sigma} + \beta_{1\sigma}\sqrt{n_{\bar{\sigma}}/n_\sigma} - \frac{\theta^2}{4}[(4\alpha_{1\sigma} - 3\beta_{1\sigma} - 3) + \sqrt{n_{\bar{\sigma}}/n_\sigma}(3\alpha_{1\sigma} - 4\beta_{1\sigma} - 3)] \Big\}. \quad (4)$$

The longitudinal collective modes have a quadratic ζ dependence. This is not surprising since the longitudinal fluctuation involve only electrons whose spin state does not change under the perturbation. As a result, the physical properties of the system should remain invariant under the change $\vec{B} \to (-\vec{B})$.

In addition to the modes described above, for $|l| > 2$, the system supports coupled cyclotron harmonics associated with the electron motion in the static magnetic field, $B_y = B\theta$. These excitations begin at

$$\omega_\pm^* = \frac{l}{2}\left[\omega_\sigma^*\alpha_{l\sigma} + \omega_{\bar{\sigma}}^*\alpha_{l\bar{\sigma}} \pm \sqrt{4\omega_\sigma^*\omega_{\bar{\sigma}}^*\beta_{l\sigma}\beta_{l\bar{\sigma}} + (\omega_\sigma^*\alpha_{l\sigma} - \omega_{\bar{\sigma}}^*\alpha_{l\bar{\sigma}})^2}\right]. \quad (5)$$

They are a first order response in θ, since $\omega_\sigma = eB\theta/m_\sigma^* c$. The coupling between the two waves is measured by $\omega_\sigma^*\omega_{\bar{\sigma}}^*\beta_{l\sigma}\beta_{l\bar{\sigma}}$, which reflects the interaction between the opposite spin electrons. At low polarization values, $(\omega_\sigma^*\alpha_\sigma - \omega_{\bar{\sigma}}^*\alpha_{\bar{\sigma}})^2 \ll 4\omega_\sigma^*\omega_{\bar{\sigma}}^*\beta_\sigma\beta_{\bar{\sigma}}$. In this approximation, the two cyclotron harmonics are

$$\omega_\pm^0 = \frac{l}{2}\left[\omega_\sigma^*(\alpha_{l\sigma} \pm \beta_{l\sigma}) + \omega_{\bar{\sigma}}^*(\alpha_{l\bar{\sigma}} \pm \beta_{l\bar{\sigma}}) \pm \frac{(\omega_\sigma^*\alpha_{l\sigma} - \omega_{\bar{\sigma}}^*\alpha_{l\bar{\sigma}})^2}{\omega_\sigma\omega_{\bar{\sigma}}\beta_{l\sigma}\beta_{l\bar{\sigma}}}\right]. \quad (6)$$

The two solutions correspond to a charge mode (+), determined by $(\alpha_l + \beta_l)$, the Fourier coefficient of the spin independent part of the interaction, and a spin mode (−), driven by $(\alpha_l - \beta_l)$. To the lowest order in ζ the spin symmetric oscillation is a linear superposition of cyclotron harmonics of each spin, $\omega_+^* = l\left[\omega_\sigma^*(\alpha_{l\sigma} \pm \beta_{l\sigma}) + \omega_{\bar{\sigma}}^*(\alpha_{l\bar{\sigma}} \pm \beta_{l\bar{\sigma}})\right]/2$. Therefore, the fundamental absorption ($l=1$)occurs at the bare cyclotron frequency, $\omega_{c\sigma}$, as required by Kohn theorem. This is possible because of the renormalization of the effective mass [5].

If the opposite-spin interaction is neglected and $\beta_{l\sigma}$ is set equal to zero in Eq. (5), the spin and charge excitations become indistinguishable.

At large polarizations, $1 - |\zeta| \ll 1$, and the opposite spin interaction, described by β_l, becomes very small. Then, $(l\omega_\sigma^*\alpha_{l\sigma} - l\omega_{\bar{\sigma}}^*\alpha_{l\bar{\sigma}})^2 \gg 4l^2\omega_\sigma^*\omega_{\bar{\sigma}}^*\beta_{l\sigma}\beta_{l\bar{\sigma}}$. The solution of Eq. (5), is $\omega_\sigma = l\omega_\sigma^*\left[\alpha_{l\sigma} + \beta_{l\sigma}/(\omega_\sigma^*\alpha_{l\sigma} - \omega_{\bar{\sigma}}^*\alpha_{l\bar{\sigma}})\right]$. The wave vector dependence of these excitations is quadratic.

Under the effect of the external perturbation, some electrons change their spin state, their fluctuations about the x and y axis being responsible for the induced transverse magnetization. The dynamics of the spin flip processes is driven by the off-diagonal components of Eq. (2). These spin waves are excited at

$$\omega_{\downarrow\to\uparrow} = -2\gamma^* B(1 + \alpha_{1\downarrow} - \mu_\downarrow) + \frac{\theta^2}{4}\omega_{p\downarrow}^2[(\alpha_\downarrow - \mu_\downarrow) - \sqrt{n_\downarrow/n_\uparrow}(\lambda_\downarrow - \beta_\downarrow)]/\omega_+,$$

$$\omega_{\uparrow\to\downarrow} = 2\gamma^* B(1 + \alpha_{1\uparrow} - \mu_\uparrow) + \frac{\theta^2}{4}\omega_{p\uparrow}^2[(\alpha_\uparrow - \mu_\uparrow) - \sqrt{n_\uparrow/n_\downarrow}(\lambda_\downarrow - \beta_\downarrow)]/\omega_+. \quad (7)$$

These SDW are spin-antisymmetric properties of the system and they depend linearly on ζ.

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